Creep and Shrinkage of Aging Concrete

By K.S. Pister, J.H. Argyris, and K.J. William

Synopsis: A thermodynamically-based creep theory is developed, in which the total incremental strain at any time is composed of mechanical, thermal, hygral and autogenous contributions. Each of these contributions (except the autogenous) consists of an instantaneous part which is reversible (in the working stress regime) and a time-dependent part which may in turn contain both reversible and irreversible components. The theory provides a unifying framework for examining existing methods of modeling creep phenomena and developing new models where experimental data reveal deficiencies in current models.

<u>Keywords: age; concretes; creep properties;</u> deformation; mass concrete; pressure vessels; <u>shrinkage</u>; strains; stresses; <u>thermodynamics</u>

Karl S. Pister is Professor of Engineering Science in the Department of Civil Engineering at the University of California, Berkeley, where during the past 25 years he has worked on problems in the mechanics of solids and structures. During 1973–74 he was a Fulbright Research Scholar at the ISD, University of Stuttgart.

John H. Argyris is Professor of Aeronautics at the University of Stuttgart, where during the past 18 years he has directed the Institut für Statik und Dynamik der Luft- und Raumfahrtkonstruktionen. He has contributed for the past 25 years to the large progress in matrix analysis of structures and in particular the development of the finite element method together with its implementation in production software packages such as ASKA and SMART.

Kaspar J. Willam is group-leader at the ISD of the University in Stuttgart, where during the past 6 years he has worked in the field of finite element analysis of concrete structures with special emphasis on the fields of creep and ultimate load behaviour under thermal environments.

INTRODUCTION

In spite of the extensive body of literature that has developed since the turn of the century, particularly during the last decade, ability to describe definitively the behaviour of concrete subjected to mechanical and environmental loading histories is at best limited. Complexity in the structure of concrete as an engineering material stands as the root cause of the often highly controversial nature of both theoretical and experimental contributions to the literature. The recent survey of Bazant [1] provides an excellent perspective in this regard. In addition it brings into sharp focus the extraordinary breadth required to deal properly with the subject-ranging from physical chemistry and continuum mechanics through numerical analysis and codified procedures for structural design.

The present paper provides a conceptual framework within which the requisite yet more or less disjoint viewpoints of physical chemistry, mechanics and structural analysis may be accommodated. Since analysis and design procedures will likely continue to be based on structural variables such as stress and strain, mechanics will play the dominant theoretical role. Nevertheless, the complex microstructure of concrete on the one hand, and the need for computationally feasible models for design purposes on the other will require that the disciplines of physical chemistry and numerical analysis play substantial roles in formulation of phenomenological models of behaviour. In the continuum theory of material behaviour a multiphase material such as concrete is properly studied under the rubrics of the theory of mixtures [2]. At present, however, such a treatment requires a degree of sophistication that seems unwarranted for engineering applications. Alternatively, we will adopt the viewpoint of constitutive theory of materials with internal variables. This will enable us to incorporate rationally

Aging Concrete

into a single-phase material model the evident multiphase microstructure of concrete. In particular the processes of heat and moisture diffusion as well as aging associated with hydration will be accounted for from the viewpoint of internal variables.

Throughout the paper our emphasis will be conceptual and qualitative so as to minimize detraction from the central ideas that otherwise are apt to become obscured by excessive symbolic and numerical detail. Applications may be found in references cited in the final section.

DEFORMATION OF CONCRETE - A THERMODYNAMIC BASIS

Roles of Theory and Experiments

Our choice of a thermodynamic theory for study of deformation of concrete is influenced by the observation that it:

- provides a mathematical structure for simultaneously treating microand macro-phenomena,
- (2) provides a proper basis for three-dimensional generalization,
- (3) incorporates irreversible phenomena,
- (4) is compatible with computer-oriented numerical analysis procedures.

