B2.6.5.2—Step E2: Isolator Sizing, Example 1.5

B2.6.5.2.1—Step E2.1: Lead Core Diameter, Example 1.5

Determine the required diameter of the lead plug, d_L, using:

$$d_L = \sqrt{\frac{Q_d}{0.9}} \qquad \text{(B2.6.5.2.1-1)}$$

See Step E2.5 in Article B2.6.5.2.5 for limitations on d_L.

B2.6.5.2.2—Step E2.2: Plan Area and Isolator Diameter, Example 1.5

Although no limits are placed on compressive stress in this Guide (maximum strain criteria are used instead, see Step E3 in Article B2.6.5.3), it is useful to begin the sizing process by assuming an allowable stress of 1.0 ksi.

Then the bonded area of the isolator is given by:

$$A_b = \frac{P_{DL} + P_{LL}}{1.0}\,\text{in.}^2 \qquad \text{(B2.6.5.2.2-1)}$$

and the corresponding bonded diameter (taking into account the hole required to accommodate the lead core) is given by:

$$B = \sqrt{\frac{4\,A_b}{\pi} + d_L^2} \qquad \text{(B2.6.5.2.2-2)}$$

Round the bonded diameter, B, to nearest quarter inch, and recalculate actual bonded area using

$$A_b = \frac{\pi}{4}(B^2 - d_L^2) \qquad \text{(B2.6.5.2.2-3)}$$

Note that the overall diameter is equal to the bonded diameter plus the thickness of the side cover layers (usually $^1/_2$ in. each side). In this case the overall diameter, B_o, is given by:

$$B_o = B + 1.0 \qquad \text{(B2.6.5.2.2-4)}$$

B2.6.5.2.3—Step E2.3: Elastomer Thickness and Number of Layers

Since the shear stiffness of the elastomeric bearing is given by:

B2.6.5.2—Step E2: Isolator Sizing, Example 1.5

Step E2.1: Lead Core Diameter, Example 1.5

$$d_L = \sqrt{\frac{Q_d}{0.9}} = \sqrt{\frac{2.34}{0.9}} = 1.61 \text{ in.}$$

Step E2.2: Plan Area and Isolator Diameter, Example 1.5

$$A_b = \frac{P_{DL} + P_{LL}}{1.0}\,\text{in.}^2 = \frac{45.52 + 15.50}{1.0} = 61.02 \text{ in.}^2$$

$$B = \sqrt{\frac{4\,A_b}{\pi} + d_L^2} = \sqrt{\frac{4\,(61.02)}{\pi} + 1.61^2} = 8.96 \text{ in.}$$

Round B up to 9.0 in. and the actual bonded area is:

$$A_b = \frac{\pi}{4}\left(9.0^2 - 1.61^2\right) = 61.57 \text{ in.}^2$$

$B_o = 9.0 + 2(0.5) = 10.0$ in.

Step E2.3: Elastomer Thickness and Number of Layers, Example 1.5

Select G, shear modulus of rubber, = 100 psi (0.1 ksi)

$$K_d = \frac{GA_b}{T_r} \qquad \text{(B2.6.5.2.3-1)}$$

where
G = shear modulus of the rubber, and
T_r = the total thickness of elastomer, it follows Eq. B2.6.5.2.2-4 may be used to obtain T_r given a required value for K_d

$$T_r = \frac{GA_b}{K_d} \qquad \text{(B2.6.5.2.3-2)}$$

A typical range for shear modulus, G, is 60–120 psi. Higher and lower values are available and are used in special applications.

If the layer thickness is t_r, the number of layers, n, is given by:

$$n = \frac{T_r}{t_r} \qquad \text{(B2.6.5.2.3-3)}$$

rounded up to the nearest integer.

Note that because of rounding the plan dimensions and the number of layers, the actual stiffness, K_d, will not be exactly as required. Reanalysis may be necessary if the differences are large.

Then

$$T_r = \frac{GA_b}{K_d} = \frac{0.1(61.57)}{1.17} = 5.27 \text{ in.}$$

$$n = \frac{5.27}{0.25} = 21.09$$

Round to nearest integer, i.e., $n = 22$

B2.6.5.2.4—Step E2.4: Overall Height

The overall height of the isolator, H, is given by:

$$H = nt_r + (n-1)t_s + 2t_c \qquad \text{(B2.6.5.2.4-1)}$$

where
t_s = thickness of an internal shim (usually about $^1/_8$ in.), and
t_c = combined thickness of end cover plate (0.5 in.) and outer plate (1.0 in.)

