Subsurface Conditions and Subgrade-Model Parameters

The key soil parameter in mat analysis is generally the compressibility. For the general case of subgrade deformation, the appropriate compressibility parameter is Young's Modulus. When dealing with a fine-grained soil (i.e., clay) as at this site, there are two limiting cases of soil behavior: the immediate (undrained) condition and the long-term (drained) condition. Only the drained condition was studied as the most-complete settlement data published were for a time well after primary consolidation was complete.

For the purposes of this paper, the subsurface conditions at the adjacent sites of the Whitaker Laboratory and Chemistry Building were assumed to be identical. Below foundation level is approximately 70 feet of clay that is known locally as the Boston Blue Clay (BBC). This is underlain by glacial outwash sands and till that was assumed to act as a rigid base. The stress at foundation level imposed by each structure was less than the vertical overburden stress prior to construction which resulted in what is frequently referred to as a compensated or floating foundation. Consequently, the buildings stressed the BBC within its reload range only. Piezometric data provided in the original paper indicated that full heave had occurred during excavation so the full stress caused by the buildings was assumed to be transmitted during the consolidation process. The drained Young's Modulus of the soil was estimated using relationships that involve the coefficient of compressibility of the soil, a_v, obtained in a standard oedometer test. Reference (21) provide a summary of techniques for estimating the drained Young's Modulus for fine-grained soils.

The 70 feet of clay was divided into several artificial layers within each Young's Modulus was assumed to be constant. A method similar to that used in (7) and described in (11) was used to calculate an equivalent average Young's Modulus (800 ksf) for the entire system. No increase in modulus was made to account for embedment of the mat below the surrounding grade. This is consistent with a recommendation in (5) that the embedment effect is small and can be ignored. The drained Poisson's Ratio was assumed to be 0.25 for all layers. The shear modulus was calculated to be 320 ksf.

Using these elastic parameters, the coefficients for the various subgrade models used in this study were calculated as follows:

1. for Winkler's Hypothesis with a constant coefficient of subgrade reaction, the Winkler-Type Simplified Continuum appears to be the most consistent method for calculating k_{Wo} (11). As derived in (10),

for an isotropic, homogeneous layer

$$k_{W_o} = \frac{E_s}{H} \tag{10}$$

where E_s is the Young's Modulus of the layer (800 ksf here) and H is the layer thickness (70 ft here);

2. for the beam-column analogy, as summarized in (15) there are at least five different ways to interpret the coefficients in Eq.8 which defines the behavior of this model. Of these, the Pasternak-Type Simplified Continuum appears to be the most logical (14). As derived in (8), for an isotropic, homogeneous layer

$$c_{P_1} = \frac{E_s}{H} \tag{11}$$

$$c_{P_2} = \frac{G_s H}{2} \tag{12}$$

Note that Eq.11 is identical to Eq.10 and represents the spring component of the subgrade. Eq.12 represents the spring-coupling effect which is interpreted as a fictitious tensile force in the coupled mat + subgrade "beam-column";

3. for the Reissner Simplified Continuum, as derived in (9) for an isotropic, homogeneous layer

$$c_{R_1} = \frac{G_s H^2}{12 E_s} \tag{13}$$

$$c_{R_3} = \frac{G_s H}{3} \tag{14}$$

$$c_{R_2} = \frac{E_s}{H} \tag{15}$$

Note that Eq.14 is identical to Eqs.10 and 11 and represents the spring component of the subgrade. The other two equations represent spring-coupling effects (note that Eq.15 is a pseudo beam-tension term similar to Eq.12).

As discussed previously, numerous methods have been proposed over the years for estimating a constant value of Winkler's coefficient of subgrade reaction, k_{W0}. Table 2 contains a summary of various k_{W0} values for the mats studied. Except for the "design" values, all were determined as part of this study. The design values were reportedly developed solely on the basis of the designers' experience and judgement and, as a result, cannot be derived from any published method. The k_{W0} values in Table 2 are presented for information and informal comparison only. Analyses performed for this study used only the design and WTSC values to limit the number of variables considered in this study.

CASE HISTORIES: PRESENTATION AND DISCUSSION OF RESULTS

Whitaker Laboratory

Introduction--Fig.7 shows the range of mat settlements in the transverse direction that were measured in early 1971, six-and-one-half years after the building superstructure was completed. The minimum values occurred at the east end of the building and the maxima at the west end. The structure had a slight rigid-body tilt toward the west. All transverse settlement patterns were qualitatively similar, i.e., slightly concave-upward ("dishing" or "sagging") although the mat behaved relatively rigidly even in this direction.