A theory is needed both to organize rationally a body of observed phenomena (set of experiments) as well as to permit cautious prediction of what might be expected to occur if certain conditions in the experiments were changed. A theory defines the structure of a mathematical model and defines critical experiments by which the limits of its validity may be established. Properly used, it minimizes the misleading practice of "curve-fitting", in which fitting of data to an empirical model, no matter how carefully done, provides no predictive quality whatsoever with respect to a new set of conditions.

Experiments, on the other hand, constitute the <u>sine qua</u> <u>non</u> of a theory. A set of experiments, used in connection with a theory:

- (1) Determines parameters in the mathematical model.
- (2) Delineates the range of validity of the theory.
- (3) Demonstrates the need for revision of a theory.

Finally, experiments usually require a theory in order to be properly understood. This results from the fact that it is often impossible to conduct experiments in which the states of strain and stress within the test specimen are spatially uniform, i.e., the stress state is statically indeterminate internally. This constitutes a problem of analysis by itself (an initial-boundary value problem for time-dependent creep experiments) in which the as yet unknown material properties are needed as part of the solution algorithm. This leads in turn to an iterative process in seeking to understand the experiment, making apparent again the complementary roles of theory and experiment. Accordingly, it is necessary that at least conceptually a theory be constructed for states of stress and strain, as well as internal variables describing micro-processes, that are spatially uniform; no spatial gradients should appear in such a theory. The problem of treating structural elements (beams,

slabs, etc.) in which non-uniform states occur constitutes a separate problem of analysis that cannot be attempted until the former problem of modeling is completed. Failure to recognize this seemingly straightforward principle has been a source of much ambiguity in interpreting results of experiments. A typical example is that of creep tests on unsealed cylinders during which uncontrolled moisture diffusion occurs. From such experiments it is impossible to isolate the creep strain associated with mechanical stress from that associated with changes in moisture concentration in the test specimen.

Decomposition of Total Strain

For simplicity we will consider a volume element of concrete subjected to uniaxial stress. Our object is to develop a theory in which the corresponding strain can be expressed in terms of the stress and the temperature, moisture content and age of the material. Generalization to a three-dimensional format is formally straight-forward using the rules of tensor algebra. It is convenient to consider the axial strain of the material in the following ways:

- (1) strain is produced by mechanical as well as non-mechanical sources,
- (2) strain contains instantaneous as well as time-dependent components,
- (3) strain components may be reversible (recoverable) or irreversible (irrecoverable),
- (4) the micro-processes responsible for strain may be interdependent (coupled).

To illustrate how these intuitive ideas may be given quantitative expression by a thermodynamic theory, consider first the calculation of instantaneous strain produced by stress, all other variables held constant. Here and in the sequel we will assume that all instantaneous effects are reversible, although this is not a requisite of the theory, merely a tentative simplification for stress levels less than 0.3 to 0.4 f_c^i . Referring to Fig. 1a, an axial strain ε is produced by a corresponding stress σ as shown. The elastic complementary energy per unit volume is defined by

$$G_{e}(\sigma) = \int \varepsilon \, \mathrm{d}\sigma \tag{1}$$

If the material is linear, i.e. $\varepsilon = J\sigma$, where J is the compliance (constant), then it follows from Eq. (1) that

$$G_{e}(\sigma) = \frac{1}{2} J \sigma^{2}$$
⁽²⁾

We observe that the complementary energy is a potential function for the strain, i.e., from Eq. (2)

$$\frac{\partial G_e}{\partial \sigma} = J\sigma = \varepsilon \tag{3}$$

Differentiating Eq. (3) again,

$$\frac{\partial \varepsilon}{\partial \sigma} = J = \frac{\partial^2 G_e}{\partial \sigma^2} \tag{4}$$

If the material is elastic but nonlinear, G_e is no longer a quadratic function of σ . Accordingly, from Eq. (4) it follows that J is no longer a constant but a function of stress level. Formally, we can write Eq. (4) in the form

$$\frac{\partial \varepsilon}{\partial \sigma} = J(\sigma) = \frac{\partial^2 G_e}{\partial \sigma^2}$$
(5)