Step E2.4: Overall Height, Example 1.5

$$H = 22(0.25) + 21(0.125) + 2(1.5) = 11.125 \text{ in.}$$

B2.6.5.2.5—Step E2.5: Size Checks

Experience has shown that for optimum performance of the lead core, it must not be too small or too large. The recommended range for the diameter is as follows:

$$\frac{B}{3} \geq d_L \geq \frac{B}{6} \qquad \text{(B2.6.5.2.5-1)}$$

Article 12.2 requires that the isolation system provides a lateral restoring force at d_t greater than the restoring force at $0.5d_t$ by not less than $W/80$. This equates to a minimum K_d of $0.025W/d$.

Step E2.5: Size Checks, Example 1.5

Since $B = 9.0$, check

$$\frac{9.0}{3} \geq d_L \geq \frac{9.0}{6}$$

i.e., $3.0 \geq d_L \geq 1.5$

Since $d_L = 1.61$, lead core size is acceptable.

$$K_{d,min} = \frac{0.025W}{d} \quad \text{(B2.6.5.2.5-2)}$$

$$K_{d,min} = \frac{0.025W}{d} = \frac{0.025(45.52)}{2.27} = 0.40 \text{ k/in.}$$

As

$$K_d = \frac{GA_b}{T_r} = \frac{0.1(61.56)}{5.5} = 1.12 \text{ k/in.} > K_{d,min}$$

B2.6.5.3—Step E3: Strain Limit Check, Example 1.5

Article 14.2 and 14.3 requires that the total applied shear strain from all sources in a single layer of elastomer should not exceed 5.5, i.e.,

$$\gamma_c + \gamma_{s,eq} + 0.5\gamma_r \leq 5.5 \quad \text{(B2.6.5.3-1)}$$

where γ_c, $\gamma_{s,eq}$, *and* γ_r are defined below.

(a) γ_c is the maximum shear strain in the layer due to compression and is given by:

$$\gamma_c = \frac{D_c \sigma_S}{GS} \quad \text{(B2.6.5.3-2)}$$

where D_c is shape coefficient for compression in circular bearings = 1.0, $\sigma_S = P_{DL}/A_b$, G is shear modulus, and S is the layer shape factor given by:

$$S = \frac{A_b}{\pi B t_r} \quad \text{(B2.6.5.3-3)}$$

(b) $\gamma_{s,eq}$ is the shear strain due to earthquake loads and is given by:

$$\gamma_{s,eq} = \frac{d_t}{T_r} \quad \text{(B2.6.5.3-4)}$$

(c) γ_r is the shear strain due to rotation and is given by:

$$\gamma_r = \frac{D_r B^2 \theta}{t_r T_r} \quad \text{(B2.6.5.3-3)}$$

where D_r is shape coefficient for rotation in circular bearings = 0.375, and θ is design rotation due to *DL*, *LL*, and construction effects. Actual value for θ may not be known at this time and a value of 0.01 is suggested as an interim measure, including uncertainties (see LRFD Article 14.4.2.1).

Step E3: Strain Limit Check, Example 1.5

Since

$$\sigma_S = \frac{45.52}{61.57} = 0.739 \text{ ksi}$$

$G = 0.1$ ksi

and

$$S = \frac{61.57}{\pi 9.0(0.25)} = 8.71$$

then

$$\gamma_c = \frac{1.0(0.739)}{0.1(8.71)} = 0.849$$

$$\gamma_{s,eq} = \frac{1.89}{5.5} = 0.344$$

$$\gamma_r = \frac{0.375(9.0^2)(0.01)}{0.25(5.5)} = 0.221$$

Substitution in Eq. B2.6.5.3-1 gives

$$\gamma_c + \gamma_{s,eq} + 0.5\gamma_r = 0.849 + 0.344 + 0.5(0.221) = 1.30 \leq 5.5 \quad OK$$

B2.6.5.4—Step E4: Vertical Load Stability Check, Example 1.5

Article 12.3 requires the vertical load capacity of all isolators be at least three times the applied vertical loads (*DL* and *LL*) in the laterally undeformed state.

