TABLE 7—LIGHT STRUCTURE, HEAVY LIVE LOAD—COMPARISON OF STRESSES AND MATERIAL QUANTITIES

Designed for *DL* & *LL*

Point	Design basis f_c = 1200, f_s = 20000				Design basis f_c = 1200, f_s = 22000			Design basis f_c = 1200, f_s = 24000		
	DLM	*LLM**	$\frac{DLM}{LLM}$	$DL+2LL=f_s$	*DLM*	$\frac{DLM}{LLM}$	$DL+2LL=f_s$	*DLM*	$\frac{DLM}{LLM}$	$DL+2LL=f_s$
$.20L_1$	80.0	1328.1	.060	38900	85.2	.064	42700	87.7	.066	46500
$.40L_1$	79.5	1864.5	.043	39200	84.4	.045	43000	87.2	.047	46900
$.50L_1$	50.7	1792.8	.028	39400	53.8	.030	43400	55.6	.031	47300
$.55L_1$	29.0	1762.8	.016	39700	30.7	.017	43600	31.7	.018	47600
$.60L_1$	3.1	1667.4	.002	40000	3.3	.002	43900	3.4	.002	47900
$.65L_1$	−28.1	1508.0	−.019	40400	−29.9	−.020	44400	−30.8	−.020	48500
$.70L_1$	−62.4	1297.8	−.048	41000	−66.2	−.051	45200	−68.4	−.053	49300
$.80L_1$	−148.0	564.7	−.262	47100	−157.2	−.278	52500	−162.3	−.288	57700
$.80L_1$	−148.0	−2111.2	.070	38700	−157.2	.074	42500	−162.3	.077	46300
$.90L_1$	−260.4	−2819.0	.092	38300	−277.4	.098	42000	−285.5	.101	45800
$1.00L_1$	−392.9	−3950.6	.099	38200	−417.1	.105	41900	−430.7	.109	45600
$.20L_2$	−73.9	−1565.1	.047	39100	−78.5	.050	42900	−81.0	.052	45400
$.20L_2$	−73.9	254.5	−.290	48100	−78.5	−.308	54000	−81.0	−.318	59200
$.30L_2$	16.0	1144.2	.014	39700	16.9	.015	43700	17.5	.015	47600
$.50L_2$	84.4	1674.6	.050	39000	89.6	.053	42800	92.5	.055	42800
Relative Quantities	Concrete 100%, Steel 100%				Concrete 106.5%, Steel 90.5%			Concrete 109.6%, Steel 80.5%		

*Live load same for all designs.

TABLE 8—LIGHT STRUCTURE, HEAVY LIVE LOAD—COMPARISON OF STRESSES AND MATERIAL QUANTITIES

Designed for *DL* & *LL*

Point	Design basis $f_c = 1200$, $f_s = 20000$				Design basis $f_c = 1050$, $f_s = 22000$			Design basis $f_c = 1050$, $f_s = 24000$		
	DLM	*LLM*†	$\frac{DLM}{LLM}$	$DL+2LL=f_s$	*DLM*	$\frac{DLM}{LLM}$	$DL+2LL=f_s$	*DLM*	$\frac{DLM}{LLM}$	$DL+2LL=f_s$
$.20L_1$	80.0	1328.1 −497.1	.060	**38900 *43800	93.1	.070	42100 49100	95.0	.072	46400 53700
$.40L_1$	79.5	1864.5 −994.2	.043	39200 41700	92.6	.050	42900 46200	94.4	.051	46800 50500
$.50L_1$	50.7	1792.8 −1242.8	.028	39400 40800	59.0	.033	43300 45300	60.2	.034	47200 49200
$.55L_1$	29.0	1762.8 −1367.1	.016	39700 40400	33.7	.019	43500 44600	34.3	.020	47500 48600
$.60L_1$	3.1	1667.4 −1491.4	.002	40000 40100	3.6	.002	44000 44000	3.6	.002	47900 48000
$.65L_1$	−28.1	1508.0 −1615.7	−.019	40400 39700	−32.7	−.022	44500 43600	−33.4	−.022	48500 47500
$.70L_1$	−62.4	1297.8 −1740.0	−.048	41000 39300	−72.6	−.056	45300 43100	−74.0	−.057	49500 47000
$.80L_1$	−148.0	564.7 −2111.2	−.262	47000 38700	−173.4	−.306	44000 42300	−176.8	−.313	58900 46200
$.90L_1$	−260.4	−2819.0 635.0	.092	38300 53900	−303.2	.107	41900 64100	−309.2	.110	47100 70800
$1.00L_1$	−392.9	−3950.6 705.6	.099	38200 65300	−457.5	.116	41700 84600	−466.4	.118	45500 94500
$.20L_2$	−73.9	564.5 −1565.1	−.131	43000 39100	−86.1	−.153	48000 42800	−87.8	−.155	52400 46700
$.30L_2$	16.0	1144.2 −991.0	.014	39700 40300	18.6	.016	43600 44400	18.9	.017	48000 48500
$.50L_2$	84.4	1674.6 −670.0	.050	39000 42900	98.3	.058	42800 47700	100.2	.060	46600 52200
Relative Quantities	Concrete 100% Steel 100%				Concrete 116.4% Steel 79.3%			Concrete 118.7% Steel 70.4%		

