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On the Verification of a Finite Element Analysis for Elastomeric Bridge Bearings

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Synopsis: A brief description is given of a recently developed nonlinear analytical model and finite element analysis for elastomeric bridge bearings. The discussion addresses the important problem of determining the stress-strain behavior of the elastomer and adopts a relatively comprehensive nonlinear elastic characterization. The analysis is applicable to arbitrarily loaded bearings of various geometries and accounts for both geometric and material nonlinearities.

The paper discusses the preliminary verification of the analysis as a reliable engineering tool. Such a study involves comparisons of analysis predictions to available experimental results. The comparisons given in the paper are for bearing compression. Such comparisons bring to light problems involving the interpretation of the experimental results due to apparent contradictions among different sources of experimental data. A study is made of possible causes of these differences and how they can be reconciled.

Keywords: bridge bearings; finite element method; plastic, polymers, and resins; structural analysis

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INTRODUCTION

A comprehensive analysis procedure for elastomeric bridge bearings is needed so that engineers can quickly and inexpensively evaluate proposed new designs and novel applications. The alternative of conducting an experimental study usually requires considerable lead time and is relatively expensive. The analysis also can be used to supplement and extend experimental results for the development of design criteria and specifications. Finally, it has proven to be extremely difficult to experimentally measure the internal stress distributions in the individual elastomeric and reinforcement layers. Such information is needed if the results of simple failure studies are to be extrapolated to complex loading states and histories for actual bearings in the field.

The extrapolation of experimental results for elastomeric bearings is made particularly difficult by their inherent nonlinear behavior, which precludes the superposition of the results of simple tests to predict behavior for complex loading histories. The development of realistic design criteria and specifications for bridge bearings requires an understanding and evaluation of this nonlinear behavior.

An analysis for bridge bearings must be capable of predicting, for arbitrary loading histories, the overall response (including possible instability phenomena) of bearings of arbitrary shapes and predict details concerning the local three-dimensional stress and strain states within the rubber and reinforcement layers. Such a broad requirement would suggest the use of a finite element analysis. The analysis must account for the nonlinear effects due to large deformations in the rubber, large displacements of the reinforcement and the nonlinear material behavior of the rubber.

In addition, the time dependent and inelastic behavior of the rubber need to be accounted for. This paper will not address the inelasticity question, instead the elastomeric material will, for the present, be modeled as a nonlinear elastic solid.

There has been a considerable amount of work done on the development of analysis procedures for elastomeric bridge bearings. Reference (1) gives a most complete summary and bibliography of this work. Elastomeric bearings find use in many other situations and have accordingly received considerable additional attention. The reader is referred to reference (2) for a comprehensive summary of these studies for elastomeric bearings for helicopters. References to other works on bearings and related problems can be found in (3-7); the last two papers also reference the previous works of the first author on this and related subjects. For the most part, past investigations have only considered linear behavior and thus are of limited value for most actual bearing problems. Some exceptions are to be found in references (2,8,9).

There are two classical approaches to the finite element analysis of composite systems (10,11). They differ in their underlying descriptions of the layered system, i.e., discrete or composite. In a discrete analysis the detailed inhomogeneous nature of the bearing is modeled so that all material interfaces fall along element boundaries (thus each finite element contains only one type of material). Because of the complicated nature of the layer interaction, in general, very fine grids are required (2,11-13). An alternative to a discrete representation is to model the layered bearing system as an equivalent homogeneous, orthotropic continuum material (3,6,7,10,13,14). An analysis using such a representation will, in this paper, be called a "composite analysis." Because the detailed geometry of the layering is not represented, rather coarse grids may be used (3,6,13,14).

The advantage of a discrete analysis is that the local stress and strain states at and near the layer interfaces, and at the edges of the bearing are directly calculated. The chief disadvantage (often rendering the method entirely impractical) is the excessive computational cost required for the analysis of two- and three-dimensional systems with large numbers of layers, and when nonlinear or inelastic effects, or both, must be included. The advantage of a composite analysis is its low computational cost. The composite approach is used in this investigation; it is a continuation of the work reported in (6,7,13).

In general, the development of an analysis procedure for a structure or a structural component involves four steps. The first is the formulation of a mathematical theory (or specialization or adaptation of an existing theory) capable of modeling the system in question. The second step is the application of an analytical or a numerical analysis procedure for the solution of the theory for specific problems. Thirdly, the predictions of the analysis are compared to available analytical solutions (when available for simple idealized examples) and experimental results for the purposes of calibrating and verifying the model. Lastly, applications of the analysis are made to relevant engineering problems.

The remainder of this paper contains brief discussions of the developed nonlinear composite theory and accompanying finite element analysis for elastomeric bridge bearings (space limitations will preclude their detailed development), and a somewhat more detailed section concerning some aspects of the verification and calibration of the analysis.

