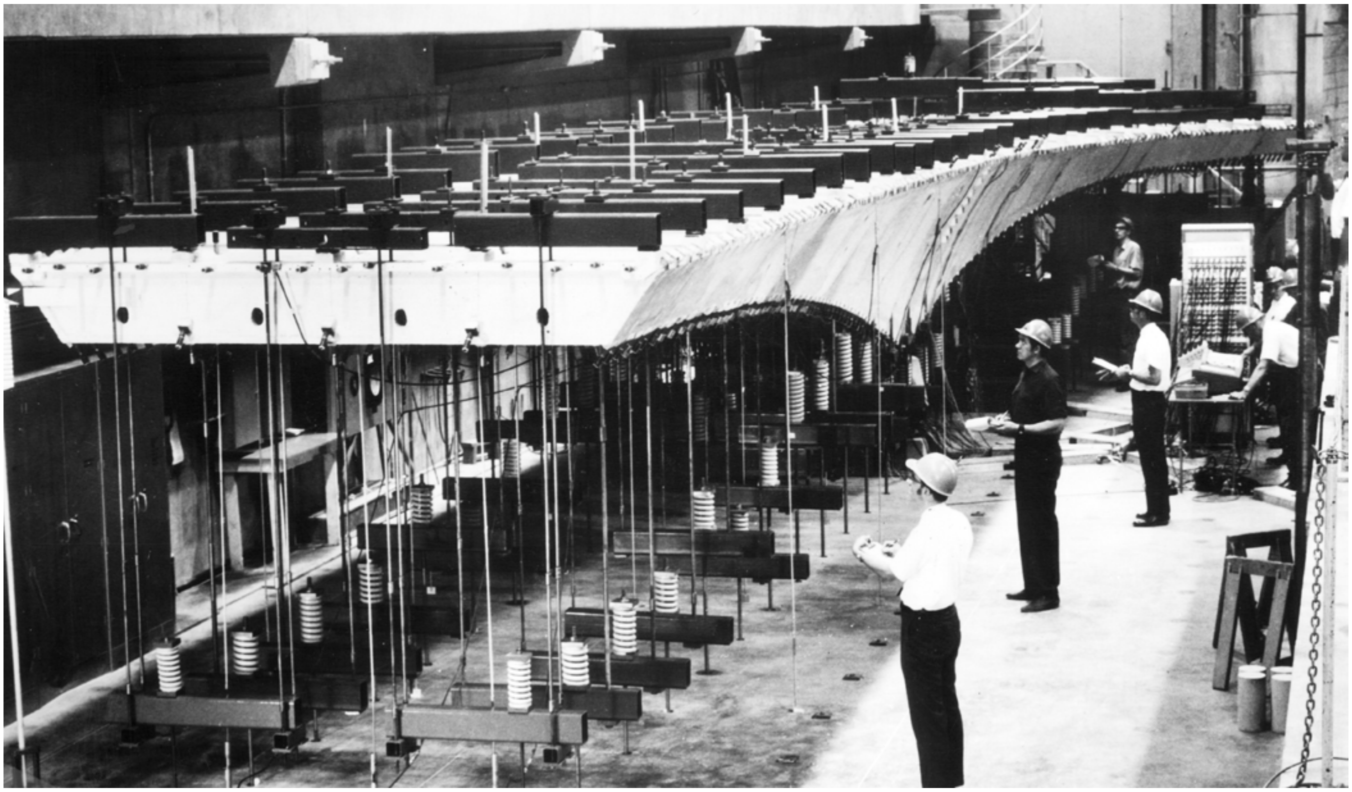

Structural tests of box girder bridge (photo courtesy of Portland Cement Association).

CHAPTER 7—STRENGTH DESIGN

7.1—Introduction

The recommendations in this chapter are intended for application to all structural elements of bridge structures, except those designed as shells, pipe culverts, and other unusual structures. They may also be applied to elements subjected to special loading conditions during the stages of fabrication and transportation of precast concrete and during placement of cast-in-place concrete.

7.2—considerations for analysis, design, and review

7.2.1 *General*—All members of statically indeterminate structures should be designed for the maximum effects of the loads specified as determined by elastic analysis, except as provided for in Section 7.2.4 and 7.2.5. Recommended loads and load combinations are discussed in Chapter 5.

7.2.2 *Stiffness*—All assumptions, adopted for computing the relative flexural and torsional stiffnesses of continuous beams and rigid frame members, should be consistent throughout the analysis. The moments of inertia to be used in obtaining the relative stiffnesses of the various members can be determined from either the uncracked concrete cross section, neglecting the reinforcement, or from the transformed cracked section as long as the same method is used throughout the analysis of a continuous or rigid frame structure.

The effect of haunches should be considered both in determining bending moments and in designing members.

7.2.3 *Span length*—The span length of members, not built integrally with their supports, should be considered as the clear span plus the depth of the member. However, the span need not exceed the distance between centers of support.

In the analysis of continuous and rigid frame members, center-to-center distances between supports should be used for the determination of moments. Moments at faces of support may be used for design of members built integrally with supports.

7.2.4 *Analysis*—Various acceptable methods of analysis are described in Section 10.3. All methods of analysis should satisfy the conditions of equilibrium, displacement compatibility, and stability at all points in the structure, along with all magnitudes of loading up to the ultimate. In addition, all serviceability recommendations of Chapter 8 should be satisfied.