Figures 1 b and 1 c give a clear geometric interpretation of Eq. (4) and Eq. (5). For the linear material the slope of the strain-stress curve is a constant, J. For the nonlinear material, Eq. (5) shows that J(G) is an incremental (tangent) compliance. For our purposes, and particularly for computation, it is convenient to write Eq. (5) in the incremental form

$$\Delta \varepsilon = J(\sigma) \Delta \sigma \tag{6}$$

We turn now to incorporation of non-mechanical sources of instantaneous strain, which will be associated with changes in temperature I and evaporable water concentration w. From thermodynamic theory we replace the complementary energy by the complementary free energy (Gibbs energy) which has the formal expression

$$G_e = G_e(\sigma, T, w) \tag{7}$$

per unit volume of material, and note that

$$\varepsilon = \frac{\partial G_e}{\partial \sigma}$$
(8)

remains valid.[×] The incremental form of Eq. (8) corresponding to Eq. (6), taking into account incremental changes in all variables, follows by using the chain rule in Eq. (8):

$$\Delta \varepsilon = d\left(\frac{\partial G_{e}}{\partial \sigma}\right) = \frac{\partial^{2} G_{e}}{\partial \sigma^{2}} \Delta \sigma + \frac{\partial^{2} G_{e}}{\partial \sigma \partial T} \Delta T + \frac{\partial^{2} G_{e}}{\partial \sigma \partial w} \Delta w \qquad (9)$$

This equation states that the total instantaneous strain increment is the sum of increments produced by changes in stress, temperature and water concentration. It is useful to introduce the following notation arising from Eq. (9):

$$\frac{\partial^2 G_e}{\partial \sigma^2} \equiv J(\sigma, T, w) , \text{ mechanical compliance}$$
(10)

$$\frac{\partial^2 G_e}{\partial \sigma \, \partial I} \equiv \alpha(\sigma, I, w) \qquad , \text{ thermal compliance} \qquad (11)$$

$$\frac{\partial^2 G_e}{\partial \sigma \partial T} \equiv \beta(\sigma, T, w) , \text{ hygral compliance}$$
(12)

The incremental compliances J, α, β are intrinsic material properties which can only be determined by appropriate experiments in which two of the variables σ, T, w are fixed while changing the third. For example, while holding Tand w constant, the strain-stress curve can be obtained as in Fig. 1 c, from which J can be found. By repeating for different values of T, w the sensitivity of J to non-mechanical variables can be established. Similar experiments can be used to evaluate α, β . It may be expected that σ will not influence these material functions appreciably, particularly if the magnitude of σ is less than 0.3 to 0.4 f_c^{-1} [1].

Our treatment of thermodynamics of solids follows the excellent expository paper of Lubliner [3]. More details can be found there.

Effect of Maturity on Material Functions

In the previous discussion it was tacitly assumed that material functions were determined at a given age of the material. However, experience indicates that the values of these functions are dependent on the maturity of the concrete, which is defined here to be a measure of extent of development of the hydration process in the cement paste. It is apparent that maturity is not really a function of the chronological age of the material; rather, it reflects how the histories of temperature and water concentration may have accelerated or decelerated the age of the concrete by affecting the rate of hydration of cement paste. It is evident that maturity is a concept associated with micro-processes whose dynamics are best discussed within the discipline of physical chemistry; obviously, the variables of continuum mechanics are inappropriate here. A thermodynamic theory incorporating internal material variables provides the vehicle for treating such a problem [3]. We introduce a measure of maturity X_{1} along with the following postulate:

"the rate of change of maturity depends on the present values of temperature, evaporable water concentration and maturity."

Expressed symbolically,

$$\dot{q}_{1} = \frac{dq_{1}}{dt} = f_{1}[T(t), w(t), q_{1}(t)]$$
(13)

where $f_1(T, w, q_1)$ is an intrinsic material property function which must be determined experimentally. It may be noted that this idea is conceptually similar to the notion of "co-age" introduced previously by Mc Henry and others [4], although it is introduced here in an entirely different context. If either T or w is fixed in time, q_1 is a surface whose coordinates are $(q_1, w \text{ or } T, t)$. For a prescribed history T(t), w(t) one would expect the solution of Eq. (13) to have the form shown in Fig. 2. Each history pair T, w would produce a different curve, i.e. T, w are the parameters of a family of maturity curves.