Further, the isolation system shall be stable under 1.2(*DL* + *SL*) at a horizontal displacement equal to either

- 2 × total design displacement, d_t, if in Seismic Zone 1 or 2, or
- 1.5 × total design displacement, d_t, if in Seismic Zone 3 or 4.

Step E4: Vertical Load Stability Check, Example 1.5

B2.6.5.4.1—Step E4.1: Vertical Load Stability in Undeformed State, Example 1.5

The critical load capacity of an elastomeric isolator at zero shear displacement is given by

$$P_{cr(\Delta=0)} = \frac{K_d H_{eff}}{2}\left[\sqrt{(1+\frac{4\pi^2 K_\theta}{K_d {H_{eff}}^2})} - 1\right] \quad \text{(B2.6.5.4.1-1)}$$

where

$H_{eff} = T_r + T_s$

T_s = total shim thickness

$K_\theta = E_b I / T_r$

$E_b = E(1+0.67S^2)$

E = elastic modulus of elastomer = $3G$

$I = \pi B^4 / 64$

It is noted that typical elastomeric isolators have high shape factors, *S*, in which case:

$$\frac{4\pi^2 K_\theta}{K_d {H_{eff}}^2} \gg 1 \quad \text{(B2.6.5.4.1-2)}$$

and Eq. B2.6.5.4.1-1 reduces to:

$$P_{cr(\Delta=0)} = \pi\sqrt{K_d K_\theta} \quad \text{(B2.6.5.4.1-3)}$$

Check that:

$$\frac{P_{cr(\Delta=0)}}{P_{DL}+P_{LL}} \ge 3 \quad \text{(B2.6.5.4.1-4)}$$

Step E4.1: Vertical Load Stability in Undeformed State, Example 1.5

$$E = 3G = 3(0.1) = 0.3 \text{ ksi}$$

$$E_b = 0.3(1+0.67(8.71^2)) = 15.55 \text{ ksi}$$

$$I = \pi\frac{9.0^4}{64} = 322.1 \text{ in.}^4$$

$$K_\theta = \frac{15.55(322.1)}{5.5} = 910.15 \text{ kin./rad}$$

$$K_d = \frac{GA_b}{T_r} = \frac{0.1(61.57)}{5.5} = 1.12 \text{ k/in.}$$

$$P_{cr(\Delta=0)} = \pi\sqrt{1.12(910.15)} = 100.27 \text{ k}$$

$$\frac{P_{cr(\Delta=0)}}{P_{DL}+P_{LL}} = \frac{100.27}{(45.52+15.5)} = 1.64 \not\ge 3 \quad \textit{NOK}$$

B2.6.5.4.2—Step E4.2: Vertical Load Stability in Deformed State, Example 1.5

The critical load capacity of an elastomeric isolator at shear displacement Δ may be approximated by:

$$P_{cr(\Delta)} = \frac{A_r}{A_{gross}} P_{cr(\Delta=0)} \quad \text{(B2.6.5.4.2-1)}$$

where

A_r = overlap area between top and bottom plates of isolator at displacement Δ (Figure 2.2-1)

$$= B^2(\delta - \sin\delta)/4$$

$$\delta = 2\cos^{-1}(\Delta/B)$$

$$A_{gross} = \pi B^2/4$$

it follows that:

$$\frac{A_r}{A_{gross}} = \frac{(\delta - \sin\delta)}{\pi} \quad \text{(B2.6.5.4.2-2)}$$

Check that:

$$\frac{P_{cr(\Delta)}}{1.2P_{DL} + P_{SL}} \geq 1 \quad \text{(B2.6.5.4.2-3)}$$

Step E4.2: Vertical Load Stability in Deformed State, Example 1.5

Since bridge is in Zone 2,

$$\Delta = 2d_t = 2(1.89) = 3.79 \text{ in.}$$

$$\delta = 2\cos^{-1}\left(\frac{3.79}{9.0}\right) = 2.27$$

$$\frac{A_r}{A_{gross}} = \frac{(2.27 - \sin 2.27)}{\pi} = 0.480$$

$$P_{cr(\Delta)} = 0.480(100.27) = 48.15 \text{ k}$$

$$\frac{P_{cr(\Delta)}}{1.2P_{DL} + P_{SL}} = \frac{48.15}{1.2(45.52) + 6.35} = 0.79 \ngeq 1 \quad \textit{NOK}$$

B2.6.5.5—Step E5: Design Review, Example 1.5

Step E5: Design Review, Example 1.5

The basic dimensions of the isolator designed above are as follows:

- 10.00 in. (OD) × 11.125 in. (high) × 1.61 in. dia. lead core, and
- the volume, excluding steel end and cover plates, is 638 in.^{3}

This design satisfies the shear strain limit criteria, but not the vertical load stability ratio in the undeformed and deformed states.

