**13.5.3.4** — Concentration of reinforcement over the column by closer spacing or additional reinforcement shall be used to resist moment on the effective slab width defined in 13.5.3.2.

**13.5.4** — Design for transfer of load from slabs to supporting columns or walls through shear and torsion shall be in accordance with Chapter 11.

### 13.6 — Direct design method

#### 13.6.1 — Limitations

Design of slab systems within the limitations of 13.6.1.1 through 13.6.1.8 by the direct design method shall be permitted.

# **COMMENTARY**

Tests of slab-column connections indicate that a large degree of ductility is required because the interaction between shear and unbalanced moment is critical. When the factored shear is large, the column-slab joint cannot always develop all of the reinforcement provided in the effective width. The modifications for edge, corner, or interior slabcolumn connections specified in 13.5.3.3 are permitted only when the reinforcement ratio (within the effective width) required to develop the unbalanced moment  $\gamma_f M_{\mu}$  does not exceed **0.375** $\rho_b$ . The use of Eq. (13-1) without the modification permitted in 13.5.3.3 will generally indicate overstress conditions on the joint. The provisions of 13.5.3.3 are intended to improve ductile behavior of the column-slab joint. When a reversal of moments occurs at opposite faces of an interior support, both top and bottom reinforcement should be concentrated within the effective width. A ratio of top to bottom reinforcement of about 2 has been observed to be appropriate.

## R13.6 — Direct design method

In multi-cell construction, the floors and walls of tank-type structures may sometimes qualify for the direct design method.

The direct design method consists of a set of rules for distributing moments to slab and beam sections to satisfy safety requirements and most serviceability requirements simultaneously. Three fundamental steps are involved as follows:

(1) Determination of the total factored static moment (see 13.6.2);

(2) Distribution of the total factored static moment to negative and positive sections (see 13.6.3);

(3) Distribution of the negative and positive factored moments to the column and middle strips and to the beams, if any (see 13.6.4 through 13.6.6). The distribution of moments to column and middle strips is also used in the equivalent frame method (see 13.7).

#### **R13.6.1** — Limitations

The direct design method was developed from considerations of theoretical procedures for the determination of moments in slabs with and without beams, requirements for simple design and construction procedures, and precedents supplied by performance of slab systems. Consequently, the

# COMMENTARY

**13.6.1.1** — There shall be a minimum of three continuous spans in each direction.

**13.6.1.2** — Panels shall be rectangular, with a ratio of longer to shorter span center-to-center of supports within a panel not greater than 2.

**13.6.1.3** — Successive span lengths center-tocenter of supports in each direction shall not differ by more than one-third the longer span.

**13.6.1.4** — Offset of columns by a maximum of 10 percent of the span (in direction of offset) from either axis between centerlines of successive columns shall be permitted.

**13.6.1.5** — All loads shall be due to gravity and fluid pressures only and shall be uniformly distributed over an entire panel. For purposes of determining the dead-to-live-load ratio of Section 13.6.1.5 and when using Eq. (13-4) to determine the column and wall moments, the full or partial portion of the liquid load that is uniform over all spans shall be considered as part of the dead load. Any non-uniform portion of the liquid load due to the slope of the floor or adjacent cells not being filled shall be considered as part of the liquid loads, whether considered live or dead, shall be multiplied by the appropriate load factor of 1.7, per Chapter 9. If fluid pressures do not act simultaneously on all panels, live load, including that resulting from fluid pressures, shall not exceed three times dead load.

**13.6.1.6** — For a panel with beams between supports on all sides, the relative stiffness of beams in two perpendicular directions

$$\frac{\alpha_1 \ell_2^2}{\alpha_2 \ell_1^2} \tag{13-2}$$

shall not be less than 0.2 nor greater than 5.0.

slab systems to be designed using the direct design method should conform to the limitations in this section.

**R13.6.1.1** — The primary reason for the limitation in this section is the magnitude of the negative moments at the interior support in a structure with only two continuous spans. The rules given for the direct design method assume that the slab system at the first interior negative moment section is neither fixed against rotation nor discontinuous.

