

Fig. 13--Decrease in strength, caused by heating, of reinforcing steels (a), and prestressing steels (b), respectively

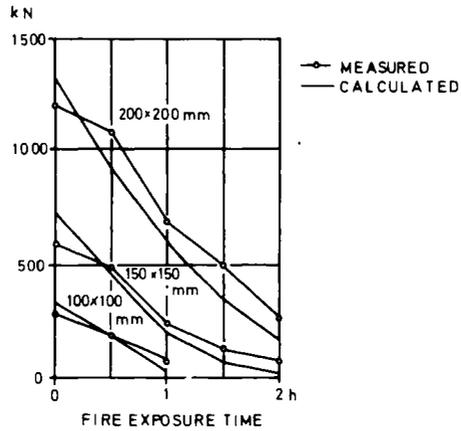


Fig. 14--Measured and calculated values of the residual load-bearing capacity of non-slender, concrete columns as function of fire exposure time at standard heating conditions

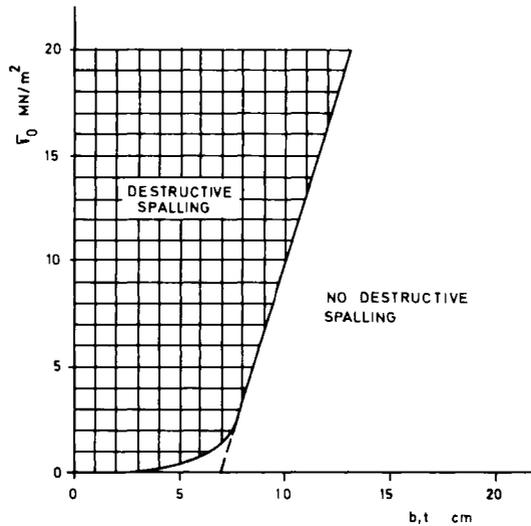


Fig. 15--Borderline between destructive and nondestructive spalling of fire exposed concrete structures with a low percentage of reinforcement.  $\sigma_0$  is the maximum compressive stress from exterior loading and prestress,  $b$  width of cross section, and  $t$  web thickness

## **Fire-Exposed Hyperstatic Concrete Structures: An Experimental and Theoretical Study**

By Y. Anderberg

Synopsis: Analytical predictions of thermal and mechanical behaviour of reinforced concrete structures exposed to differentiated complete fire processes including the cooling phase are presented and verified by tests. The modelling of the fire response comprises a heat flow analysis in the first step and a structural analysis in the second step, based on two separate computer programs. The evaluated structural fire response is compared with the measured behaviour in a great number of experimental tests in which, the fire process and the external load level are widely varied. The experimental investigation refers to a well-defined hyperstatic structure, viz. a reinforced concrete plate strip fire-exposed on one side and completely fixed against rotation at both ends while axial movement is free to develop. The outline of the project is built on the philosophy of a functionally based, differentiated design procedure for fire exposed, load-carrying and separating structures. Such a design procedure refers to performance criteria and postulates that the real physical processes with respect to fire exposure, heat transfer and structural behaviour are predicted as far as possible.

Keywords: computer programs; fire tests; heat transfer; limit state design; loads (forces); moments; plates (structural members); reinforced concrete; restraints; structural analysis; thermal properties

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## INTRODUCTION

Most experimental investigations, published in literature of fire-exposed, reinforced and prestressed concrete structures have been performed under statically determinate conditions and by the use of an internationally standardized fire process. Additionally, the effect of cooling, which follows a heating phase, is ignored and the objective in most cases has been limited to determining the fire resistance time. Furthermore published laboratory tests on fire-exposed structures have not hitherto been combined with a complete analytical behaviour study. One reason for that is the lack of knowledge about the mechanical behaviour of different structural materials at transient, high temperature conditions. However, valuable contributions within the fire research field concerning concrete structures can be found in /1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13/.

This paper presents a detailed analysis of the complex structural behaviour and load-carrying capacity of hyperstatic concrete members, exposed to a complete process of a fire development. The analysis consists of a theoretical study, combined with laboratory tests performed under well-defined conditions. The theoretical study comprises a calculation of temperature distribution of the reinforced concrete structure, used in the experiment, and based on these temperature-time fields, an evaluation of the structural response. The calculation of structural behaviour at fire is based on recently developed material behaviour models of concrete in compression /14/ and tension /15/ valid at transient as well as steady state high-temperature conditions. A correspondent constitutive law is also used for the reinforcing steel. A more thorough presentation of the whole study is given in /15/.