A redesign is therefore required and the easiest way to increase the P_{cr} is to increase the shape factor, S, since the bending stiffness of an isolator is a function of the shape factor squared. See equations in Step E4.1 in Article B2.6.5.4.1. To increase S, increase the bonded area, A_b, while keeping t_r constant (Eq. B2.6.5.2.3-2). To keep K_d constant, while increasing A_b and keeping T_r constant, decrease the shear modulus, G (Eq. B2.6.5.2.3-1).

This redesign is outlined below. After repeating the calculation for the diameter of the lead core, the process begins by reducing the shear modulus to 60 psi (0.06 ksi) and increasing the bonded diameter to 12 in.

E2.1

$$d_L = \sqrt{\frac{Q_d}{0.9}} = \sqrt{\frac{2.34}{0.9}} = 1.61 \text{ in.}$$

E2.2

$$A_b = \frac{T_r K_d}{G} \text{in.}^2 = \frac{5.5(1.17)}{0.06} = 107.25 \text{ in.}^2$$

$$B = \sqrt{\frac{4\,A_b}{\pi} + d_L^2} = \sqrt{\frac{4\,(107.25)}{\pi} + 1.61^2} = 11.80 \text{ in.}$$

Round B to 12 in. and the actual bonded area becomes:

$$A_b = \frac{\pi}{4}\left(12^2 - 1.61^2\right) = 111.06 \text{ in.}^2$$

$B_o = 12 + 2(0.5) = 13$ in.

E2.3

$$T_r = \frac{GA_b}{K_d} = \frac{0.06(111.06)}{1.17} = 5.71 \text{ in.}$$

$$n = \frac{5.71}{0.25} = 22.82$$

Round to nearest integer, i.e., $n = 23$.

E2.4

$$H = 23(0.25) + 22(0.125) + 2(1.5) = 11.5 \text{ in.}$$

E2.5

Since B=12, check

$$\frac{12}{3} \geq d_L \geq \frac{12}{6}$$

i.e., $4 \geq d_L \geq 2$

Since d_L = 1.61, the size of the lead core is too small, and there are two options: (1) Accept the undersize and check for adequate performance during the Quality Control Tests required by Article 15.2.2; or (2) Only have lead cores in every second isolator, in which case the core diameter, in those isolators with cores, will be √2 × 1.61 = 2.27 in. (which satisfies above criterion).

$$K_d = \frac{GA_b}{T_r} = \frac{0.06(111.06)}{5.75} = 1.16 \text{ k/in.} > K_{d,min}$$

E3

$$\sigma_S = \frac{45.52}{111.06} = 0.41 \text{ ksi}$$

$$S = \frac{111.06}{\pi 12(0.25)} = 11.78$$

$$\gamma_c = \frac{1.0(0.41)}{0.06(11.78)} = 0.580$$

$$\gamma_{s,eq} = \frac{1.89}{5.75} = 0.329$$

$$\gamma_r = \frac{0.375(12^2)(0.01)}{0.25(5.75)} = 0.376$$

$$\gamma_c + \gamma_{s,eq} + 0.5\gamma_r = 0.580 + 0.329 + 0.5(0.376) = 1.10 \le 5.5 \quad OK$$

E4.1

$$E = 3G = 3(0.06) = 0.18 \text{ ksi}$$

$$E_b = 0.18(1 + 0.67(11.78^2)) = 16.93 \text{ ksi}$$

$$I = \pi \frac{12^4}{64} = 1017.88 \text{ in.}^4$$

$$K_\theta = \frac{16.93(1017.88)}{5.75} = 2996.42 \text{ kin./rad}$$

$$K_d = \frac{GA_b}{T_r} = \frac{0.06(111.06)}{5.75} = 1.159 \text{ k/in.}$$

$$P_{cr(\Delta=0)} = \pi\sqrt{1.159(2996.42)} = 185.13 \text{ k}$$

$$\frac{P_{cr(\Delta=0)}}{P_{DL} + P_{LL}} = \frac{185.13}{(45.52 + 15.50)} = 3.03 \ge 3 \quad OK$$