*Stresses in top steel. **Stresses in bottom steel. †Live load same for all designs.

TABLE 9—LIGHT STRUCTURE, HEAVY LIVE LOAD—COMPARISON OF STRESSES AND MATERIAL QUANTITIES

Designed for *DL* & *LL*

Point	Design basis f_c = 1200, f_s = 20000				Design basis f_c = 975, f_s = 22000			Design basis f_c = 975, f_s = 24000		
	DLM	*LLM**	$\frac{DLM}{LLM}$	$DL+2LL=f_s$	*DLM*	$\frac{DLM}{LLM}$	$DL+2LL=f_s$	*DLM*	$\frac{DLM}{LLM}$	$DL+2LL=f_s$
$.20L_1$	80.0	1328.1	.060	38900	103.6	.078	42400	107.5	.081	46200
$.40L_1$	79.5	1864.5	.043	39200	103.0	.055	42800	106.9	.057	46700
$.50L_1$	50.7	1792.8	.028	39400	65.7	.037	43200	68.1	.038	47100
$.55L_1$	29.0	1762.8	.016	39700	37.5	.021	43500	38.9	.022	47500
$.60L_1$	3.1	1667.4	.002	40000	4.0	.002	44000	4.1	.002	47900
$.65L_1$	−28.1	1508.0	−.019	40400	−36.4	−.024	44500	−37.8	−.025	48600
$.70L_1$	−62.4	1297.8	−.048	41000	−80.8	−.062	45500	−83.9	−.064	46500
$.80L_1$	−148.0	564.7	−.262	47000	−191.7	−.339	55300	−198.9	−.352	61000
$.80L_1$	−148.0	−2111.2	.070	38700	−191.7	.091	42200	−198.9	−.092	45900
$.90L_1$	−260.4	−2819.0	.092	38300	−337.2	.119	41600	−350.0	−.124	45300
$1.00L_1$	−392.9	−3950.6	.099	38200	−508.7	.113	41500	−528.0	−.133	45200
$.20L_2$	−73.9	−1565.1	.047	39100	−95.7	.061	42700	−99.3	−.064	46600
$.20L_2$	−73.9	254.5	−.290	48100	−95.7	−.376	42200	−99.3	−.390	46100
$.30L_2$	16.0	1144.2	.014	39700	20.7	.018	43600	21.5	.019	47600
$.50L_2$	84.4	1674.6	.050	39000	109.3	.065	42800	113.4	.068	46400
Relative Quantities	Concrete 100% Steel 100%				Concrete 129.5% Steel 77.9%			Concrete 134.4% Steel 77.0%		

*Live load same for all designs.

TABLE 10—LIGHT STRUCTURE HEAVY LIVE LOAD

Comparison of Concrete and Steel Quantities

	Conventional Design						Balanced Design	
Designed for $f_c = 1{,}200$ $f_s = 20{,}000$	$f_c = 1{,}200$ $f_s = 22{,}000$	$f_c = 1{,}200$ $f_s = 24{,}000$	$f_c = 1{,}050$ $f_s = 22{,}000$	$f_c = 1{,}050$ $f_s = 24{,}000$	$f_c = 975$ $f_s = 22{,}000$	$f_c = 975$ $f_s = 24{,}000$	$.5DL + LL$ $f_c = 1{,}200$ $f_s = 20{,}000$	$.5DL + LL$ $f_c = 1{,}050$ $f_s = 20{,}000$
Concrete = 100%	106.5	109.6	116.4	118.7	129.5	134.4	99	113
Steel = 100%	90.5	80.5	79.3	70.4	77.9	70.0	99	90

all of the load is transferred to the steel. Therefore in this type of design it seems logical to assume that the concrete is not taking the diagonal tension and compute the stirrups on the basis of carrying all the shear at the same permissible stress as used in the main steel.