Level 1 Analyses--A comparison of measured versus calculated settlements for the Level 1 analyses is shown in Fig.8. Settlements calculated using the design value of k_{W0} are significantly less than the observed values, by about one order of magnitude. However, in fairness to the original designers, they followed the traditional philosophy and did not use the results from their analyses using Winkler's Hypothesis as an estimate of expected mat settlements. Settlements calculated using a value of k_{W0} from the WTSC method are somewhat greater than the observed which is consistent with results obtained for other case histories (11). Although the WTSC method provides results reasonably close to the actual, the dishing pattern of settlement was not indicated. Of the variable Winkler (pseudo-coupled) methods, only the simplest (doubling the WTSC k_{W0} value at the edge) is shown here. For all analyses performed for this study, using a Winkler coefficient of subgrade reaction that increased gradually from center to edge produced essentially identical result to those obtained by simply doubling the edge values. Consequently, only the doubled-edge values will be shown. The calculated settlements are slightly closer to the actual but not significantly different from those obtained assuming a constant value of Winkler's coefficient of subgrade reaction. The results from the beam-column analogy (Pasternak subgrade) are similar to the Winkler subgrade results, but with a flatter settlement pattern that more closely matches the observed. The RSC model is seen to produce the best agreement with the observed settlements, although the calculated dishing is less than observed.

A comparison of calculated moments is shown in Fig.9. As is typical, the relative range in moments is significantly less than the relative range in settlements despite the wide variation in subgrade models and parameters considered. Moments from the assumption of a perfectly rigid mat are also included for comparison. It is interesting to note the very good agreement between the results from RSC model and the designer's uniform Winkler coefficient of subgrade reaction. Note also that moments using the RSC model exceed the assumed cracked-section value (150 kip-ft/ft) only in the vicinity of Column Line B near the center of the mat.

The calculated subgrade reaction (mat-subgrade contact stresses) are shown in Fig.10. The effect of the simplest pseudo-coupled method where the coefficient of subgrade reaction is doubled at the edge is clearly seen with the peak edge stresses that attempt to duplicate the peaks obtained using the RSC.

<u>Level 2 Analyses</u>--The settlements for the Level 2 analyses are shown in Fig.11. In general, they were found to be only slightly different from the Level 1 results (Fig.8), so the results from some subgrade models have been omitted for clarity. The most noteworthy difference is the flattening of the slopes of the deformed mat at the edges. This is the result of approximately modeling the rotational restraint provided by the exterior below-grade walls. Calculated moments are shown in Fig.12. The positive moments at the edge are greater than in the Level 1 case because of the assumed rotational restraint.

<u>Level 3 Analyses</u>--The Levels 3a and 3b results were very similar, so only the 3b results are presented. This outcome is not surprising because relatively little cracking is calculated for this mat, so whether or not cracked-section behavior is included has little effect. Settlements from representative subgrade models are shown in Fig.13. For this case, somewhat greater edge settlements are calculated for all subgrade models compared to the Levels 1 and 2 results (the Level 2 results for the RSC model are shown for comparison). Calculated moments are shown for the RSC subgrade model only in Fig.14, with the Level 2 results shown for

comparison. The major difference is the noticeable increase in negative moments, as the frame action of the superstructure tends to reduce the positive moments caused by the earth and water pressures on the basement walls.

The final issue considered was the effect of assuming a two-thirds reduction in the stiffness of the foundation and superstructure concrete as an approximation of time-related effects. Fig.15 compares actual and calculated settlements using the RSC model only (the Level 3b structural model was assumed for both analyses). It can be seen that the reduced modulus results, as expected, in more-flexible behavior and slightly better agreement with observed settlements.

Chemistry Building

Introduction--An identical suite of analyses was performed for the Chemistry Building. Observed settlements in the transverse direction are shown in Fig.16. These data were also obtained in early 1971, approximately two years after the superstructure was completed. Although the minimum values were at the north end of the building and the maxima toward the south end, there was no rigid-body tilt of this building. Note that the settlement pattern here is slightly concave-downward ("hogging"), although the lack of measurements along the exterior columns (lines A and E) precludes a complete picture of the actual settlement pattern. Overall, this mat is relatively more flexible than the Whitaker Laboratory.