7.2.5 *Redistribution*—Negative moments calculated by elastic analysis at the supports of continuous nonprestressed flexural members for any assumed loading arrangement can be increased or decreased as follows (ACI 318)

$$20\left(1 - \frac{\rho - \rho'}{\rho_b}\right) \text{, percent} \qquad (7\text{-}1)$$

where

ρ = ratio of tension reinforcement = A_s/bd

ρ' = ratio of compression reinforcement = A_s'/bd

ρ_b = reinforcement producing balanced condition

The modified negative moments should be used for calculating moments at sections within the spans. Redistribution of negative moments should be made only when the section, at which the moment is reduced, is so designed that ρ or $\rho - \rho'$ is not greater than $0.50\rho_b$, where ρ_b is calculated as follows (ACI 318)

$$\rho_b = \frac{0.85\,\beta_1 f_c'}{f_y}\,\frac{87{,}000}{87{,}000 + f_y} \qquad (7\text{-}2)$$

$$\rho_b = \frac{0.85\,\beta_1 f_c'}{f_y}\,\frac{600}{600 + f_y}$$

where

f_c' = specified compressive strength of concrete

f_y = design yield strength of nonprestressed reinforcement

β_1 = factor used to determine the stress block in ultimate load analysis and design

Negative moments calculated by elastic analysis at the supports of continuous prestressed flexural members, where bonded reinforcement is provided at supports in accordance with Section 9.11, can be increased or decreased by not more than

$$(30 - 47c/d)\text{ percent} \qquad (7\text{-}3)$$

for any assumed gravity loading arrangement. In the previous expression, c is the distance from the extreme compression fiber to the neutral axis, and d is the distance from the extreme compression fiber to the centroid of the tension steel.[7-1]

The modified negative moments should be used for calculating moments at sections within spans for the same loading arrangement. In no case should the negative moments be increased or decreased by more than 20 percent.

Negative moments should not be redistributed where fatigue of reinforcement is a governing factor. This condition can occur where moving live loads contributes a significant part of the stresses in the reinforcement.

7.2.6 *Composite concrete construction*

7.2.6.1 *General considerations*—The recommendations of this section provide for the design of composite flexural members consisting of concrete elements constructed in separate placements, but so interconnected that the elements respond to superimposed loads as a unit.

The entire composite member, or portions thereof, may be used in resisting the shear and the bending moment. The individual elements should be investigated for all critical stages of loading.

If the specified strength, unit weight, or other properties of the various components are different, the properties of the individual components, or the most critical values, should be used in design.

In calculating the flexural strength of a composite member, no distinction should be made between shored and unshored members. However, unshored construction leads to higher stresses at service loads, and may pose a problem where fatigue is a major consideration.

All elements should be designed to support all loads introduced prior to the full development of the design strength of the composite member. Reinforcement should be provided as necessary to prevent separation of the components and to control cracking.

7.2.6.2 *Shoring*—When used, shoring should not be removed until the supported elements have developed the strength required to support the prevailing loads and to limit deflections and cracking at the time of shore removal.

7.2.6.3 *Vertical shear*—When an entire composite member is assumed to resist vertical shear, design should be in accordance with the requirements of Section 7.3.7, as for a monolithically cast member of the same cross-sectional shape.

Web reinforcement should be fully anchored into interconnected elements in accordance with Section 13.2.

Extended and anchored shear reinforcement may be included as ties for horizontal shear.

7.2.6.4 *Horizontal shear*—In a composite member, full transfer of horizontal shear forces should be assured at contact surfaces of interconnected elements. Design for horizontal shear should be in accordance with the recommendations of Section 7.3.15.

7.2.7 *T-girder construction*—In T-girder construction, the girder web and slab should be effectively bonded together. Full transfer of shear forces should be assured at the interface of the web and the slab.

The effective slab width used as a girder flange should not exceed one-fourth of the girder span; its overhanging width on either side of the web should not exceed six times the thickness of the slab nor one-half the clear distance to the next girder.

For girders having a slab on one side only, the effective overhanging slab width used as a girder flange should not exceed one-twelfth of the girder span, nor six times the thickness of the slab, nor one-half the clear distance to the next girder.

For isolated T-girders, where the flange is used to provide additional compression area, only that part of the flange adjacent to the girder web, with a thickness at least one-half the width of the girder web, should be used as compression area. Also, the total width of the flange used as compression area should not exceed four times the width of the girder web.

Load distributing diaphragms should be placed between the girders at span ends and within the spans at intervals not exceeding 40 ft (12 m). Diaphragms may be omitted where tests or structural analysis show adequate strength. Diaphragms for curved girders should be given special consideration.

7.2.8 *Box girder construction*

7.2.8.1 *General*—This section pertains to the design of simple and continuous spans of single and multiple cell box girder bridges of moderate span lengths (see Section 6.5 for typical span lengths and depth-to-span ratios).

Box girders consist of girder webs and top and bottom slabs. To insure full transfer of shear forces, the girder web and top and bottom flanges should be effectively bonded together at their interfaces.

For curved girder bridges, torsion should be considered, and exterior girder shears should be increased to account for torsion.

7.2.8.2 *Lateral distribution of loads for bending moment*—The live load bending moment for each interior beam in a prestressed box beam superstructure should be determined using the method given in Section 10.5.

7.2.8.3 *Effective compression flange width*—The effective width of slab used as a girder flange should not exceed one-fourth of the girder span; the overhanging width used as a flange on either side of the web should not exceed six times the least thickness of the slab, nor one-half the clear distance to the next web.

For webs having a slab on one side, only the effective overhanging width of slab used as a girder flange should not exceed one-twelfth of the girder span, nor six times the least thickness of the slab, nor one-half the clear distance to the next web.

7.2.8.4 Slab and web thickness—The thickness of the top slab for highway bridges should be at least 6 in. (150 mm) for nonprestressed construction and at least 5-1/2 in. (140 mm) for prestressed construction.

For highway bridges, the thickness of the bottom slab should be at least one-sixteenth of the clear span between webs, or 5-1/2 in. (140 mm), whichever is greater. The minimum thickness may be reduced to 5 in. (127 mm) for factory-produced precast elements.

The thickness of the bottom slab need not be greater than the top slab, unless required by design.

If required for shear, the web may be thickened in the area adjacent to the supports. The change in web thickness should be tapered over a minimum distance equal to twelve times the difference in web thickness.

For post-tensioned box girders, in order to accommodate the post-tensioning ducts, the webs should be at least 1.0 ft (300 mm).

The designer should note, however, that these minimum thickness recommendations may not be adequate for heavily reinforced members. For the top and bottom slabs in particular, where more than three layers of reinforcing are provided, the thickness should be sufficient to provide adequate clear cover, construction tolerances, and design depth.

7.2.8.5 *Top and bottom slab reinforcement*—Uniformly distributed reinforcement of at least 0.4 percent of the flange area should be placed in the bottom slab parallel to the girder span. A single layer of reinforcement may be provided. The spacing of such reinforcement should not exceed 18 in. (500 mm).