E4.2

$$\delta = 2\cos^{-1}\left(\frac{3.79}{12}\right) = 2.50$$

$$\frac{A_r}{A_{gross}} = \frac{(2.50 - \sin 2.50)}{\pi} = 0.605$$

$$P_{cr(\Delta)} = 0.605(185.13) = 111.96 \text{ k}$$

$$\frac{P_{cr(\Delta)}}{1.2P_{DL}+P_{SL}}=\frac{111.96}{1.2(45.52)+6.35}=1.84\geq 1 \quad OK$$

E5

The basic dimensions of the redesigned isolator are as follows:

- 13.0 in. (od) × 11.5 in. (high) × 1.61 in. dia. lead core, and
- its volume (excluding steel end and cover plates) is 1128 in.^{3}

This design meets all the design criteria but is about 75 percent larger by volume than the previous design. This increase in size is driven by the need to satisfy the vertical load stability ratio of 3.0 in the undeformed state.

B2.6.5.6—Step E6: Minimum and Maximum Performance Check, Example 1.5

Article 8 requires the performance of any isolation system be checked using minimum and maximum values for the effective stiffness of the system. These values are calculated from minimum and maximum values of K_d and Q_d, which are found using system property modification factors, λ, as indicated in Table B2.6.5.6-1.

Table B2.6.5.6-1—Minimum and Maximum Values for K_d and Q_d

Eq. 8.1.2-1	$K_{d,max} = K_d \lambda_{max,Kd}$	(B2.6.5.6-1)
Eq. 8.1.2-2	$K_{d,min} = K_d \lambda_{min,Kd}$	(B2.6.5.6-2)
Eq. 8.1.2-3	$Q_{d,max} = Q_d \lambda_{max,Qd}$	(B2.6.5.6-3)
Eq. 8.1.2-4	$Q_{d,min} = Q_d \lambda_{min,Qd}$	(B2.6.5.6-4)

Determination of the system property modification factors should include consideration of the effects of temperature, aging, scragging, velocity, travel (wear), and contamination as shown in Table B2.6.5.6-2. In lieu of tests, numerical values for these factors can be obtained from Appendix A.

Step E6: Minimum and Maximum Performance Check, Example 1.5

Minimum Property Modification factors are:
$\lambda_{min,Kd} = 1.0$
$\lambda_{min,Qd} = 1.0$

which means there is no need to reanalyze the bridge with a set of minimum values.

Maximum Property Modification factors are:
$\lambda_{max,a,Kd} = 1.1$
$\lambda_{max,a,Qd} = 1.1$

$\lambda_{max,t,Kd} = 1.1$
$\lambda_{max,t,Qd} = 1.4$

$\lambda_{max,scrag,Kd} = 1.0$
$\lambda_{max,scrag,Qd} = 1.0$

Table B2.6.5.6-2—Minimum and Maximum Values for System Property Modification Factors

Eq. 8.2.1-1	$\lambda_{min,Kd} = (\lambda_{min,t,Kd})(\lambda_{min,a,Kd})(\lambda_{min,v,Kd})(\lambda_{min,tr,Kd})(\lambda_{min,c,Kd})(\lambda_{min,scrag,Kd})$	(B2.6.5.6-5)
Eq. 8.2.1-2	$\lambda_{max,Kd} = (\lambda_{max,t,Kd})(\lambda_{max,a,Kd})(\lambda_{max,v,Kd})(\lambda_{max,tr,Kd})(\lambda_{max,c,Kd})(\lambda_{max,scrag,Kd})$	(B2.6.5.6-6)
Eq. 8.2.1-3	$\lambda_{min,Qd} = (\lambda_{min,t,Qd})(\lambda_{min,a,Qd})(\lambda_{min,v,Qd})(\lambda_{min,tr,Qd})(\lambda_{min,c,Qd})(\lambda_{min,scrag,Qd})$	(B2.6.5.6-7)
Eq. 8.2.1-4	$\lambda_{max,Qd} = (\lambda_{max,t,Qd})(\lambda_{max,a,Qd})(\lambda_{max,v,Qd})(\lambda_{max,tr,Qd})(\lambda_{max,c,Qd})(\lambda_{max,scrag,Qd})$	(B2.6.5.6-8)