**R13.6.1.2** — If the ratio of the two spans (long span/short span) of a panel exceeds 2, the slab resists the moment in the shorter span essentially as a one-way slab.

**R13.6.1.3** — The limitation in this section is related to the possibility of developing negative moments beyond the point where negative moment reinforcement is terminated, as prescribed in Fig. 13.3.8.

**R13.6.1.4** — Columns can be offset within specified limits from a regular rectangular array. A cumulative total offset of 20 percent of the span is established as the upper limit.

**R13.6.1.5** — The direct design method is based on tests<sup>13.18</sup> for uniform gravity loads and resulting column reactions determined by statics. Lateral loads such as wind or seismic require a frame analysis. Inverted foundation mats designed as two-way slabs (see 15.10) involve application of known column loads. Therefore, even where the soil reaction is assumed to be uniform, a frame analysis is required.

Two-way slab systems are sometimes used for tank bottoms where they are subjected to uniform fluid pressures many times larger than the dead load. As long as the fluid pressures are uniform and act on all panels, they need not be included in the limiting live-to-dead-load ratio, as they cannot produce pattern loading effects. When the fluid pressures vary significantly, such as when slabs have pronounced slope or contain cells where one may be full while the adjacent one is empty, the equivalent frame, or other method of analysis, should be used.

Sediments, which can accumulate in some tanks, should be treated as live loads because there can be pronounced pattern loading effects when tanks are drained and cleaned.

**R13.6.1.6** — The elastic distribution of moments will deviate significantly from those assumed in the direct design method unless the requirements for stiffness are satisfied.

**13.6.1.7** — Moment redistribution as permitted by 8.4 shall not be applied for slab systems designed by the direct design method. See 13.6.7.

**13.6.1.8** — Variations from the limitations of 13.6.1 shall be permitted if demonstrated by analysis that requirements of 13.5.1 are satisfied.

#### 13.6.2 — Total factored static moment for a span

**13.6.2.1** — Total factored static moment for a span shall be determined in a strip bounded laterally by centerline of panel on each side of centerline of supports.

**13.6.2.2** — Absolute sum of positive and average negative factored moments in each direction shall not be less than

$$M_{o} = \frac{w_{u}\ell_{2}\ell_{n}^{2}}{8}$$
(13-3)

**13.6.2.3** — Where the transverse span of panels on either side of the centerline of supports varies,  $\ell_2$  in Eq. (13-3) shall be taken as the average of adjacent transverse spans.

**13.6.2.4** — When the span adjacent and parallel to an edge is being considered, the distance from edge to panel centerline shall be substituted for  $l_2$  in Eq. (13-3).

**13.6.2.5** — Clear span  $\ell_n$  shall extend from face to face of columns, capitals, brackets, or walls. Value of  $\ell_n$  used in Eq. (13-3) shall not be less than **0.65** $\ell_1$ . Circular or regular polygon shaped supports shall be treated as square supports with the same area.

### COMMENTARY

**R13.6.1.7** — Moment redistribution as permitted by 8.4 is not intended where approximate values for bending moments are used. For the direct design method, 10 percent modification is allowed by 13.6.7.

**R13.6.1.8** — The designer is permitted to use the direct design method even if the structure does not fit the limitations in this section, provided it can be shown by analysis that the particular limitation does not apply to that structure. For a slab system carrying a non-movable load (such as a water reservoir in which the load on all panels is expected to be the same), the designer need not satisfy the live load limitation of 13.6.1.5.

#### R13.6.2 — Total factored static moment for a span

**R13.6.2.2** — Equation (13-3) follows directly from Nichol's derivation<sup>13.19</sup> with the simplifying assumption that the reactions are concentrated along the faces of the support perpendicular to the span considered. In general, the designer will find it expedient to calculate static moments for two adjacent half panels that include a column strip with a half middle strip along each side.

**R13.6.2.5** — If a supporting member does not have a rectangular cross section or if the sides of the rectangle are not parallel to the spans, it is to be treated as a square support having the same area, as illustrated in Fig. R13.6.2.5.



Fig. R13.6.2.5—Examples of equivalent square section for supporting members.