THEORY

There are two fundamental steps in developing the desired composite model for elastomeric bridge bearings. The first is the statement of conventional continuum theory in a form capable of representing all the nonlinear aspects of bearing behavior. The second step involves the homogenization of these equations to yield a composite model for the layered bearing system.

Nonlinear Continuum Theory

Of the three basic sets of equations of mechanics, equilibrium, compatibility and constitutive laws, the first two are well understood and can be written without approximation (assuming that micropolar effects (15) are negligible). The only real choice involves whether one wishes to use the deformed or undeformed body as the reference state.

Because the expressions of equilibrium and compatibility are well established even in the presence of large deformations and deflections, they are only briefly discussed here. An updated Lagrangian coordinate system (16) is used for each element and is defined by the average rotation of the reinforcement layers contained in the element. Thus, the assumption of "moderate rotations" is adequate for modeling the large deflections of the reinforcement layers. However, because the elastomeric layers may experience large deflections relative to the reinforcement layers no approximations are possible in the description of their deformations. The Lagrangian description used for the elastomeric component is expressed in terms of Green strains and Kirchoff stresses. Before stresses are printed they are converted to the Cauchy (true) stress components. These quantities are all well documented in reference (17) and thus are not discussed further here. It is emphasized that these concepts can be considered to be exact. Unfortunately, the same is not true for the third basic element of solid mechanics, i.e., constitutive equations.

As is the case in many engineering projects, theoretical work on elastomeric bridge bearings involves mathematical manipulations which require material properties. Thus, the successful verification and ultimate use of the proposed analysis depends on the ability to determine the constitutive (stress-strain) properties of the constituent materials (temperature effects are not considered in the present paper).

Much of the current industrial emphasis would appear to be on the use of steel reinforcement whose engineering properties are relatively well known. There is also interest in using materials such as fiberglass and fabrics (18) as possible reinforcement; the relatively difficulty question of determining their constitutive properties will not be addressed in this paper (there is, however, a vast literature on the subject, e.g., see (11)).

Unlike steel, with its longer history of engineering use and therefore a better understanding of its material properties, research on rubber is in somewhat of an early stage and often suffers from a lack of common understanding and communication between the producers and engineers. However, for the successful analysis of bridge bearings it is essential that a comprehensive and usable constitutive model be available for the elastomer.

For this study the elastomer will be modeled as a nonlinear elastic material. The neglection of the inelastic effects is usually acceptable, except in cyclic loading cases where material damping is important (19,20). Future work will model the elastomer as a nonlinear viscoelastic solid.

In the following discussion use will be made of the concept of strain invariants. The symbols I_1, I_2, and I_3 will be used for these quantities; their precise definitions may be found in most texts on continuum mechanics, e.g., see (17). At this point it is only necessary to note that they are sufficient (in the absence of temperature effects) for expressing the free energy function of a nonlinear elastic body that is isotropic in its initial (undeformed) configuration. The derivatives of the free energy function with respect to the components of strain yield the stress-strain law.

The constitutive model for rubber must be valid for three-dimensional conditions and very large deformations (even though a bearing may be only compressed to a nominal strain of 10 to 12 percent, the local strains in the rubber may exceed 100 percent (9)). The earliest successful three-dimensional characterization for rubber, the "statistical theory of rubber", was developed from a theoretical consideration of the molecular structure of an elastomeric solid. (For the derivation of this theory, and a history of its development, the reader is referred to the classical book by Treloar (19).) The resulting free energy expression is:

$$U = \frac{1}{2} G (I_1 - 3) \qquad (1)$$

The material coefficient G is the shear modulus of classical linear elasticity. This expression has two shortcomings. The first is that the predicted nonlinearities are less than measured in actual tests. This deficiency is illustrated, for example, in Figures 5.4 and 5.6 of (19). In viewing these figures it is to be remembered that local strains of magnitudes an order larger than the average bearing strain can occur. The second shortcoming results from the assumption of incompressibility, which was used in the development of the statistical theory for rubber.

In an attempt to improve predictions for large deformations an empirical addition was made to the above expression by Mooney (19), (the results are often referred to as the Mooney-Rivlin model), i.e.,

$$U = C_1 (I_1 - 3) + C_2 (I_2 - 3) \qquad (2)$$

where C_1 and C_2 are two material parameters that must be selected, for the particular rubber in question, from a consideration of experimental results. The stress-strain law obtained from this expression is still somewhat deficient in its prediction of material nonlinearity (19) and does not include material compressibility. It is the latter problem that is particularly significant for elastomeric bearings (1,6). Because of the confinement provided by the reinforcement and the large compressive loads applied to bridge bearings, very substantial mean-pressure are developed within the rubber. Thus, even though the bulk modulus of rubber is relatively high, significant volume changes can occur.