Minimum distributed reinforcement of 0.5 percent of the cross-sectional area of the slab, based on the least slab thickness, should be placed in the bottom slab transverse to the girder span. Such reinforcement should be distributed over both surfaces with a maximum spacing of 18 in. (500 mm). All transverse reinforcement in the bottom slab should extend to the exterior face of the outside web in each group and be anchored by a standard 90 deg hook.

At least one-third of the bottom layer of the transverse reinforcement in the top slab should extend to the exterior face of the outside web in each group and be anchored by a standard 90 deg hook. If the slab extends beyond the last girder web, such reinforcement should extend into the slab overhang and have an anchorage beyond the exterior face of the web not less than that provided by a standard hook.

7.2.8.6 *Diaphragms*—Load distributing diaphragms or spreaders should be placed at 60 ft (18 m) intervals, maximum, unless tests or structural analysis show adequate strength. In addition, diaphragms should be placed at main supports to provide transfer of transverse wind loads to the substructure. On curved box girders the need for diaphragms and spacing requirements should be given special consideration.

7.2.9 *Limiting dimensions for members*

7.2.9.1 *General*—Because of the difficulty in placing concrete and the increase in maintenance costs, the use of thin or small members is seldom economically justifiable. The designer should exercise good judgment in choosing the optimum size of member.

7.2.9.2 *Compression members*—Circular compression members, constituting the principal supports of a structure, should have a diameter of at least 12 in. (300 mm). Rectangular compression members should have a thickness of at least 10 in. (250 mm) and a gross area not less than 100 in.^{2} (62500 mm^2). Auxiliary supports should be not less than 6 in. (150 mm) minimum dimension.

7.2.9.3 *Flexural members*—The width of the compression face of flexural members should not be less than 6 in. (150 mm). Structural slabs, including the flanges of T-girders, should not be less than 4 in. (102 mm) thick; however, in many situations, especially where more than two layers of reinforcing are required, a greater thickness may be needed to meet minimum cover recommendations of Section 13.8, and to provide required design depth.

7.3—Strength requirements

7.3.1 *Required strength*—Bridge structures and structural members should be designed to have strength at all sections sufficient to safely resist the structural effects of the load groups which represent various combinations of loads and forces to which the structure may be subjected, as stipulated in Section 5.12. Each part of such structures should be proportioned for the group loads that are applicable, and the maximum design required should be used. The serviceability requirements of Chapter 8 should also be satisfied to insure adequate performance at service load levels.

7.3.2 *Strength*—The design strength provided by a member or cross-section in terms of load, moment, shear, or stress should be taken as the nominal strength calculated in accordance with the recommendations and assumptions of this Section 7.3, multiplied by a strength reduction factor ϕ. Strength reduction factor ϕ should be as follows (ACI 318):

Flexural, without axial load	0.90
Axial tension, and axial tension with flexure	0.90
Axial compression, and axial compression with flexure:	
Members with spiral reinforcement conforming to Section 13.3	0.75
Members with ties conforming to Section 13.3	0.70

Except for low values of axial load, ϕ may be increased in accordance with the following:

For members in which f_y does not exceed 60,000 psi (410 MPa) with symmetric reinforcement, and with $(h - d' - d_s)/h$ not less than 0.70, ϕ may be increased linearly to 0.90 as ϕP_n decreases from $0.10 f_c' A_g$ to zero.

For other reinforced members, ϕ may be increased linearly to 0.90, as ϕP_n decreases from $0.10 f_c' A_g$ or ϕP_b, whichever is smaller, to zero.

Shear and torsion	0.85
Bearing on concrete	0.70
Flexure in plain concrete	0.65

For prestressed members produced in plants meeting the requirements of PCI *Manual* MNL-116, the following strength reduction factors should be used (AASHTO *Standard Specifications for Highway Bridges* and AREA *Manual for Railway Engineering*, Chapter 8):

Flexure, with or without axial tension and for axial tension	0.95
Shear and torsion	0.90
Compression members, with prestress exceeding 225 psi (1.55 MPa) and with spiral reinforcement conforming to Section 13.3.2	0.80
Compression members, with prestress exceeding 225 psi (1.55 MPa) without spiral reinforcement	0.75
Bearing on concrete	0.75

For all other prestressed members not specifically covered, the factors for nonprestressed concrete should be used. Development lengths specified in Chapter 13 do not require a ϕ factor.

7.3.3 *Design assumptions*—The strength design of members for flexural and axial loads should be based on assumptions given in this section, and on satisfaction of the applicable conditions of equilibrium and compatibility of strains.

Strain in reinforcement and concrete should be assumed directly proportional to the distance from the neutral axis, except that, for deep flexural members with overall depth-to-clear span ratios greater than 2/5 for continuous spans and 4/5 for simple spans, a nonlinear distribution of strain should be considered (ACI 318).

Maximum usable strain at the extreme concrete compression fiber should be assumed equal to 0.003, excluding shrinkage, creep, and temperature strains (ACI 318).

Stress in reinforcement, below specified yield strength f_y for the grade of reinforcement used, should be taken as E_s times steel strain. For strains greater than that corresponding to f_y, stress in reinforcement should be considered independent of strain and equal to f_y (ACI 318).

Tensile strength of concrete should be neglected in axial tension strength calculations and in flexural tension strength calculations of reinforced concrete, except when meeting the requirements of Section 8.7.

The relationship between the concrete compressive stress distribution and the concrete strain may be assumed to be rectangular, trapezoidal, parabolic, or any other shape that results in prediction of strength in substantial agreement with the results of comprehensive tests. These recommendations may be considered satisfied by an equivalent rectangular concrete stress distribution defined by the following (ACI 318):

a. The concrete stress of $0.85 f_c'$ is assumed, uniformly distributed over an equivalent compression zone, bounded by the edges of the cross section and a straight line located parallel to the neutral axis at a distance $a = \beta_1 c$ from the fiber of maximum compressive strain.

b. Distance c from the fiber of maximum strain to the neutral axis is measured in a direction perpendicular to that axis.

c. Factor β_1 is taken as 0.85 for concrete strengths f_c', up to and including 4000 psi (27.6 MPa). For strengths above 4000 psi (27.6 MPa), β_1 is reduced at a rate of 0.05 for each 1000 psi (6.89 MPa) of strength in excess of 4000 psi (27.6 MPa), but β_1 need not be less than 0.65.