To incorporate the effect of maturity in the thermodynamic theory, we must replace Eq. (7) by

$$G_e = G_e(\sigma, T, w, q_1) \tag{14}$$

in order to introduce the dependence of instantaneous response on age at time of testing. This produces two effects in Eq. (9) for the total incremental instantaneous strain; first an additional term appears:

$$\Delta \varepsilon_a = \frac{\partial^2 G_e}{\partial \sigma \partial q_1} \Delta q_1 = \lambda_1 \Delta q_1 \tag{15}$$

where

$$\lambda_1 \equiv \frac{\partial^2 G_e}{\partial \sigma \, \partial q_1} \tag{16}$$

defines the autogenous incremental compliance, representing a strain increment produced solely by the phenomenon of maturation. Secondly, the material functions

This viewpoint was suggested by Professor J. Lubliner in a personal communication.

Aging Concrete

 $J, \alpha, \beta, \lambda$ now must be regarded as dependent on the present values of the macroscopic (observable) variables σ , T, w as well as the present value of the (internal) micro-variable q_1 which reflects maturity. Since q_1 is computed from a differential equation, Eq. (13), whose solution reflects how the past values of temperature and water concentration affect maturity, it is clear that the instantaneous response function $J, \alpha, \beta, \lambda$ are thereby dependent on the past history of T, w as well through their dependence on q_1 . Observe that this is an additional functional dependence beyond that already explicit in their dependence on present values T(t), w(t) as seen in Eqs. (10), (11), (12). Observe, however, that $J, \alpha, \beta, \lambda$ do not depend on the past history of stress in the theory considered here.^X Using Eqs. (9) - (12) along with (15) and (16) the total instantaneous incremental strain can be written.

$$\Delta \varepsilon = J \Delta \sigma + \alpha \Delta T + \beta \Delta w + \lambda_1 \Delta q_1 \tag{17}$$

In summary we have illustrated qualitatively how an internal variable measuring micro-processes associated with hydration can be incorporated in a continuum theory of behaviour to account for the effect of maturity on mechanical, thermal and hygral instantaneous response function. We turn now to a similar exposition to account for time-dependent strain phenomena.

TIME-DEPENDENT STRAIN - INELASTIC BEHAVIOUR AND

Complementary Free Energy

Up to this point we have accounted for instantaneous strain resulting from stress, temperature and water concentration changes, including the effect of maturity. This has been accomplished by utilizing a complementary free energy function G_e depending on the present values of σ , T, W, q_1 as shown in Eq. (14), resulting in the total instantaneous incremental strain expression given by Eq. (17). We must now incorporate into the complementary free energy necessary structure to account for time-dependent contributions to the total strain. Such contributions may be of mechanical origin, e.g., viscoelastic or viscoplastic, or they may be associated with thermal or hygral phenomena. To accomplish this we introduce a complementary free energy function [3]

$$G = G_e(\sigma, T, w, q_1) + \sigma \varepsilon_i(q_w) \tag{18}$$

where G_e has been previously defined and ε_i is the total time-dependent, inelastic strain. The inelastic strain ε_i depends on a set of internal variables q_{α} ($\alpha = 2, 3, ..., n$) which is used to describe the dissipative, time-dependent microprocesses responsible for viscoelastic, delayed thermal and other related types of inelastic response. Then, using Eq. (18) it follows that

$$\boldsymbol{\varepsilon} = \frac{\partial G}{\partial \sigma} = \frac{\partial G_e}{\partial \sigma} + \boldsymbol{\varepsilon}_i(\boldsymbol{q}_{\alpha}) \tag{19}$$

^x This is only an assumption which can be removed if necessary. However, it seems plausible for stress levels less than 0.3 to 0.4 f_c^{1} .