In recognition of the need, in certain circumstances, to account for the slight compressibility of rubber and also to improve the nonlinear aspects of the resulting stress-strain law, a more comprehensive expression for the free energy expression has been developed and used in Europe (21):

$$U = \frac{1}{2} B (I_3^{1/2} - 1)^2 + \frac{1}{2} G \left[\left(\frac{I_2}{I_3^{2/3}} - 3\right) + \mu \left(\frac{I_1}{I_3^{1/3}} - 3\right)^m\right] \qquad (3)$$

The quantities B and G are the classical bulk and shear moduli of linear elasticity, and μ and m are two additional material parameters whose presence strengthens the nonlinearity of the model. It is to be noted that this function includes the previous expressions as special cases ($I_3 = 1$ for assumed incompressible behavior) and yields linear elastic behavior for infinitesimal strains. The Mooney-Rivlin expression is obtained when $m = 1$, $G\mu/2 = C_1$ $G/2 = C_2$ and $I_3 = 1$; the typical values suggested in (21) of $m = 2 \rightarrow 3$ and $\mu \approx .01 \rightarrow .05$, however, are quite different than suggested in (19) where values of $C_2 \approx .05$ and $C_1 + C_2 = G$ are reported.

The importance of accounting for the compressibility of the rubber when analyzing elastomeric bearings has also been recognized by Simo and Taylor (22), who have developed an alternative expression for the free energy of rubber.

In this work an approximation of Equation 3 is adopted. In the formulation of the composite theory the fact that the power "m", of the last term, can take on non-integer values causes some difficulties. To avoid this problem the last term is approximated by the first two terms of a Taylor series expansion about a reference value F_o of the bracketed quantity in question.

$$U = \frac{1}{2} B(I_3^{1/2} - 1)^2 + \frac{G}{2} \{(\frac{I_1}{I_3^{2/3}} - 3) + \mu F_o^m [(\frac{2-m}{F_o})(\frac{I_1}{I_3^{1/3}} - 3) + (\frac{m-1}{F_o^2})(\frac{I_1}{I_3^{1/3}} - 3)^2] \tag{4}$$

This expression can be simplified and its relationship to Equation (2) clarified by renaming the material parameters:

$$U = \frac{1}{2} B(I_3^{1/2} - 1)^2 + \frac{G}{2}[(1 - \mu_1)(\frac{I_2}{I_3^{1/3}} - 3) + \mu_1(\frac{I_1}{I_3^{1/3}} - 1) + \mu_2(\frac{I_1}{I_3^{1/3}} - 1)^2] \tag{5}$$

The use, for either predictive or verification purposes, of the resulting rubber stress-strain law in the analysis of a given bearing will require a knowledge of the four material parameters, B, G, μ_1 and μ_2 that describe the free energy function. It would be ideal if their values could be determined independently from simple homogeneous tests on samples of the elastomer in question (such a process is not at all well defined (19)). Unfortunately, this is not possible for bearings that have been tested in the past and is probably out of the question for future commercial bearings; however, it should be a required part of any future research program. Some information concerning rubber properties; however, must be available (or inferred) before an analysis can be performed.

From a survey of the literature of analytical and experimental work on elastomeric bridge bearings it was found that the kind of material properties necessary for the engineering design of bearings are not commonly considered by the rubber industry when developing and evaluating a new compound (23). The shear modulus, bulk modulus, and the effects of temperature and creep are usually essential for the designer's work, whereas most producers describe their products in terms of chemical compounds, hardness, tensile stress at some elongation, compressive set, etc., and state that their products meet AASHTO material specifications. However, it is generally agreed that AASHTO specifications are only established to control quality and do not lead to information that can be used by either the producer to develop a new material, or the designer for an analytical study (23). In order to achieve a better understanding of rubber behavior, more standard test procedures with meaningful and informative stress-strain results would seem essential.

Attempts have been made to establish relationships between the reported hardness of rubber samples and their engineering properties (i.e., stiffness, or modulus, in compression or shear). If such a correlation were possible it would be extremely useful. Unfortunately, there is no agreement as to its feasibility and/or the number of properties that can be so treated. It is stated in ASTM designations (24) that there is no simple relationship between durometer hardness values and any fundamental property of natural rubber or neoprene. Nevertheless, a number of empirical relationships are to be found in the literature.

For example, Figure 11 of (25) reports experimentally observed effects of temperature on the shear modulus of neoprene for different values of hardness. In Figure 4.8 of (20) the tangent shear modulus is related (experimentally) to durometer hardness values for natural rubber. Figure 1 of (23) compares the experimental curve for shear modulus used by researchers in NCHRP and other curves which were found in the course of their literature survey; substantial differences in the reported values of shear modulus for a given hardness are apparent.