7.3.4 *Flexure*

7.3.4.1 *Minimum reinforcement of nonprestressed flexural members*—At any section of a flexural member where tension reinforcement is required by analysis, the reinforcement provided should be adequate to develop a factored moment at least 1.2 times the cracking moment calculated on the basis of the modulus of rupture for normal weight concrete. The modulus of rupture should be obtained from tests, or may be taken as

$$7.5\sqrt{f_c'}\ (0.623\sqrt{f_c'})\ \text{(ACI 318)}$$

The previous recommendations may be waived if the area of reinforcement provided at a section is at least one-third greater than that required by analysis based on the critical loading combinations.

As an aid to the designer, the minimum recommended reinforcement ratio ρ_{min}[7-2] may be obtained from the following approximate expressions

$$\rho_{min} = \left[10 + \frac{(I/y_t)}{bd^2}\right]\left[\frac{(I/y_t)}{bd^2}\right]\frac{\sqrt{f_c'}}{f_y} \tag{7-4}$$

$$\rho_{min} = \left\{0.83\left[10 + \frac{(I/y_t)}{bd^2}\right]\left[\frac{(I/y_t)}{bd^2}\right]\frac{\sqrt{f_c'}}{f_y}\right\}$$

$$\rho_{min} = 10.2\left[\frac{(I/y_t)}{bd^2}\right]\frac{\sqrt{f_c'}}{f_y} \tag{7-5}$$

$$\rho_{min} = 0.847\left[\frac{(I/y_t)}{bd^2}\right]\frac{\sqrt{f_c'}}{f_y}$$

$$\rho_{min} = 1.7(h/d^2)\frac{\sqrt{f_c'}}{f_y}$$

$$\rho_{min} = \left[0.141(h/d^2)\frac{\sqrt{f_c'}}{f_y}\right] \tag{7-6}$$

Of the previous expressions, Eq. (7-4) is the most accurate and can be used for T-beams, box girders, and rectangular sections. Eq. (7-5) is a somewhat simpler expression that can be used for box girders or rectangular sections. The simplest expression, Eq. (7-6), can be used for the special case of rectangular sections.

These minimum reinforcement recommendations should be followed, even where analysis shows that the calculated moment would be resisted solely by the tensile strength of the concrete.

These minimum reinforcement recommendations do not apply to footings.

7.3.4.2 *Maximum reinforcement of nonprestressed flexural members*—For flexural members, the reinforcement ratio ρ provided should not exceed 0.75 of that ratio ρ_b, which would produce balanced strain conditions for the section under flexure (ACI 318).

Balanced strain conditions exist at a cross section when the tension reinforcement reaches its yield strength f_y, just as the concrete in compression reaches its ultimate strain of 0.003 (ACI 318).

7.3.4.3 *Rectangular sections with nonprestressed tension reinforcement only*—For rectangular sections, the design moment strength can be computed as follows (ACI 318R)

$$M_n = A_s f_y d\,(1 - 0.59\rho f_y / f_c')$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) \qquad (7\text{-}7)$$

where

$$a = A_s f_y / (0.85\, f_c' cb) \qquad (7\text{-}8)$$

The balanced reinforcement ratio for rectangular sections with tension reinforcement only may be calculated as follows (ACI 318R)

$$\rho_b = \frac{0.85\,\beta_1 f_c'}{f_y} \frac{87{,}000}{87{,}000 + f_y}$$

$$\left(\rho_b = \frac{0.85\,\beta_1 f_c'}{f_y} \frac{600}{600 + f_y}\right) \qquad (7\text{-}9)$$

7.3.4.4 *Flanged sections with tension reinforcement only*—When the compression flange thickness is equal to or greater than the depth of the equivalent rectangular stress block, the design moment strength M_n may be computed by the equations given in Section 7.3.4.3. When the compression flange thickness is less than a, the design moment strength can be computed as follows (ACI 318R)

$$M_n = \left[(A_s - A_{sf}) f_y \left(d - \frac{a}{2}\right) + A_{sf} f_y (d - 0.5 h_f)\right] \qquad (7\text{-}10)$$

where

$$A_{sf} = \frac{0.85 f_c' (b - b_w) h_f}{f_y} \qquad (7\text{-}11)$$

and

$$a = \frac{(A_s - A_{sf}) f_y}{0.85 f_c' c b_w} \qquad (7\text{-}12)$$

The balanced reinforcement ratio for flanged sections with tension reinforcement only can be computed as follows (ACI 318R)

$$\rho_b = \frac{b_w}{b}\left(\frac{0.85\beta_1 f_c'}{f_y} \frac{87{,}000}{87{,}000 + f_y} + \frac{A_{sf}}{b_w d}\right)$$

$$\left[\rho_b = \frac{b_w}{b}\left(\frac{0.85\beta_1 f_c'}{f_y} \frac{600}{600 + f_y} + \frac{A_{sf}}{b_w d}\right)\right] \qquad (7\text{-}13)$$

For T-girder and box-girder construction the width of the compression face b should be equal to the effective slab width as defined in Section 7.2.7.

7.3.4.5 *Rectangular sections with compression reinforcement*—For rectangular sections and flanged sections in which the neutral axis lies within the flange, the design moment strength can be computed as follows (ACI 318R)

$$M_n = \left[(A_s - A_s') f_y \left(d - \frac{a}{2}\right) + A_s' f_y (d - d')\right] \qquad (7\text{-}14)$$

where

$$a = \frac{(A_s - A_s') f_y}{0.85 f_c' b} \qquad (7\text{-}15)$$

and the following condition should be satisfied

$$\frac{A_s - A_s'}{bd} \ge 0.85\beta_1 \frac{f_c' d'}{f_y d} \frac{87{,}000}{87{,}000 - f_y}$$

$$\left[\frac{(A_s - A_s')}{bd} \ge 0.85\beta_1 \frac{f_c' d'}{f_y d} \frac{600}{600 - f_y}\right] \qquad (7\text{-}16)$$

When the value of $(A_s - A_s')/bd$ is less than the value given by Eq. (7-17), so that the stress in the compression reinforcement is less than the yield strength, or when the effects of compression reinforcement are neglected, the design moment strength can be computed by the equations in Section 7.3.4.3. In these cases, the section is treated as if reinforced with tension steel only. Alternatively, a general analysis can be made based on stress and strain compatibility using the assumptions given in Section 7.3.3.