#### 13.6.3 — Negative and positive factored moments

**13.6.3.1** — Negative factored moments shall be located at face of rectangular supports. Circular or regular polygon shaped supports shall be treated as square supports with the same area.

**13.6.3.2** — In an interior span, total static moment  $M_{o}$  shall be distributed as follows:

Negative factored moment...... 0.65

Positive factored moment ...... 0.35

**13.6.3.3** — In an end span, total factored static moment  $M_{o}$  shall be distributed as follows:

-					
	(1)	(2)	(3)	(4)	(5)
	Exterior	Slab with beams	Slab without beams between interior supports		Exterior
	edge unrestrained	between all supports	Without edge beam	With edge beam	edge fully restrained
Interior negative factored moment	0.75	0.70	0.70	0.70	0.65
Positive factored moment	0.63	0.57	0.52	0.50	0.35
Exterior negative factored moment	0	0.16	0.26	0.30	0.65

**13.6.3.4** — Negative moment sections shall be designed to resist the larger of the two interior negative factored moments determined for spans framing into a common support unless an analysis is made to distribute the unbalanced moment in accordance with stiffnesses of adjoining elements.

**13.6.3.5** — Edge beams or edges of slab shall be proportioned to resist in torsion their share of exterior negative factored moments.

# COMMENTARY

#### R13.6.3 — Negative and positive factored moments

**R13.6.3.3** — The moment coefficients for an end span are based on the equivalent column stiffness expressions from References 13.20, 13.21, and 13.22. The coefficients for an unrestrained edge would be used, for example, if the slab were simply supported on a masonry or concrete wall. Those for a fully restrained edge would apply if the slab were constructed integrally with a concrete wall having a flexural stiffness so large compared with that of the slab that little rotation occurs at the slab-to-wall connection.

For other than unrestrained or fully restrained edges, coefficients in the table were selected to be near the upper bound of the range for positive moments and interior negative moments. As a result, exterior negative moments were usually closer to a lower bound. The exterior negative moment capacity for most slab systems is governed by minimum reinforcement to control cracking. The final coefficients in the table have been adjusted so that the absolute sum of the positive and average moments equal  $M_{o}$ .

For two-way slab systems with beams between supports on all sides (two-way slabs), moment coefficients of Column (2) apply. For slab systems without beams between interior supports (flat plates and flat slabs), the moment coefficients of Column (3) or (4) apply, without or with an edge (spandrel) beam, respectively.

In the 1977 ACI Building Code, distribution factors defined as a function of the stiffness ratio of the equivalent exterior support were used for proportioning the total static moment  $M_o$  in an end span. The approach may be used in place of values in 13.6.3.3.

**R13.6.3.4** — The differences in slab moment on either side of a column or other type of support should be accounted for in the design of the support. If an analysis is made to distribute unbalanced moments, flexural stiffness may be obtained on the basis of the gross concrete section of the members involved.

**R13.6.3.5** — Moments perpendicular to, and at the edge of, the slab structure should be transmitted to the supporting columns or walls. Torsional stresses caused by the moment assigned to the slab should be investigated.

# COMMENTARY

# CODE

**13.6.3.6** — The gravity load moment to be transferred between slab and edge column in accordance with 13.5.3.1 shall be  $0.3M_o$ .

#### 13.6.4 — Factored moments in column strips

**13.6.4.1** — Column strips shall be proportioned to resist the following portions in percent of interior negative factored moments:

$\ell_2/\ell_1$	0.5	1.0	2.0
$(\alpha_1\ell_2/\ell_1)=0$	75	75	75
$(\alpha_1 \ell_2 / \ell_1) \geq 1.0$	90	75	45

Linear interpolations shall be made between values shown.

**13.6.4.2** — Column strips shall be proportioned to resist the following portions in percent of exterior negative factored moments:

l2/l1		0.5	1.0	2.0
$(\alpha_1 \ell_2 / \ell_1) = 0$	β <sub>t</sub> = 0	100	100	100
	β <sub>t</sub> ≥2.5	75	75	75
$(\alpha_1 \ell_2 / \ell_1) \ge 1.0$	β <sub>t</sub> = 0	100	100	100
	β <b>t</b> ≥2.5	90	75	45

Linear interpolations shall be made between values shown.