Adjustment factors are applied to individual λ-factors (except λ_v) to account for the likelihood of occurrence of all of the maxima (or all of the minima) at the same time. These factors are applied to all λ-factors that deviate from unity, but only to the portion of the λ-factor that is greater than, or less than, unity. Article 8.2.2 gives these factors as follows:

1.00 for critical bridges,
0.75 for essential bridges, and
0.66 for all other bridges.

As required in Section 7, and shown in Figure C7-1 , the bridge should be reanalyzed for two cases: once with $K_{d,min}$ and $Q_{d,min}$, and again with $K_{d,max}$ and $Q_{d,max}$. As indicated in Figure C7-1, maximum displacements will probably be given by the first case ($K_{d,min}$ and $Q_{d,min}$) and maximum forces by the second case ($K_{d,max}$ and $Q_{d,max}$).

Applying a system adjustment factor of 0.66 for an "other" bridge, the maximum property modification factors become:

$\lambda_{max,a,Kd} = 1.0 + 0.1(0.66) = 1.066$
$\lambda_{max,a,Qd} = 1.0 + 0.1(0.66) = 1.066$

$\lambda_{max,t,Kd} = 1.0 + 0.1(0.66) = 1.066$
$\lambda_{max,t,Qd} = 1.0 + 0.4(0.66) = 1.264$

$\lambda_{max,scrag,Kd} = 1.0$
$\lambda_{max,scrag,Qd} = 1.0$

Therefore, the maximum overall modification factors

$\lambda_{max,Kd} = 1.066(1.066)1.0 = 1.14$
$\lambda_{max,Qd} = 1.066(1.264)1.0 = 1.35$

Since the possible variation in upper bound properties exceeds 15 percent, a reanalysis of the bridge is required to determine performance with these properties.

The upper-bound properties are:

$Q_{d,max} = 1.35\ (2.34) = 3.16$ k

and

$K_{d,max} = 1.14(1.16) = 1.32$ kips/in.

B2.6.5.7—Step E7: Design and Performance Summary, Example 1.5

B2.6.5.7.1—Step E7.1: Isolator Dimensions, Example 1.5

Summarize final dimensions of isolators:

- Overall diameter (includes cover layer)
- Overall height
- Diameter lead core
- Bonded diameter
- Number of rubber layers
- Thickness of rubber layers
- Total rubber thickness
- Thickness of steel shims
- Shear modulus of elastomer

Check all dimensions with manufacturer.

Step E7: Design and Performance Summary, Example 1.5

Step E7.1: Isolator dimensions, Example 1.5

Isolator dimensions are summarized in Table B2.6.5.7.1-1.

Table B2.6.5.7.1-1—Isolator Dimensions

Isolator Location	Overall size including mounting plates (in.)	Overall size without mounting plates (in.)	Diam. lead core (in.)
Under edge girder on Pier 1	17.0 × 17.0 × 11.50(H)	13.0 dia. × 10.0(H)	1.61

Isolator Location	No. of rubber layers	Rubber layers thick-ness (in.)	Total rubber thick-ness (in.)	Steel shim thick-ness (in.)
Under edge girder on Pier 1	23	0.25	5.75	0.125

Shear modulus of elastomer = 60 psi.

B2.6.5.7.2—Step E7.2: Bridge Performance, Example 1.5

Summarize bridge performance:

- Maximum superstructure displacement (longitudinal)
- Maximum superstructure displacement (transverse)
- Maximum superstructure displacement (resultant)
- Maximum column shear (resultant)
- Maximum column moment (about transverse axis)

Step E7.2: Bridge Performance, Example 1.5

Bridge performance is summarized in Table B2.6.5.7.2-1, where it is seen that the maximum column shear is 19.56 k. This is less than the column plastic shear strength (25 k) and therefore the required performance criterion is satisfied (fully elastic behavior). The maximum longitudinal displacement is 2.07 in., which is slightly more than the 2.0 in. available at the abutment expansion joints, and is barely acceptable (light pounding may occur but not likely to cause damage to the back wall).