The balanced reinforcement ratio for rectangular sections with compression reinforcement can be calculated as follows (ACI 318R)

$$\rho_b = \frac{0.85\beta_1 f_c'}{f_y}\frac{87{,}000}{87{,}000 + f_y} + \frac{f_{sb}'}{f_y}\rho'$$

$$\left(\rho_b = \frac{0.85\beta_1 f_c'}{f_y}\frac{600}{600 + f_y} + \frac{f_{sb}'}{f_y}\rho'\right) \qquad (7\text{-}17)$$

where f_{sb}' = stress in compression reinforcement at balanced conditions

$$f_{sb}' = 87{,}000\left(1 - \frac{d'}{d}\frac{87{,}000 + f_y}{87{,}000}\right) \geq f_y$$

$$\left[f_{sb}' = 600\left(1 - \frac{d'}{d}\frac{600 + f_y}{600}\right) \geq f_y\right] \qquad (7\text{-}18)$$

7.3.4.6 *Other nonprestressed cross sections*—For other cross sections and for conditions of nonsymmetrical bending, the design moment strength ϕM_n should be computed by a general analysis based on stress and strain compatibility, using the assumptions given in Section 7.3.3. The recommendations of Section 7.3.4.2 should also be satisfied.

7.3.4.7 *Prestressed concrete members*—The design moment strength for prestressed flexural members can be computed by the same strength design procedures and equations recommended for nonprestressed members. For prestressing tendons, f_{ps} should be substituted for f_y.

In lieu of a more accurate determination of f_{ps} based on strain compatibility, and provided that f_{se} is not less than $0.5f_{pu}$, the following approximate values should be used:[7-3]

a. For members with bonded prestressing tendons,[7-3] the equation $f_{ps} = f_{pu}(1 - 0.3\, c/d)$ can be approximated by

$$f_{ps} = f_{pu}(1 - 0.3c/d_u) \qquad (7\text{-}19)$$

where

$$d_\mu = \frac{A_{ps}f_{pu}d_p + A_s f_y d}{A_{ps}f_{pu} + A_s f_y} \qquad (7\text{-}20)$$

and

$$c = \frac{A_{ps}f_{pu} + A_s f_y - A_s' f_y}{0.85\beta_1 f_c' b + 0.3A_{ps}f_{pu}/d_u} \text{ for } \beta_1 c \leq h_f \qquad (7\text{-}21)$$

or

$$c = \frac{A_{ps}f_{pu} + A_s f_y - A_s' f_y - T_f}{0.85\beta_1 f_c' b_w + 0.3A_{ps}f_{pu}/d_u} \text{ for } \beta_1 c \geq h_f \qquad (7\text{-}22)$$

in which case

$$T_f = 0.85 f_c'(b - b_w)h_f \qquad (7\text{-}23)$$

In the previous expressions, d_p is the effective depth of the prestressing steel. Design examples are given in Reference 7-3.

b. For members with unbonded prestressing tendons and a span-to-depth ratio of 35 or less (ACI 318)

$$f_{ps} = f_{se} + 10{,}000 + \frac{f_c'}{100\rho_p}$$

$$\left(f_{ps} = f_{se} + 69 + \frac{f_c'}{100\rho_p}\right) \qquad (7\text{-}24)$$

but f_{ps} in Eq. (7-24) should not be greater than f_{py} or (f_{se} + 60,000) [(f_{se} + 410)].

c. For members with unbonded prestressing tendons and with a span-to-depth ratio greater than 35 (ACI 318)

$$f_{ps} = f_{se} + \frac{f_c'}{300\rho_p} + 10{,}000$$

$$\left(f_{ps} = f_{se} + \frac{f_c'}{300\rho_p} + 69\right) \qquad (7\text{-}25)$$

but f_{ps} in Eq. (7-25) should not be greater than f_{py}, or (f_{se} + 30,000) [(f_{se} + 205)].

Nonprestressed reinforcement conforming to Section 3.2, when used in combination with prestressing tendons, may be considered to contribute to the tensile force and may be included in the moment strength calculations at a strength equal to the specified yield strength f_y.

The index of prestressed and nonprestressed reinforcement used for the computation of the moment strength of a member should be such that $c/d_u \leq 0.42$.[7-3] For flanged sections, the steel area should be required to develop the compressive strength of the web only.

When a reinforcement index in excess of that previously recommended is provided, design moment strength should be based on the compression portion of the internal moment resisting couple. The following expressions satisfy the intent of this recommendation:

a. For rectangular or flanged sections in which the neutral axis is within the flange (ACI 318R)

$$M_n = \left[f_c' b d_p^2(0.36\beta_1 - 0.08\beta_1^2)\right] \qquad (7\text{-}26)$$

b. For flanged sections in which the neutral axis falls outside the flange (ACI 318R)

$$M_n = [f_c' b_w d_p^2(0.36\beta_1 - 0.08\beta_1^2) +$$

$$0.85 f_c'(b - b_w)h_f(d_p - 0.5h_f)] \qquad (7\text{-}27)$$

The total amount of prestressed and nonprestressed reinforcement should be adequate to develop a factored load at least 1.2 times the cracking load, computed on the basis of the modulus of rupture f_r.

7.3.4.8 *Special recommendations for slabs*—The minimum area of flexural reinforcement for one-way, nonprestressed slabs in the direction of the span should be as recommended in Section 7.3.4.1. The maximum area of flexural reinforcement for one-way, nonprestressed slabs in the direction of the span should be as recommended in Section 7.3.4.2. The design moment strength ϕM_n of one-way, nonprestressed slabs may be computed as recommended in Section 7.3.4.3. The design moment strength ϕM_n of one-way, prestressed slabs may be computed as recommended in Section 7.3.4.7.