**13.6.4.3** — Where supports consist of columns or walls extending for a distance equal to or greater than three-quarters the span length  $l_2$  used to compute  $M_o$ , negative moments shall be considered to be uniformly distributed across  $l_2$ .

**13.6.4.4** — Column strips shall be proportioned to resist the following portions in percent of positive factored moments:

$\ell_2/\ell_1$	0.5	1.0	2.0
$(\alpha_1\ell_2/\ell_1)=0$	60	60	60
$(\alpha_1 \ell_2 / \ell_1) \ge 1.0$	90	75	45

Linear interpolations shall be made between values shown.

**13.6.4.5** — For slabs with beams between supports, the slab portion of column strips shall be proportioned to resist that portion of column strip moments not resisted by beams.

# R13.6.4, R13.6.5, and R13.6.6 — Factored moments in column strips, beams, and middle strips

The rules given for assigning moments to the column strips, beams, and middle strips are based on studies of moments in linearly elastic slabs with different beam stiffness<sup>13.23</sup> tempered by the moment coefficients that have been used successfully.

For the purpose of establishing moments in the half column strip adjacent to an edge supported by a wall,  $\ell_n$  in Eq. (13-3) may be assumed equal to  $\ell_n$  of the parallel adjacent column to column span, and the wall may be considered as a beam having a moment of inertia  $I_b$  equal to infinity.

**R13.6.4.2** — The effect of the torsional stiffness parameter  $\beta_t$  is to assign all of the exterior negative factored moment to the column strip, and none to the middle strip, unless the beam torsional stiffness is high relative to the flexural stiffness of the supported slab. In the definition of  $\beta_t$ , the shear modulus has been taken as  $E_{cb}/2$ .

Where walls are used as supports along column lines, they can be regarded as very stiff beams with an  $\alpha_1 t_2 / t_1$  value greater than one. Where the exterior support consists of a wall perpendicular to the direction in which moments are being determined,  $\beta_t$  may be taken as zero if the wall is of masonry without torsional resistance, and  $\beta_t$  may be taken as 2.5 for a concrete wall with great torsional resistance which is monolithic with the slab.

#### 13.6.5 — Factored moments in beams

**13.6.5.1** — Beams between supports shall be proportioned to resist 85 percent of column strip moments if  $(\alpha_1 \ell_2 / \ell_1)$  is equal to or greater than 1.0.

**13.6.5.2** — For values of  $\alpha_1 \ell_2 / \ell_1$  between 1.0 and zero, proportion of column strip moments resisted by beams shall be obtained by linear interpolation between 85 and zero percent.

**13.6.5.3** — In addition to moments calculated for uniform loads according to 13.6.2.2, 13.6.5.1, and 13.6.5.2, beams shall be proportioned to resist all moments caused by concentrated or linear loads applied directly to beams, including weight of projecting beam stem above or below the slab.

#### 13.6.6 — Factored moments in middle strips

**13.6.6.1** — That portion of negative and positive factored moments not resisted by column strips shall be proportionately assigned to corresponding half middle strips.

**13.6.6.2** — Each middle strip shall be proportioned to resist the sum of the moments assigned to its two half middle strips.

**13.6.6.3** — A middle strip adjacent to and parallel with a wall-supported edge shall be proportioned to resist twice the moment assigned to the half middle strip corresponding to the first row of interior supports.

#### 13.6.7 — Modification of factored moments

Modification of negative and positive factored moments by 10 percent shall be permitted provided the total static moment for a panel in the direction considered is not less than that required by Eq. (13-3).

#### 13.6.8 — Factored shear in slab systems with beams

**13.6.8.1** — Beams with  $\alpha_1 \ell_2 \ell_1$  equal to or greater than 1.0 shall be proportioned to resist shear caused by factored loads on tributary areas that are bounded by 45 deg lines drawn from the corners of the panels and the centerlines of the adjacent panels parallel to the long sides.