It will be seen from the shear diagram that the different shear curves intersect at point "A." This would be so no matter what value of k were used. Therefore the factor of safety for shear is the same at this point for all values of k.

The curve marked S-60bjd shows the shear for which steel would be provided by present methods of design assuming the concrete to carry 2 per cent of f'_c. It can be seen that an inadequate amount of steel has been provided for ultimate design from the left support to about $0.5L_1$ and at the point "A" none would be provided except for code provisions which require a minimum spacing based on depth.

From this diagram it can be seen that the total amount of steel required by either design is practically the same but that balanced design probably gives a better distribution.

No shear diagram for the light structure is shown but the heavy structure would present the more unfavorable case for balanced design.

The actual bond stresses can be computed from the equation

$$u' = u \frac{DL + nLL}{kDL + LL}$$

where u = allowable bond stress as specified in codes.

This is the same equation as No. (3) for moment stress in steel except that u has been substituted for f_s and DL and LL represent shears rather than moments.

LARGE $\frac{DL}{LL}$ RATIOS

In some types of structures the $\frac{DL^*}{LL}$ ratio may be great enough so that the stress due to DL alone exceeds that which would be deemed permissible. For instance, assume that a factor of safety F_D on the dead load is desired when dead load alone is acting. Then the ratio of $\frac{DL}{LL}$ beyond which it will be necessary to design for dead load alone may be found as follows:

$$DL + F_L LL = f_{yp}, \text{ and } F_D DL = f_{yp}$$

then

$$\frac{DL + F_L LL}{DL} = F_D$$

*This ratio, as stated in footnote of the introductory paragraphs, is the ratio of the stresses produced by dead load and live load respectively.

since $$k = \frac{1}{F_L}, \quad \frac{DL + \frac{LL}{k}}{DL} = F_D \quad \therefore \quad \frac{DL}{LL} = \frac{1}{k(F_D - 1)}$$

Assuming $k = 0.5$ and $F_D = 1.5$, then where $\frac{DL}{LL} > 4$ the structure should be examined for dead load only. In such a case the design would be based on a stress of

$$f_s = k\,f_{yp}$$

and the load used for obtaining moment would be $1.5DLk = .75DL$.

CONCLUSIONS

That failures of reinforced concrete designed by conventional methods are relatively few considering the great amount of such construction does not necessarily prove the adequacy of the method. It does prove that the carrying capacity of the structures so designed is great enough to carry the loads to which they have been subjected. The actual factor of safety may be 1.1 or 10 and may vary widely from point to point in any structure or in any member of that structure. It may be true that loading to cause excessive stresses near the point of counter-flexure is more improbable than at other points where the factor of safety is greater; nevertheless, such loading is possible, and if it did occur the safety of the structure would depend upon the excess strength known to exist in reinforced concrete when sections are chosen by the present theories of straight line stress distribution or upon a redistribution of stress.

It is felt that the first approach to a restudy of the problem of reinforced concrete design should begin by eliminating the most conspicuous deficiencies such as have been pointed out in this study. This discussion and study have been prepared in the hope that they would hasten the adoption of a more rational and consistent method of design than now in use by removing the anomoly of having a factor of safety of about unity at some points in a member and of two, three, four or five at other places. Such a method will give a better distribution of materials by thus balancing the design; at the same time the amount of work required of the designer is not increased nor is he forced to learn new methods or construct new design aids in the form of charts and tables.

Discussion of this paper should reach the ACI Secretary, in triplicate, by April 1, 1943, for publication in the JOURNAL for June, 1943

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A Semi-Circular Arched Conduit With Uniform Symmetrical Loading*

By STANLEY U. BENSCOTER†

SYNOPSIS

The conduit is first considered to be divided into two parts, the base slab and the arched frame. The fixed end moments, fixed-end shears and stiffness value for the arched frame are presented by formulas and graphs. From these values and similar well known values for the base slab we may determine the final moments in the conduit by a single distribution of moments at one corner by the usual method of Moment Distribution. A "shear correction factor" is given to change the fixed-end shear of the frame to the final shear. The formulas and graphs take exact account of the conduit wall thickness and special considerations are given to the indeterminate state of strain in the corner region.