The minimum area of shrinkage and temperature reinforcement for one-way slabs transverse to the direction of the span should be as follows:

Slabs where Grade 40 or 50 deformed bars are used	0.0020
Slabs where Grade 60 deformed bars are used	0.0018

7.3.5 *Nonprestressed compression members with or without flexure*

7.3.5.1 *General requirements*—The design of members subject to combined flexure and axial load should be based on stress and strain compatibility using the assumptions given in Section 7.3.3. For prestressed members, refer to Chapter 9. Slenderness effects should be evaluated following the recommendations of Section 7.3.6.

Members subject to compression load only, or to combined axial load and flexural load, should be designed according to the recommendations of Section 7.3.5.3.

7.3.5.2 *Limits for reinforcement of compression members*—The area of longitudinal reinforcement for compression members should not be less than 0.01 or more than 0.08 times the gross area of A_g of the section.

The minimum number of longitudinal reinforcing bars in compression members should be four for bars within rectangular ties six for bars within circular ties, and six for bars enclosed by spirals conforming to Section 13.3.

For compression members with a larger cross section than required by considerations of loading, a reduced effective area A_g may be used to determine the minimum longitudinal reinforcing. In no case, however, should this effective area be taken as less than one-half the total area.

7.3.5.3 *Compression member strength*—The following provisions may be used as a guide to define the range of the load-moment interaction relationship for members subject to axial load only or combined axial load and flexural load.

The design axial load strength ϕP_n of compression members with spiral reinforcement as recommended in Section 13.3 should not be greater than (ACI 318)

$$\phi P_n = 0.85\phi[0.85 f_c'(A_g - A_{st}) + f_y A_{st}] \tag{7-28}$$

The design axial load strength ϕP_n of compression members with tie reinforcement as recommended in Section 13.3 should not be greater than (ACI 318)

$$\phi P_n = 0.80\phi\ [0.85 f_c'(A_g - A_{st}) + f_y A_{st}] \tag{7-29}$$

All members subjected to a compression load should be designed for the maximum effects of factored moments and axial loads. The factored axial load P_u at the given eccentricity should not exceed ϕP_n as calculated in Eq. (7-28) or (7-29).

The balanced strain conditions for a cross section are defined in Section 7.3.4.2. For a rectangular section with reinforcement in one or two faces and located at approximately the same distance from the axis of bending, the balanced load strength ϕP and balanced moment strength ϕM_n can be computed as follows (ACI 318R)

$$\phi P_b = \phi(0.85 f_c' b a_b + A_s' f_{sb}' - A_s f_y) \tag{7-30}$$

and

$$\phi M_b = [0.85 f_c' b a_b (h/2 - a_b/2) + A_s' f_{sb}'(h/2 - d') + A_s f_y (h/2 - d_s)] \tag{7-31}$$

where

$$a_b = \left(\frac{87{,}000}{87{,}000 + f_y}\right)\beta_1 d$$

$$\left[a_b = \left(\frac{600}{600 + f_y}\right)\beta_1 d\right] \tag{7-32}$$

and

$$f_{sb}' = 87{,}000 - (d'/d)(87{,}000 + f_y) \le f_y$$

$$[f_{sb}' = 600 - (d'/d)(600 + f_y) \le f_y] \tag{7-33}$$

The design strength under combined flexure and axial load should be based on stress and strain compatibility using the assumptions given in Section 7.3.3. The strength of a cross section is controlled by tension when the nominal axial load strength P_n is less than P_b. The strength of a cross section is controlled by compression when the axial load design strength P_n is greater than P_b. The combined axial load and moment strength should be multiplied by the appropriate capacity factor reduction ϕ as recommended in Section 7.3.2.

For members subject to combined flexure and compressive axial load, when the design axial load strength ϕP_n is less than the smaller of $0.10 f_c' A_g$ or ϕP_b, the ratio of reinforcement ρ provided should not exceed 0.75 of the ratio ρ_b that would produce balanced strain conditions for the section under flexure without axial load. For members with compression reinforcement, that portion of ρ_b, equalized by compression reinforcement, need not be reduced by the 0.75 factor.

7.3.5.4 *Biaxial loading*—In lieu of a general section analysis based on stress and strain compatibility for a condition of biaxial flexural loading, the design strength of rectangular members under such loading conditions may be approximated using the charts, tables, and formulas given in Reference 7-4, or the following expressions can be used to approximate the axial design strength of noncircular members subject to biaxial bending[7-5]

$$\phi P_{nxy} = \frac{1}{1/\phi P_{nx} + 1/\phi P_{ny} + 1/\phi P_o} \qquad (7\text{-}34)$$

when

$$P_u \geq 0.10 f_c' A_g \qquad (7\text{-}35)$$

or

$$M_{ux}/\phi M_{nx} + M_{uy}/\phi M_{ny} = 1 \qquad (7\text{-}36)$$

when the applied axial design load

$$P_u < 0.10 f_c' A_g \qquad (7\text{-}37)$$

7.3.6 *Slenderness effects in compression members*

7.3.6.1 *General*—Wherever possible, the design of compression members should be based on a comprehensive analysis of the structure. Such analysis should be a second-order analysis, taking into account the deformation of the structure and the duration of the loads. When axial loads are of sufficient magnitude to reduce stiffness or increase fixed-end moments, such effects should be included.

In lieu of a second-order analysis, the design of compression members can be based on the approximate procedure recommended in the following sections.

7.3.6.2 *Unsupported length*—For purposes of determining the limiting dimensions of compression members, the unsupported length l_u should be the clear distance between lateral supports, except as recommended in Subsections (a) through (e) below:

a. In pile bent construction, l_u for the pile should be the clear distance between the lowest lateral support, as described in Subsection (c) below, and the lower extremity at which the pile may be assumed to be fixed, depending on the soil conditions.

b. For compression members supported on spread or pile footings, l_u should be the clear distance between the top of the footing and the underside of the deepest flexural member, framing into the compression member in the direction of potential translation at the next higher level.

c. For members restrained laterally by struts, ties, or beams, l_u should be the clear distance between consecutive struts in the vertical plane, provided that two such struts should meet the compression member at approximately the same level, and the angle between vertical planes through the struts should not vary more than 15 deg from a right angle. Such struts should be of adequate dimensions and have sufficient anchorage to restrain the member against lateral deflection.

d. For compression members restrained laterally by struts or beams with brackets used at the junction, l_u should be the clear distance between the lower support or fixity of the base and the lower edge of the bracket, provided that the bracket width equals that of the strut or beam, and is at least one-half the width of the compression member.

e. Where haunches are present, l_u should be measured to the lower extremity of the haunch in the plane considered.