Denoting the instantaneous response by

$$\varepsilon_{e} = \frac{\partial G_{e}}{\partial \sigma}$$
(20)

we can rewrite Eq. (19) in the form

$$\varepsilon = \varepsilon_e + \varepsilon_i$$
 (21)

In incremental form Eq. (21) is

$$\Delta \varepsilon = \Delta \varepsilon_e + \Delta \varepsilon_i \tag{22}$$

We have already expressed $\Delta \varepsilon_e$ in Eq. (17). Noting that ε_i depends only on q_{α} , we can write the inelastic strain increment

$$\Delta \epsilon_{i} = \sum_{\alpha=2}^{n} \frac{\partial \epsilon_{i}}{\partial q_{\alpha}} \Delta q_{\alpha} = \sum_{\alpha=2}^{n} \lambda_{\alpha} \Delta q_{\alpha}$$
(23)

. where

$$\lambda_{\alpha} = \frac{\partial \varepsilon_{i}}{\partial q_{\alpha}} = \frac{\partial^{2} G}{\partial \sigma \partial q_{\alpha}} , \qquad \alpha = 2, 3, \dots n$$
(24)

denote incremental inelastic compliance functions associated with each internal variable q_{α} . As noted, the set of internal variables has the task of representing at the macroscopic, continuum level the effects of inelastic micro-processes. We may therefore expect to require mechanical internal variables q_{α}^{m} , thermal internal variables q_{α}^{m} and hygral internal variables q_{α}^{w} .

As in the case of the internal variables q_1 , used to account for maturity, it is necessary to postulate rate equations from which the internal variables can be calculated when the macroscopic state of the material (expressed in terms of σ , τ , w) is prescribed.

Rate Equations for Internal Variables

The following postulate is introduced:

"the rate of change of internal variables accounting for inelastic phenomena depends on the present values of stress, temperature, evaporable water concentration, maturity and the internal variables themselves." Expressed symbolically,

$$\dot{q}_{\alpha} = f_{\alpha} \left[\sigma(t), T(t), w(t), q_{1}(t), q_{\alpha}(t) \right], \alpha = 2, 3, \dots n \quad (25)$$

where the functions f_{α} reflect intrinsic material properties. A special, though very general form of Eq. (25) results if it is rewritten

$$\dot{q}_{\alpha} = Q_{\alpha}(q_1) f_{\alpha}[\sigma, T, w, q_{\alpha}] , \qquad \alpha = 2, 3, \ldots n \quad (26)$$

This permits us to write Eq. (26) as a set of rate equations in terms of equivalent, temperature- and water concentration – compensated time measures $\zeta_{\alpha}(t)$. To see this let

$$\zeta_{\alpha}(t) \equiv \int_{0}^{t} Q_{\alpha}[q_{1}(s)] ds , \qquad \alpha = 2, 3, \dots n$$
(27)

Using this result and changing variables in Eq. (27) yields

$$\frac{\mathrm{d}q_{\alpha}}{\mathrm{d}\zeta_{\alpha}} = f_{\alpha}\left[\sigma, T, w, q_{\alpha}\right] , \qquad \alpha = 2, 3, \dots, n \qquad (28)$$

where it is understood that σ , Γ , w, q_{α} are now expressed as functions of the time variable ζ_{α} . Observe that the evolution of inelastic behaviour (as measured by q_{α}) is dependent on the history of σ , Γ , w through the solution of Eq.(28). Several important points should be noted:

- (1) each internal variable (or each set q_{α}^{m} , q_{α}^{T} , q_{α}^{w}) may have its own time scale defined by Eq. (27).
- (2) rate equations (28) for the internal variables may be coupled or uncoupled, i.e., the equations for mechanical internal variables q_{α}^{m} may depend on non-mechanical internal variables q_{α}^{I} , q_{α}^{w} and vice versa.
- (3) the number of internal variables required to describe a particular inelastic phenomenon (permanent flow, delayed elastic recovery, delayed dilatation, etc.) will depend on the intrinsic complexity of the phenomenon and the degree of approximation desired. At least one variable of each type is the obvious minimum.