INTRODUCTION

The shape of conduit herein considered is shown by the line diagram in Fig. 1(a). The arched roof and sidewall are considered to be of constant thickness. The loading on top and sides is considered to be uniform as indicated in Fig. 1. This loading condition should prove adequate for the design of conduits beneath high earth fills. The distribution of the base reaction will not be discussed but is merely assumed to be symmetrical. The effects of the shortening of the perimeter of the conduit due to thrust are neglected.

The above conditions permit direct and simple integration of the integral equations of continuity which govern the moments in the continuous frame in accord with elastic theory. Fixed-end moments and shears, and stiffness, can thus be determined for the frame shown in Fig. 1(b.

Conduits beneath high fills such as earth dams must have very thick walls. This thickness sometimes exceeds 25 per cent of the span of the

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conduit. A consideration of wall thickness in writing the expression for bending moment at any point in the frame leads to a second method of analysis. Considering the sidewall and base slab each to have an infinite I in the region of the corner leads to a third method. Method 1 should be satisfactory for a preliminary analysis. Methods 2 and 3 may be regarded as upper and lower bounds on the solution, the exact solution lying somewhere in between.

It becomes convenient in the analysis and design of single barrel conduits to use a "modified bending moment" sign convention which states that positive bending moment causes maximum compressive stress in the outer fiber.

The scope of this paper has been limited to accomplish three objects: (1) to illustrate the possibility of determining and using the stiffness and fixed-end moments for a frame, (2) to present the graphs required for actual design of this shape of conduit, (3) to show a reasonable method of taking account of the effect of wall thickess in large conduits.

NOTATION

C = reciprocal of end-rotation constant
C_H = shear correction coefficient
E = modulus of elasticity
h = thickness of base slab
H = shear at corner
I = moment of inertia
K_H = shear with unit rotation (See Fig. 4)
K_M = stiffness
m = bending moment
M = bending moment at corner
$q = 1 - \left(\frac{t}{2R}\right)^2$
r = carry-over factor
R = radius of roof to center line
S_H = a shear coefficient
S_M = stiffness coefficient
t = thickness of roof and side wall
T = thrust at corner
w = uniform load (See Fig. 1)
$\alpha = \frac{h}{2R}$
β = joint rotation angle
Φ = fixed-end shear coefficient
Ψ = fixed-end moment coefficient
ϵ = ratio of side wall height to R
Θ = angle of polar coordinates

METHOD 1

Let us consider the conduit shown in Fig. 1(a) to be divided into two parts, the frame and the base slab, as shown in (b) and (c). If we could find readily values for fixed-end moment and stiffness for the frame, and for the slab, we could find the final moment at point G by a single balancing of moments according to the method of Moment Distribution. Fixed-end moments and stiffness for the base slab may be determined from graphs or formulas which are available in many publications.

The determination of fixed-end moment and stiffness values for the frame is an interesting exercise in application of the elastic theory of continuous structures. The values thus determined for fixed-end thrust. shear and moment will be as follows:

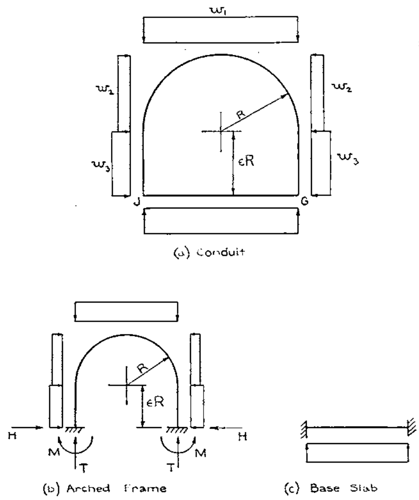


Fig. 1—Line diagram of conduit with loads

$$T = w_1 R \quad (1)$$

$$H = [\phi_1 (w_1 - w_2) - \phi_3 w_3] R \quad (2)$$

$$M = [\psi_1 (w_1 - w_2) - \psi_3 w_3] R^2 \quad (3)$$

where,

$$\phi_1 = \frac{1}{2}\left[\frac{3\pi\epsilon^2 + 16\epsilon + 2\pi}{2\epsilon^4 + 4\pi\epsilon^3 + 24\epsilon^2 + 6\pi\epsilon + 3(\pi^2 - 8)}\right] \quad (4)$$