The experimental results on the measured behaviour verify the analytical study for a great number of cases i.e. the fire response of hyperstatic concrete structures is realistically predicted.

The present study can be employed for theoretical design purposes and be systematically used for computing tables and diagrams, facilitating the practical design. Furthermore expensive fire testings of structural members can be replaced by analytical predictions of thermal and structural response.

## EXPERIMENTAL STUDY

The experimental part of the project is mainly focused on a fundamental study of the behaviour of a well-defined type of hyperstatic structure, namely a reinforced concrete plate strip exposed to fire on one side and completely fixed against rotation at both ends but free to move longitudinally (Fig. 1).

The plate strip has a span length of 2.5 m and a total length of 3.5 m, while the cross-sectional area  $b \times h$  is  $0.3 \times 0.15 \text{ m}^2$ . Furthermore the plate strip is reinforced with 5  $\phi 10$  KS 40 bars placed symmetrically at top and bottom along the strip with a concrete layer of 0.025 m.

The experimental investigation embraced 81 tests performed for varying types of fire exposure i.e. different combinations of opening factor ( $0.01 - 0.08 \text{ m}^{1/2}$ ) and fire load ( $31 - 2010 \text{ MJ} \cdot \text{m}^{-2}$ ). The differentiated fire exposure applied is in accordance with a real fire situation and determined by the opening factor,  $A\sqrt{H}/A_t$  ( $\text{m}^{1/2}$ ), the fire load density  $q$  ( $\text{MJ}/\text{m}^2$  of the total surrounding surface area of the compartment), and the thermal properties of the surrounding structures.  $A$  = opening area ( $\text{m}^2$ ),  $H$  = opening height (m), and  $A_t$  = total interior surface, bounding the compartment, opening area included ( $\text{m}^2$ ).

One part of the investigation was devoted to a profound study of the structural response for a pure thermal exposure. The other part comprised a study of the simultaneous effect of an external transverse load consisting of two symmetrically placed concentrated loads 0.76 m apart, and a fire exposure. The loading levels 1/4, 1/2, 3/4 and 1/1 of  $P_{a11}$  were used, where  $P_{a11} = 16 \text{ kN}$  denotes the allowable load at ambient conditions according to Swedish Concrete Standards. Furthermore the flexural rigidity during fire exposure was determined for nil-loaded plate strips. As a complementary study on residual state, the residual flexural rigidity and load-carrying capacity was also measured.

In considering the influence of differentiated fire processes and in following the structural behaviour during a complete fire, until the residual state of stresses and deformations is reached, the investigation reported in /15, 16/ constitutes the first systematic one of its kind.

## THEORETICAL STUDY

Thermal Analysis

The transient temperature-time fields of the fire-exposed concrete structure are calculated in the actual application by a finite element computer program, developed from a program library constructed by

U. Wickström at Lund Institute of Technology. The program library consists of a supply of permanent system routines and problem adapted routines, which must be constructed by the user. One-dimensional, two-dimensional and axi-symmetrical problems with various boundary conditions as prescribed heat flow, temperature and adiabatic state can be solved.

The current program, which solves the one-dimensional case, considers the temperature dependence of thermal properties, viz. thermal conductivity and enthalpy even during a cooling phase, but disregards the simultaneous moisture transport within the structure. In the experimental investigation of the plate strip the fire exposure was unilateral and approximately symmetrical but varied longitudinally. Therefore, the simulation of thermal exposure was divided into four fire zones each with a characteristic temperature-time curve and owing to symmetry only half the structure was studied. Due to the discretization of the structure into segments, where each segment has a mean thermal exposure the heat flow in axial direction is of secondary importance and is therefore neglected in the calculations. The measured temperature distribution along the plate strip on surface and 3 cm inside the structure is given in Fig. 2 at a fire exposure, characterized by the opening factor  $0.04 \text{ m}^{1/2}$  and the fire load  $502 \text{ MJ} \cdot \text{m}^{-2}$ . This diagram illustrates how the temperature outside a mid-region is decreasing towards the supports.

The prediction of temperature distribution history for fire-tested concrete plate strips was in all cases in a very good agreement with measurements under heating as well as during cooling phase. As the temperature-time fields serve as input in the structural analysis program these have successfully contributed for further progress in calculations of mechanical response.

### Structural Analysis

Research contributions /14, 15, 17/ enabling the development of functionally based, realistic behaviour models, valid for concrete at transient thermal exposures have opened for practically reliable computations of structural behaviour. Computer-oriented material behaviour models of concrete and steel valid during heating and a subsequent cooling phase taking into account the history of temperature and stress as well as any process of unloading used in the study are here briefly described.