We can now summarize the contributions of mechanical and non-mechanical effects to the total incremental strain. Using Eqs. (17), (22), (23) we can write

$$\Delta \varepsilon = J \Delta \sigma + \sum_{\alpha=2}^{k} \lambda_{\alpha} \Delta q_{\alpha}^{m} + \alpha \Delta I + \sum_{\alpha=2}^{l} \lambda_{\alpha} \Delta q_{\alpha}^{T}$$

mechanical thermal
+ $\beta \Delta w + \sum_{\alpha=2}^{r} \lambda_{\alpha} \Delta q_{\alpha}^{w} + \lambda_{1} \Delta q_{1}$
hygral autogenous (29)

Eq. (29) shows that the total strain increment at any time is composed of mechanical, thermal, hygral and autogenous contributions. Each of these contributions (except the autogenous) consists of an instantaneous part which is reversible (for working stress regimes) and a time-dependent part which may in turn contain both reversible and irreversible (permanent) components. The time-dependent contributions are determined by internal variables whose values are the solutions of rate equations (28), which may or may not couple together the mechanical, thermal and hygral internal variables depending on the type of model desired.

It may be noted that discontinuous phenomena such as plastic flow, transitional thermal or hygral creep, etc. may be accounted for by requiring that the rate equations (28) display a discontinuity in the function f_{α} , where the discontinuity would depend on an appropriate threshold ("yield") condition.

Finally, in connection with the thermodynamic theory, it should be noted that the second law of thermodynamics (in the form of the local Clausius-Duhem inequality) must be satisfied for the complementary free energy in every deformation process. In incremental form this leads to [3]:

$$\frac{\partial G}{\partial q_1} \varDelta q_1 + \sum_{\alpha=2}^n \frac{\partial G}{\partial q_\alpha} \varDelta q_\alpha \ge 0$$
(30)

From Eq. (18) this can be written

$$\frac{\partial G_{\mathbf{e}}}{\partial q_{1}} \varDelta q_{1} + \sum_{\alpha=2}^{n} \sigma \; \frac{\partial \varepsilon_{i}}{\partial q_{\alpha}} \varDelta q_{\alpha} \ge 0 \tag{31}$$

Eq. (31) expresses the requirement that micro-processes characterized by internal variables remove free energy from the material. The consequences in the present context is to place restrictions on the manner in which the function G may depend on q_1 and q_n .

This completes our formal treatment of a theory for creep and shrinkage of concrete. To proceed further one must make explicit assumptions, based on intuition or experiments, about the structure of the complementary free energy function G and the rate equations (28). We observe that all of the well-known linear creep models appear as special cases if the function G is taken to be a quadratic polynomial function of variables σ , T, W, q_{α} and the rate equations are linear with age-dependent coefficients. A summary of these results may be found in [5] and [6]. More important the present theory provides a vehicle for developing models that reflect nonlinearity in any or all variables. However, the planning and execution of a coordinated research program with balanced emphasis on the complementary roles of theory and experiment is essential for further progress in this area.

AGING MODELS FOR CONCRETE CREEP

Different Forms of Aging

We assign now specific structure to the form of aging in order to reproduce familiar creep models of concrete literature. To this end we consider the internal variable description of the time dependent strain increment, Eq. (23)

$$\dot{\epsilon}_{i} = \sum_{\alpha=2}^{n} \lambda_{\alpha} \dot{q}_{\alpha}$$
(32)

where λ_{α} transforms internal variables into inelastic deformations. For uniaxial conditions a positive value will satisfy the dissipation inequality (31), thus we can assign to a_{α} .

$$\lambda_{\alpha} = \frac{\partial \varepsilon_i}{\partial q_{\alpha}} = 1 \tag{33}$$

without being too restrictive. For multiaxial conditions, the proper choice for λ_{α} is very complex and is equivalent to the flow rule postulate in plasticity.

For an assessment of different forms of aging we can now restrict our discussion to specific mathematical forms of the evolution law (25).

Delayed Elastic Model -- To this end we recall the classical formulation of linear viscoelasticity which interprets creep as a delayed elastic phenomenon.