**13.6.8.2** — In proportioning of beams with  $\alpha_1 \ell_2 / \ell_1$  less than 1.0 to resist shear, linear interpolation, assuming beams carry no load at  $\alpha_1 = 0$ , shall be permitted.

**13.6.8.3** — In addition to shears calculated according to 13.6.8.1 and 13.6.8.2, beams shall be proportioned to resist shears caused by factored loads applied directly on beams.

# COMMENTARY

#### **R13.6.5** — Factored moments in beams

Loads assigned directly to beams are in addition to the uniform dead load of slab, uniform superimposed dead loads such as the ceiling, floor finish, or assumed equivalent partition loads, and uniform live loads. All of these loads are normally included with  $w_u$  in Eq. (13-3). Linear loads applied directly to beams include partition walls over or along beam centerlines and additional dead load of the projecting beam stem. Concentrated loads include posts above or hangers below the beams. For the purpose of assigning directly applied loads, only loads located within the width of the beam stem should be considered as directly applied to the beams. (The effective width of a beam as defined in 13.2.4 is solely for strength and relative stiffness calculations.) Line loads and concentrated loads located on the slab away from the beam stem require special consideration to determine their apportionment to slab and beams.

#### R13.6.8 — Factored shear in slab systems with beams

The tributary area for computing shear on an interior beam is shown shaded in Fig. R13.6.8. If the stiffness for the beam  $\alpha_1 \ell_2 / \ell_1$  is less than 1.0, the shear on the beam may be obtained by linear interpolation. In such cases, the beams framing into the column will not account for all the shear force applied on the column. The remaining shear force will produce shear stresses in the slab around the column that should be checked in the same manner as for flat slabs, as required by 13.6.8.4. Sections 13.6.8.1 through 13.6.8.3 do not apply to the calculation of torsional moments on the beams. These moments should be based on the calculated flexural moments acting on the sides of the beam.

245

# **COMMENTARY**

# CODE

**13.6.8.4** — Computation of slab shear strength on the assumption that load is distributed to supporting beams in accordance with 13.6.8.1 or 13.6.8.2 shall be permitted. Resistance to total shear occurring on a panel shall be provided.

**13.6.8.5** — Shear strength shall satisfy requirements of Chapter 11.

#### 13.6.9 — Factored moments in columns and walls

**13.6.9.1** — Columns and walls built integrally with a slab system shall resist moments caused by factored loads on the slab system.

**13.6.9.2** — At an interior support, supporting elements above and below the slab shall resist the moment specified by Eq. (13-4) in direct proportion to their stiffnesses unless a general analysis is made.

$$M = 0.07[(w_d + 0.5w_\ell)\ell_2 \ell_n^2 - w_d' \ell_2' (\ell_n')^2] \quad (13-4)$$

where  $w_{d'}$ ,  $\ell_{2'}$ , and  $\ell_{n'}$  refer to shorter span.

#### 13.7 — Equivalent frame method

**13.7.1** — Design of slab systems by the equivalent frame method shall be based on assumptions given in 13.7.2 through 13.7.6, and all sections of slabs and supporting members shall be proportioned for moments and shears thus obtained.

**13.7.1.1** — Where metal column capitals are used, it shall be permitted to take account of their contributions to stiffness and resistance to moment and to shear.



Fig. R13.6.8—Tributary area for shear on interior beam.

#### R13.6.9 — Factored moments in columns and walls

Equation (13-4) refers to two adjoining spans, with one span longer than the other, with full dead load plus one-half live load applied on the longer span and only dead load applied on the shorter span.

The term  $w_{\ell}$  in Eq. (13-4) need not include uniform fluid pressures that act simultaneously on both spans. Where there is a variation in fluid pressure due to a moderate slope, or due to adjacent cells being full or empty, the full value of the difference in average pressure between adjacent spans should be used in place of  $0.5w_{\ell}$  in Eq. (13-4).

Design and detailing of the reinforcement transferring the moment from the slab to the edge column is critical to both the performance and the safety of flat slabs or flat plates without edge beams or cantilever slabs. It is important that complete design details be shown on design drawings, such as concentration of reinforcement over the column by closer spacing or additional reinforcement.