### Material Behaviour Models

The complete constitutive law of concrete in compression as well as tension under transient high temperature conditions is developed on a purely phenomenological level. The behaviour model for concrete in compression is already presented in another symposium paper "A Constitutive Law for Concrete at Transient High-Temperature Conditions", and details behind the formulation are therefore omitted as regards

this matter.

The total deformation is seen as the sum of four different components and introducing  $\bar{\sigma}$ , which denotes the history of stress the following relation is established

$$\epsilon = \epsilon_{th}^C(T) + \epsilon_{\sigma}^C(\bar{\sigma}, \sigma, T) + \epsilon_{Cr}^C(\sigma, T, t) + \epsilon_{tr}^C(\sigma, T) \quad (1)$$

where

$$\begin{aligned} \epsilon^C &= \text{total strain of concrete} \\ \epsilon_{th}^C &= \text{thermal strain of concrete} \\ \epsilon_{\sigma}^C &= \text{instantaneous, stress-related strain of concrete} \\ \epsilon_{Cr}^C &= \text{creep strain of concrete} \\ \epsilon_{tr}^C &= \text{transient strain of concrete} \end{aligned}$$

Furthermore there exists an interrelationship between the stress-related and the transient strain component.

In a time step calculation of the structural response at fire, the total strain is known, which means that the instantaneous stress-related strain is directly assessed from eq. (1). The evaluation of current stress is then based on a stress-strain relationship valid at every state. The complete stress-strain loop can be studied in Fig. 3. On tension side relatively simple assumptions have been made and two linear branches have here been chosen to describe the behaviour. The slope of the second descending branch is evaluated from measured moment-curvature relationship of a segment of the actual structure taking into account the integrated effect of the tensile carrying capacity between the cracks. The unloading branch on tension side always leads to origin or to  $\epsilon_0$  (permanent inelastic strain) depending on the prehistory and any unloading process.

When tension stresses are considered the creep strain is evaluated as in compression, but the transient strain is assumed to be zero as experimental data are lacking.

As regards the model on compression side the reader is referred to the aforementioned paper.

The constitutive law of steel is far from that complicated as concerns concrete but has a more traditional definition. Only three components are involved as shown in the following equation.

$$\epsilon^S = \epsilon_{th}^S(T) + \epsilon_{\sigma}^S(\sigma, T) + \epsilon_{Cr}^S(\sigma, T, t) \quad (2)$$

where

$\epsilon^S$  = total strain of steel

$\epsilon_{th}^S$  = thermal strain of steel

$\epsilon_{\sigma}^S$  = instantaneous, stress-related strain of steel

$\epsilon_{Cr}^S$  = creep strain of steel

This law is further described in /15, 18/.

### Analytical Study

The structural analysis program is a modified version of "FIRES-RC", originally constructed at University of California, Berkeley /18/. For the present application, this version can be characterized as an extension of the Berkeley program, reconstructed for use on a Univac 1108 Computer, which furthermore can be used also for frame analyses. The program is based on a non-linear direct stiffness formulation coupled with a time-step integration and functionally reliable analytical models here briefly described are utilized. The continual change and redistribution of forces and moments in a fire-exposed hyperstatic structure are evaluated for each time step by an iterative approach. This approach is used to find incremental changes in deformed shape, which results in equilibrium between external and internal forces. Secondary order effects and shear forces are not considered in the program.

The computer program is capable of evaluating a detailed structural behaviour and extensive comparisons with measured behaviour for a great number of tests in terms of time-history of bending restraint moments and deflections have verified the analytical predictions. A complementary test carried out on a fire-exposed, simply supported plate strip without external load is also studied theoretically, and a good agreement was attained. In view of the good agreement, which will be demonstrated in this paper a behaviour study of axially and rotationally restrained plate strips at fire under external load is furthermore made.

## RESULTS

### Predicted and Observed Structural Behaviour

An example of a temperature-time field calculation compared with experimental measurements is given in Fig. 4 at six depths of the mid-section of the fire-exposed plate strip. This fire zone is characterized by the furnace and surface temperatures also illustrated in the  $\bar{q}$ -figure and the fire exposure corresponds to a fire load,  $q = 502 \text{ MJ}\cdot\text{m}^{-2}$  and an opening factor,  $A\sqrt{H}/A_t = 0.04$  (cf. Fig. 2). The calculation agrees very well with the measurements except at temperatures below  $100^\circ\text{C}$ , where moisture transport disturbs the concordance. This example

is a typical result for the calculations.