Despite the uncertainties involved in relating hardness to engineering properties, Figure 1 of (26) gives a set of curves which reportedly can be used to determine a number of elastic constants (for natural rubber) from a knowledge of the hardness value. The information of interest for this study is reproduced here in Figure 1. The hardness is described in terms of ^{o}BS, rather than standard shore A durometer readings. A comparison of Figure 4.8 of (20) and Figure 1 of (26) indicates the possibility of a linear dependence between the two hardness measures; however, it appears that no such relationship has received universal acceptance. For example, it is stated in ASTM (24) that no simple relationship exists between hardness measurements with different instruments. For lack of better evidence it is assumed that they are interchangeable.

An additional difficulty arises when attempting to correlate data from several researchers. That is, natural rubber or neoprene samples with similar hardness values exhibit different behaviors for similar tests. An example of this difficulty was noted by CALTRANS (27) where similar samples were made from 53 hardness neoprene, supplied by different producers. Compressive load-deflection curves of bearing with a shape factor of 3 show difference of up to 4 percent strain for the same stress level (3000 psi). Inasmuch as present AASHTO design specifications recommend limiting the compressive strain to 7 percent, such differences cannot be ignored.

Another important question that enters this discussion is whether or not the bulk (B) and Shear (G) moduli can vary independently for the classes of elastomeric materials used in bridge bearings. If they are in fact independent, then it would most certainly be impossible to correlate both their values to a single measured quantity such as hardness. That is, various combinations of B and G would correspond to the same hardness value and thus could not be uniquely determined by it.

There are two scenarios in which the hardness correlations have value. One possibility is that hardness is at most only a weak function of B and thus could be used to determine G; this possibility is suggested by the approximate theoretical relationship expressing hardness as a function of Young's modulus given in (28); for a nearly-incompressible material Young's modulus is approximately 3G. The other is, for a given type of rubber, that B bears a fixed functional relationship to G (i.e., B = B(G)) that is independent of hardness. If this relationship can be established by some independent means, then hardness would be sufficient to determine both G and B. Although the literature review has produced no evidence to suggest the latter condition, for the present it will be assumed that such a relationship is embedded in the curves for B and G given in Figure 1 (it is further assumed that the relationship is the same for both natural rubber and neoprene). Based on this assumption both B and G can be found from Figure 1 once the hardness is known. If G is known (for example measured in a shear test), it can be entered into Figure 1 to yield a hardness, which in turn can be used to find B. The selection of the parameters μ_1 and μ_2 of Equation 5 will be discussed in a later section.

In addition to the above difficulties, it is generally accepted that hardness measurements are not highly accurate. For example, a material of stated 60 hardness should be considered to be 60 $\pm$ 5 in hardness (18), etc. This, as can be seen from Figure 1, results in a significant uncertainty in the shear modulus value. Further, it should be remembered that a hardness measurement, which is a surface interaction (24), may not necessarily be indicative of averaged properties of the material. For example, the shear modulus of a rubber sample could be significantly different at or near the surface than in the interior. This variation may occur because it is not always possible to obtain the same degree of cure throughout the body during the vulcanization process (23).

In general, the uncertainties in rubber properties made it difficult to quantitatively interpret much of the experimental work on bearings available in the literature. In the section describing model verification the method used in inferring rubber properties will be described.

Composite Theory

The first step in the development of a "composite analysis" is the modeling of the elastomeric bearing as an "equivalent, homogeneous, orthotropic solid". The basis for such a representation involves extending the continuum concept to a level of observation that does not distinguish individual reinforcing layers or individual rubber pads. Classically, the continuum concept has been applied to materials which are inhomogeneous and discontinuous at the microstructure and/or macrostructure levels (e.g., the molecular level for amorphous materials such as glass, crystalline level for metals, aggregate level for concrete, etc.). While the consideration of an elastomeric bearing as homogeneous, as compared to a similar treatment of steel, requires several orders of shift in our scale of observation and thinking, it requires little or no shift for concrete (where the maximum aggregate size is usually in the same range as the thickness of the rubber pads for bridge bearings).

One of the fundamental differences between the modeling of a material such as concrete and an elastomeric bearing as homogeneous is the importance of edge effects for the latter. Edge effects result from the interaction of the structure of the composite with the boundaries of the system. For many composite systems, such as concrete, edge effects are of little consequence; however for others, including elastomeric bearings, they are of major importance (5,6,10,13,14). Thus, in the development of a composite theory for elastomeric bearings special care must be taken to include edge effects; the nature of these effects is described in (3,6,7,13,14).

The development of a nonlinear composite theory for elastomeric bridge bearings parallels the development of the linear theory in (7). Because of space limitations the details of the derivation cannot be presented here, instead only a brief outline