#### **R13.7** — Equivalent frame method

The equivalent frame method involves the representation of the three-dimensional slab system by a series of two-dimensional frames that are then analyzed for loads acting in the plane of the frames. The negative and positive moments so determined at the critical design sections of the frame are distributed to the slab sections in accordance with 13.6.4 (column strips), 13.6.5 (beams), and 13.6.6 (middle strips). The equivalent frame method is based on studies reported in References 13.20, 13.21, and 13.22. Many of the details of the equivalent frame method given in the Commentary in the ACI 318-89 code were removed in the ACI 318-95 code.

**R13.7.1.1** — Metal column capitals (that is, shear heads) are seldom used in liquid-containing structures.

**13.7.1.2** — It shall be permitted to neglect the change in length of columns and slabs due to direct stress, and deflections due to shear.

#### 13.7.2 — Equivalent frame

**13.7.2.1** — The structure shall be considered to be made up of equivalent frames on column lines taken longitudinally and transversely through the building.

**13.7.2.2** — Each frame shall consist of a row of columns or supports and slab-beam strips, bounded laterally by the centerline of panel on each side of the centerline of columns or supports.

**13.7.2.3** — Columns or supports shall be assumed to be attached to slab-beam strips by torsional members (see 13.7.5) transverse to the direction of the span for which moments are being determined and extending to bounding lateral panel centerlines on each side of a column.

**13.7.2.4** — Frames adjacent and parallel to an edge shall be bounded by that edge and the centerline of adjacent panel.

**13.7.2.5** — Analysis of each equivalent frame in its entirety shall be permitted. Alternatively, for gravity loading, a separate analysis of each floor or roof with far ends of columns considered fixed shall be permitted.

**13.7.2.6** — Where slab-beams are analyzed separately, determination of moment at a given support assuming that the slab-beam is fixed at any support two panels distant therefrom shall be permitted, provided the slab continues beyond that point.

# COMMENTARY

#### R13.7.2 — Equivalent frame

Application of the equivalent frame to a regular structure is illustrated in Fig. R13.7.2. The three-dimensional building is divided into a series of two-dimensional frame bents (equivalent frames) centered on column or support centerlines with each frame extending the full height of the building. The width of each equivalent frame is bounded by the centerlines of the adjacent panels. The complete analysis of a slab system for a building consists of analyzing a series of equivalent (interior and exterior) frames spanning longitudinally and transversely through the building.

The equivalent frame comprises three parts: (1) the horizontal slab strip, including any beams spanning in the direction of the frame; (2) the columns or other vertical supporting members, extending above and below the slab; and (3) the elements of the structure that provide moment transfer between the horizontal and vertical members.



Fig. R13.7.2—Definitions of equivalent frame.

### 13.7.3 — Slab-beams

**13.7.3.1** — Determination of the moment of inertia of slab-beams at any cross section outside of joints or column capitals using the gross area of concrete shall be permitted.

**13.7.3.2** — Variation in moment of inertia along axis of slab-beams shall be taken into account.

**13.7.3.3** — Moment of inertia of slab-beams from center of column to face of column, bracket, or capital shall be assumed equal to the moment of inertia of the slab-beam at face of column, bracket, or capital divided by the quantity  $(1 - c_2/\ell_2)^2$  where  $c_2$  and  $\ell_2$  are measured transverse to the direction of the span for which moments are being determined.

#### 13.7.4 — Columns

**13.7.4.1** — Determination of the moment of inertia of columns at any cross section outside of joints or column capitals using the gross area of concrete shall be permitted.

**13.7.4.2** — Variation in moment of inertia along axis of columns shall be taken into account.

**13.7.4.3** — Moment of inertia of columns from top to bottom of the slab-beam at a joint shall be assumed infinite.

# **COMMENTARY**

#### R13.7.3 — Slab-beams

**R13.7.3.3** — A support is defined as a column, capital, bracket, or wall. Note that a beam is not considered to be a support member for the equivalent frame.