Predicted structural response here presented is pervadingly based on the temperature-time fields illustrated above. The calculation of structural behaviour is in Fig. 5 compared with observations in terms of the restraint bending moment and the mid-section deflection. Comparisons are made at these load levels viz.  $P = 0$ ,  $P = 1/2 \cdot P_{a11}$  and  $P = P_{a11}$ . For the calculation of the substructured plate strip shown in the figure the concordance in behaviour is quite satisfactory under heating as well as during cooling phase. At the load level  $P = P_{a11}$  the yield moment is reached.

A more detailed analysis of structural response to fire is described by the distribution of all strain components inherent in the material behaviour model, which together constitute the evaluated behaviour and the stress distribution. Such a strain and stress distribution is in Fig. 6 reproduced for segment 5 for the test illustrated in Fig. 5 and characterized by the load level  $P = P_{a11}$ . Segment 5 is representative for the mid-section and the result is shown at selected times. The total strain distribution is also indicated as a dashed line giving the curvature of the section. Furthermore the stress distribution is accounted both for concrete and steel subslices. Due to the imposed thermal gradient excessive thermal strains, which successively are compensated by the transient strain and the creep strain, are developing in portions close to the fire-exposed surface. A severe degradation due to cracking rapidly takes place inside the cross-section and after 1 hour even subslice ① is cracked. Consequently the bottom as well as the top reinforcing bars and their thermal behaviour signify a dominating effect on the structural response of the strip. During cooling great stress-related positive strains develop and the curvature continues to increase somewhat, while stresses are redistributed. The redistribution after 4.8 hours indicate that all subslices are cracked except No. ⑦ which is transferred into compression. The detailed explanation behind the structural response (Fig. 5) can be found by studying the strain and stress distribution of each section during the fire process.

Another example illustrating the significance of the computer model is given in Fig. 7 for a simply supported plate strip solely thermally exposed as in the previous example. The predicted thermal mid-point deflection is here in an extraordinary agreement with the measurements.

#### Further Calculations

In order to illustrate how an axial restraint added to the rotational restraint on a fire-exposed plate strip changes the structural behaviour two calculations are presented at the load levels  $P = 0$  and  $P = 1/2 P_{a11}$ . Results are reproduced in Fig. 8 as regards time-history of thermal restraint moment, axial compression force and deflection process at midsection. The increase in thermal restraint moment is similar to comparative tests in Fig. 5, but the maximum value is here somewhat higher. However, after 0.7 hours the behaviour is quite

different and a sudden decrease in restraint moment is characteristic in both cases and after 5 hours considerable positive values are further attained. This dramatic change is owing to the axial force, which reaches as maximum 40% of the ultimate compressive load at ambient temperature. This thermal force also seems to be almost independent of external loading which to some extent is noticed for the deflection process at mid-section. The calculated time histories of restraint moment for the two tests are in conformity with each other and the difference between the curves is approximately the same as the initial value due to load. The different mode of behaviour can be analyzed from the time-variation of the strain and stress distribution at different cross-sections of the plate strip. An example of the stress-distribution is in Fig. 9 given for the end and the mid-section of the nil-loaded plate strip. The sudden decrease of restraint moment illustrated above is due to excessive transient strains and the decrease in compressive strength above 400°C for concrete layers close to the fire-exposed surface. After 1 hour in subslice ① at mid-section the decrease in stress is obvious as a result of the aforementioned reasons.

#### CONCLUSIONS

1. For investigated plate strips the residual bending stiffness and strength after fire were found to have a decrease within the range 0-30% and 0-20% respectively, for fire loads  $q$  less than  $1000 \text{ MJ}\cdot\text{m}^{-2}$ .
2. Reliable predictions of thermal response of concrete structures at fire, carried out by a finite element computer program, are demonstrated for a great number of tests at varying fire exposure conditions.
3. Based on predictions in 2 and functionally trustworthy analytical models of material behaviour valid at transient, high temperature conditions computerized calculations of fire-exposed, hyperstatic concrete structures have been validated for a great number of cases. The computer program used was originally developed in Berkeley /18/.
4. Material behaviour models based on experimental data obtained at steady state conditions applicated in the prediction of structural response to fire results in unrealistically great thermal restraint forces and moments (if less than yield moment) and sometimes in an erroneous collapse state.
5. Thermal restraint forces never exceed 45% of the load-carrying capacity at ambient conditions, in analysed fire-exposed hyperstatic concrete structures, axially restrained.
6. The influence of restraint forces and moments on the load-carrying capacity of a fire-exposed, hyperstatic concrete structure depends on the deformation capacity of concrete at high temperatures.