$$\phi_3 = \frac{\epsilon}{2}\left[\frac{2\epsilon^4 + 5\pi\epsilon^3 + 40\epsilon^2 + 12\pi\epsilon + 6(\pi^2 - 8)}{2\epsilon^4 + 4\pi\epsilon^3 + 24\epsilon^2 + 6\pi\epsilon + 3(\pi^2 - 8)}\right] \quad (5)$$

$$\psi_1 = \frac{1}{4}\left[\frac{2\pi\epsilon^3 + 16\epsilon^2 + 4\pi\epsilon - 3\pi^2 + 32}{2\epsilon^4 + 4\pi\epsilon^3 + 24\epsilon^2 + 6\pi\epsilon + 3(\pi^2 - 8)}\right] \quad (6)$$

$$\psi_3 = \frac{\epsilon^2}{6}\left[\frac{\epsilon^4 + 3\pi\epsilon^3 + 30\epsilon^2 + 12\pi\epsilon + 9(\pi^2 - 8)}{2\epsilon^4 + 4\pi\epsilon^3 + 24\epsilon^2 + 6\pi\epsilon + 3(\pi^2 - 8)}\right] \quad (7)$$

A positive sign for H, T or M means that they have direction as indicated in Fig. 1(b). The fixed-end shear coefficients ϕ_1 and ϕ_3 are represented graphically in Fig. 2. The fixed-end moment coefficients ψ_1 and ψ_3 are represented graphically in Fig. 3.

The stiffness* of a beam has been defined as the moment required to produce a unit rotation at one end while the other end is held fixed. This implies that there shall be no relative translation of the ends of the beam. We may use the same definition of stiffness in the case of a frame. (See Fig. 4(a)). However, in dealing with symmetrical loads it is more convenient to evaluate a "modified stiffness" which may be defined as the

*"Continuous Frames of Reinforced Concrete," by H. Cross and N. D. Morgan, John Wiley and Sons, Inc., New York, N. Y., 1932, p. 83.

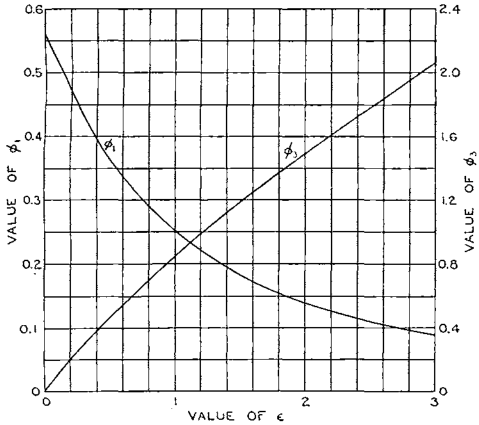
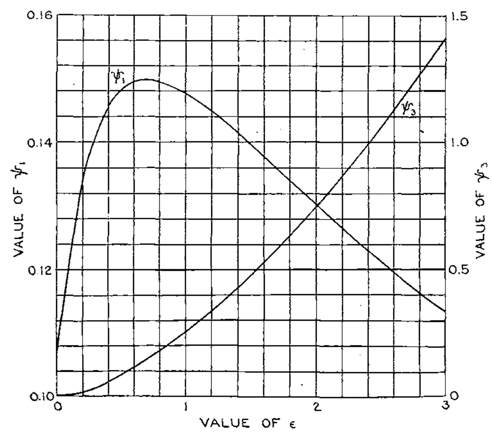


Fig. 2—Fixed-end shear coefficients

Fig. 3—Fixed-end moment coefficients



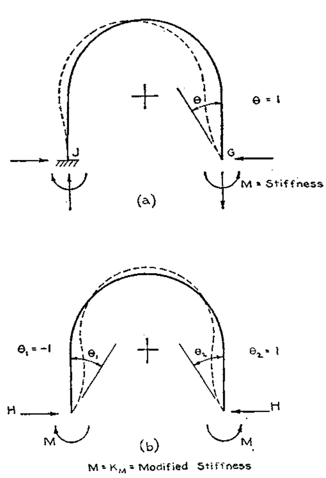


Fig. 4—Stiffness of arched frame