The length which produces the greatest ratio of length to the radius of gyration of the section should be considered, as well as compatibility with the loading and support conditions.

7.3.6.3 *Radius of gyration*—The radius of gyration r may be equal to 0.30 times the overall dimension in the direction in which stability is being considered for rectangular members, and 0.25 times the diameter for circular members. For other shapes, r may be computed for the gross concrete section.

7.3.6.4 *Effective length factor and lateral stability*—The effective length of a compression member should be Kl_u, where the effective length factor K should not be less than (ACI 318)

a. For compression members braced against side-sway, the effective length factor K should be 1.0, unless analysis shows that a lower value may be used.

b. For compression members not braced against side-sway, lateral instability should be considered in determining the effective length Kl_u for all structures in which the total translational stiffness of the bracing elements is less than six times the sum of the combined translational stiffnesses of all the compression members in the level under consideration. All other structures may be assumed to be restrained against lateral instability. For structures subject to side-sway, the value of Kl_u should be determined with due consideration of cracking and reinforcement on relative stiffness. When determining the stiffness ratio of beams and columns to obtain K, the beam stiffness should be obtained using Eq. (7-39), with β_d as zero. In no case should K be less than 1.0.

For compression members braced against side-sway, the effects of slenderness may be neglected when Kl_u/r is less than $34 - 12M_1/M_2$. For compression members not braced against side-sway, the effects of slenderness may be neglected when Kl_u/r is less than 22. For all compression members with Kl_u/r greater than 100, an analysis should be made as defined in Section 7.3.6.1.

For evaluation of slenderness effects, M_1 is defined as the smaller factored end moment, calculated by conventional frame analysis. M_1 is positive if the member is bent in single curvature and negative if bent in double curvature. M_2 is defined as the larger factored end moment, calculated by conventional frame analysis, always positive.

7.3.6.5 *Moment magnification*—Compression members should be designed using the factored axial load P_u from a conventional frame analysis and a magnified factored moment M_c defined as follows (ACI 318)

$$M_c = \delta_b M_{2b} + \delta_s M_{2s} \quad (7\text{-}38)$$

where

$$\delta_b = \frac{C_m}{1 - P_u/\phi P_c} \geq 1.0 \quad (7\text{-}39)$$

$$\delta_s = \frac{1}{1 - \Sigma P_u/\phi \Sigma P_c} \geq 1.0 \quad (7\text{-}40)$$

and

$$P_c = \pi^2 EI/(Kl_u)^2 \quad (7\text{-}41)$$

In the previous expressions, ΣP_u and ΣP_c are the summations for all columns in a story. For frames not braced against side-sway, both δ_b and δ_s should be computed. For frames braced against side-sway, δ_s should be zero. In calculating P, K should be computed according to Section 7.3.6.4(a) for δ_b and according to Section 7.3.6.4(b) for δ_s.

In lieu of a more accurate calculation, EI in Eq. (7-41) may be either

$$EI = \frac{E_c I_g/5 + E_s I_{se}}{1 + \beta_d} \quad (7\text{-}42)$$

or conservatively

$$EI = \frac{E_c I_g/2.5}{1 + \beta_d} \quad (7\text{-}43)$$

where β_d is defined as the ratio of maximum factored dead load moment to maximum factored total load moment, always positive (ACI 318).

In Eq. (7-39) C_m, for members braced against side-sway and without transverse loads between supports, may be

$$C_m = 0.6 + 0.4M_1/M_2 \quad (7\text{-}44)$$

but not less than 0.4. For all other cases, C_m should be 1.0 (ACI 318).

If the computations show that there is no moment at both ends of a compression member, or that the computed end eccentricities are less than $(0.6 + 0.03h)$ in. [$(15 + 0.03h)$ mm], M_2 in Eq. (7-38) should be based on a minimum eccentricity of $(0.6 + 0.03h)$ in. [$(15 + 0.03h)$ mm] about each principal axis separately. The ratio M_1/M_2 in Eq. (7-41) should be determined as follows (ACI 318):

a. When the computed end eccentricities are less than $(0.6 + 0.03h)$ in. [$(15 + 0.03h)$ mm], computed end moments may be used to evaluate M_1/M_2.
b. If computations show that there is essentially no moment at both ends of a compression member, the ratio M_1/M_2 should be equal to one.

When compression members are subject to bending about principal axes, the moment about each axis should be magnified by δ_b and δ_s and computed from corresponding conditions of restraint about that axis. In structures which are not braced against side-sway, the flexural members should be designed for the total magnified end moments of the compression members at the joint.

When a group of compression members on one level comprise a bent, or when they are connected integrally to the same superstructure and collectively resist the side-sway of the structure, δ_b and δ_s should be computed for the member group, as described in Section 7.3.6.5.

7.3.7 *Shear strength required*—The design of cross sections subject to shear should be based on

$$V_u \leq \phi V_n \quad (7\text{-}45)$$

where V_u is the factored shear force at the section under consideration, and V_n is the nominal shear strength computed by

$$V_n = V_c + V_s \quad (7\text{-}46)$$

where V_c is the nominal shear strength provided by the concrete in accordance with Section 7.3.8 or 7.3.9, and V_s is the nominal shear strength provided by the shear reinforcement in accordance with Section 7.3.11.

In determining the shear strength V_c whenever applicable, the effects of axial forces due to creep, shrinkage, and temperature changes, should be considered in restrained members, and the effects of inclined flexural compression in variable-depth members may be included.