#### R13.7.4 — Columns

Column stiffness is based on the length of the column from mid-depth of slab above to mid-depth of slab below. Column moment of inertia is computed on the basis of its cross section, taking into account the increase in stiffness provided by the capital, if any.

When slab-beams are analyzed separately for gravity loads, the concept of an equivalent column, combining the stiffness of the slab-beam and torsional member into a composite element, is used. The column flexibility is modified to account for the torsional flexibility of the slab-tocolumn connection that reduces its efficiency for transmission of moments. The equivalent column consists of the actual columns above and below the slab-beam plus "attached" torsional members on each side of the columns extending to the centerline of the adjacent panels as shown in Fig. R13.7.4.



Fig. R13.7.4—Equivalent column (column plus torsional members).

248

#### 13.7.5 — Torsional members

**13.7.5.1** — Torsional members (see 13.7.2.3) shall be assumed to have a constant cross section throughout their length consisting of the largest of (a), (b), and (c):

(a) A portion of slab having a width equal to that of the column, bracket, or capital in the direction of the span for which moments are being determined;

(b) For monolithic or fully composite construction, the portion of slab specified in (a) plus that part of the transverse beam above and below the slab;

(c) The transverse beam as defined in 13.2.4.

**13.7.5.2** — Where beams frame into columns in the direction of the span for which moments are being determined, the torsional stiffness shall be multiplied by the ratio of the moment of inertia of the slab with such a beam to the moment of inertia of the slab without such a beam.

#### 13.7.6 — Arrangement of live load

**13.7.6.1** — When loading pattern is known, the equivalent frame shall be analyzed for that load.

**13.7.6.2** — When live load is variable but does not exceed three-quarters of the dead load, or the nature of live load is such that all panels will be loaded simultaneously, it shall be permitted to assume that maximum factored moments occur at all sections with full factored live load on entire slab system.

**13.7.6.3** — For loading conditions other than those defined in 13.7.6.2, it shall be permitted to assume that maximum positive factored moment near midspan of a panel occurs with three-quarters of the full factored live load on the panel and on alternate panels; and it shall be permitted to assume that

# COMMENTARY

#### R13.7.5 — Torsional members

Computation of the stiffness of the torsional member requires several simplifying assumptions. If no transversebeam frames into the column, a portion of the slab equal to the width of the column or capital is assumed to be the torsional member. If a beam frames into the column, Tbeam or L-beam action is assumed, with the flanges extending on each side of the beam a distance equal to the projection of the beam above or below the slab, but not greater than four times the thickness of the slab. Furthermore, it is assumed that no torsional rotation occurs in the beam over the width of the support.

The member sections to be used for calculating the torsional stiffness are defined in 13.7.5.1. In the ACI 318-89 code, Eq. (13-6) specified the stiffness coefficient  $K_t$  of the torsional members. The approximate expression for  $K_t$  has been moved to the commentary, and the expression for the torsional constant (Eq. (13-7) in the ACI 318-89 code) is now defined in 13.0.

Studies of three-dimensional analyses of various slab configurations suggest that a reasonable value of the torsional stiffness can be obtained by assuming a moment distribution along the torsional member that varies linearly from a maximum at the center of the column to zero at the middle of the panel. The assumed distribution of unit twisting moment along the column centerline is shown in Fig. R13.7.5.

An approximate expression for the stiffness of the torsional member, based on the results of three-dimensional analyses of various slab configurations (References 13.18, 13.19, and 13.20) is given below as

$$K_t = \sum \frac{9E_{cs}C}{\ell_2 \left(1 - \frac{c_2}{\ell_2}\right)^3}$$

where an expression for C is given in 13.0.

#### R13.7.6 — Arrangement of live load

The use of only three-quarters of the full-factored live load for maximum moment loading patterns is based on the fact that maximum negative and maximum positive live load moments cannot occur simultaneously and that redistribution of maximum moments is thus possible before failure occurs. This procedure, in effect, permits some local overstress under the full-factored live load if it is distributed in the prescribed manner, but still ensures that the ultimate capacity of the slab



Fig. R13.7.5—Distribution of unit twisting moment along column centerline AA shown in Fig. R13.7.4.