The maximum factored shear force V_u at supports may be computed in accordance with (c) or (d) below when both of the following conditions (a) and (b) are satisfied (ACI 318):

a. The support reaction in the direction of the applied shear introduces compression into the end regions of the member.
b. No concentrated load occurs between the face of the support and the location of critical section as defined below in (c) or (d).
c. For nonprestressed members, sections located less than a distance d from the face of the support may be designed for the same shear V_u as that computed at a distance d.
d. For prestressed members, sections located less than a distance $h/2$ from the face of the support may be designed for the same shear V_u as that computed at a distance $h/2$.

For deep flexural members, brackets, corbels, slabs and footings, the recommendations of Section 7.3.14 through 7.3.16 should be followed.

7.3.8 *Shear strength provided by concrete for nonprestressed members*

7.3.8.1 *Simplified strength calculations*—Shear strength V_c should be computed by the provisions of (a) through (d) below, unless a more detailed calculation is made (ACI 318).

a. For members subject to shear and flexure only

$$V_c = 2\sqrt{f_c'}\,b_w d\ (0.17\sqrt{f_c'}\,b_w d) \quad (7\text{-}47)$$

b. For members subject to axial compression

$$V_c = 2(1 + 0.005\,N_u/A_g)\ \sqrt{f_c'}\,b_w d$$

$$[V_c = 0.17\,(1 + 0.073N_u/A_g)\sqrt{f_c'}\,b_w d] \quad (7\text{-}48)$$

with N_u/A_g expressed in psi (MPa).

c. For members subject to significant axial tension, the shear reinforcement should be designed to carry the total shear.

d. At sections of members where the torsional moment T_u exceeds

$$\phi(0.5\sqrt{f_c'}x^2y) \qquad [\phi(0.04\sqrt{f_c'}x^2y)]$$

$$V_c = \frac{2\sqrt{f_c'}b_w d}{\sqrt{1 + (2.5C_tT_u/V_u)^2}}$$

$$\left[V_c = \frac{0.17\sqrt{f_c'}b_w d}{\sqrt{1 + (2.5C_tT_u/V_u)^2}}\right] \quad (7\text{-}49)$$

where x is the smaller overall dimension, and y is the larger overall dimension of the rectangular cross section.

7.3.8.2 *Detailed strength calculations*—Alternatively, shear strength V_c may be computed by the more detailed provisions of (e) through (g) below (ACI 318):

e. For members subject to shear and flexure only

$$V_c = [1.9\sqrt{f_c'} + 2500\rho_w(V_ud/M_u)]b_w d$$

$$\{V_c = [0.16\sqrt{f_c'} + 17.2\rho_w(V_ud/M_u)]b_w d\} \quad (7\text{-}50)$$

but not greater than $3.5\sqrt{f_c'}\,b_w d$ or $(0.29\sqrt{f_c'}\,b_w d)$. The quantity V_ud/M_u should not be greater than 1.0 for computing V_c by Eq. (7-50), where M_u is the factored moment occurring simultaneously with V_u at the section under consideration.

f. For members subject to axial compression, Eq. (7-50) may be used to compute V_c with a modified moment M_m, M_m substituted for M_u, and V_{ud}/M_u not limited to 1.0, where

$$M_m = M_u - N_u(4h - d)/8 \quad (7\text{-}51)$$

However, V_c should not be greater than

$$V_c = 3.5\sqrt{f_c'}\quad b_w d\sqrt{1 + N_u/500A_g}$$

$$(V_c = 0.29\sqrt{f_c'}\quad b_w d\sqrt{1 + 0.29N_u/A_g}) \quad (7\text{-}52)$$

Quantity N_u/A_g should be expressed in psi (MPa). When M_m as computed by Eq. (7-51), is negative, V_c should be computed by Eq. (7-52).

g. For members subject to significant axial tension

$$V_c = 2(1 + N_u/500A_g)\sqrt{f_c'}\,b_w d$$

$$[V_c = 0.17\,(1 + 0.29N_uA_g)\sqrt{f_c'}\,b_w d] \quad (7\text{-}53)$$

where N_u is negative for tension. Quantity N_u/A_g should be expressed in psi (MPa).

7.3.9 *Shear strength provided by concrete for prestressed members*

7.3.9.1 *Basic strength calculation*—For members with an effective prestress force no less than 40 percent of the tensile strength of the flexural reinforcement, the shear strength V_c should be computed by equation (7-54), unless a more detailed calculation is made (ACI 318)

$$V_c = (0.6\sqrt{f_c'} + 700\,V_ud/M_u)b_w d$$

$$[V_c = (0.05\sqrt{f_c'} + 4.8\,V_ud/M_u)b_w d] \quad (7\text{-}54)$$

But V_c need not be less than $2\sqrt{f_c'}\,b_w d\ (0.17\sqrt{f_c'}\,b_w d)$, nor should V_c be greater than $5\sqrt{f_c'}\,b_w d\ (0.42\sqrt{f_c'}\,b_w d)$, or the value given in Section 7.3.9.3. The quantity V_ud/M_u should not be greater than 1.0, where M_u is the factored moment occurring simultaneously with V_u at the section considered. When applying Eq. (7-54), d in the quantity V_ud/M_u should be the distance from the extreme compression fiber to the centroid of the prestressed reinforcement.

7.3.9.2 *Detailed strength calculations*—Alternatively, the shear strength V_c may be computed in accordance with (a) through (c) below, where V_c should be the lesser of V_{ci} or V_{cw}.[7-3]

a. Shear strength V_{ci} should be computed by

$$V_{ci} = 0.6\sqrt{f_c'}\,b_w d + Vd + V_iM_{cr}/M_{max}$$

$$(V_{ci} = 0.05\sqrt{f_c'}\,b_w d + Vd + V_iM_{cr}/M_{max}) \quad (7\text{-}55)$$

but V_{ci} need not be less than

$$1.7\sqrt{f_c'}\,b_w d\ (0.14\sqrt{f_c'}\,b_w d)$$

where

$$M_{cr} = (I/yt)\,(6\sqrt{f_c'} + f_{pe} - f_d)$$

$$[M_{cr} = (I/yt)\,(0.5\sqrt{f_c'} + f_{pe} - f_d)] \quad (7\text{-}56)$$