Level 1 Analyses--A comparison of measured versus calculated settlements for the basic analyses is shown in Fig.17. Settlements calculated using the design value of k_{W0} are again significantly less than the observed values and actually indicate a net uplift near the centerline of the mat. For this problem, the agreement between observed settlements and those calculated using the RSC subgrade model were not as good as for the Whitaker Laboratory, although the RSC model results are significantly better than those using the constant-value Winkler (WTSC) and variable Winkler (WTSC k_{W0} doubled at edge) subgrade models. In this problem, the beam-column analogy produces results significantly better than Winkler's Hypothesis and quite close to the observed.

Calculated moments are shown in Fig.18. It is interesting to note the implication (using the RSC-model results) that cracked-section behavior would occur over much of this mat (estimated cracking moment=70 kip-ft/ft). The constant-value Winkler (WTSC) and variable Winkler (WTSC doubled at edge) methods appear to be conservative in the negative moment range, again with little difference between these two methods. The assumption of a rigid mat produces grossly conservative negative moments, not surprising in view of the relatively flexible nature of this mat.

The calculated subgrade reactions are shown in Fig.19. No attempt was made to iterate the analysis for the constant-value Winkler (design value) case to eliminate the negative subgrade reaction, which is physically impossible, that was calculated.

Level 2 Analyses--Settlements for the Level 2 analyses are shown in Fig.20. In general, they are little different from the Level 1 results (Fig.17), so the results from some subgrade models have been omitted for clarity. The most noteworthy difference is the flattening of the slopes of the deformed mat at the edges, especially for Winkler's Hypothesis with a constant (WTSC) Winkler coefficient of subgrade reaction. Again, this is the result of the rotational restraint provided by the exterior below-grade walls. Calculated moments are shown in Fig.21. The positive moments at the edge are somewhat greater than in the Level 1 case because of the assumed rotational restraint.

Level 3 Analyses--Again, only the Level 3b results are shown. Settlements are shown in Fig.22, with the RSC model results from the Level 2 analyses included for comparison. Modeling variable loads and mat stiffness appears to have little influence on the calculated settlements even though extensive cracking is calculated for this mat. A comparison of calculated moments is shown in Fig.23 using the RSC model only. Finally, the influence of assumed concrete modulus is shown in Fig.24. In this case, somewhat poorer correlation with observed settlements was obtained using a 50% reduction in modulus.

CONCLUSIONS

Subgrade Models

The primary conclusions drawn with respect to subgrade models are:

1. the Conventional Method of Static Equilibrium, which assumes a rigid mat, provides poor approximation of observed behavior. Virtually all mats exhibit some flexibility relative to the subgrade. As a consequence, moments calculated assuming mat rigidity can be seriously in error;
2. use of Winkler's Hypothesis with a constant Winkler coefficient of subgrade reaction does not produce accurate estimates of moments and settlements from a single value of k_{W0}. For example, the original design values for k_{W0}, which were chosen solely on the

basis of engineering judgement, provided surprisingly good agreement with the RSC model for moments, but underestimated settlements by an order of magnitude. Conversely, the WTSC values for k_{WO} produced better estimates of settlement, but poorer moment agreement compared to the RSC results;

3. in general, results from using a variable Winkler coefficient of subgrade reaction (pseudo-coupled concept) are inconsistent. The simplest approach of doubling the value of k_{WO} along the edges of the mat produced only modest improvement in both buildings compared to using Winkler's Hypothesis with a constant value for k_{WO}, with overall poor comparison to observed behavior and results from the RSC model. On the other hand, (24) has demonstrated that by uniquely determining the magnitude and variation in Winkler's coefficient of subgrade reaction for a given project, good results can be obtained. The conclusion is that variations in the Winkler coefficient of subgrade reaction that are based on simple rules, e.g., doubling the values at the mat edges, cannot be expected to be accurate for the infinite range in combinations of loads, etc. that occur in mat design practice. As stated previously, the accuracy of results from the pseudo-coupled method in general is directly related to how well the Winkler coefficient of subgrade reaction assumed matches the actual. This is illustrated in Figs.25 and 26 where the coefficient of subgrade reaction calculated using the RSC model is compared to the values assumed using Winkler's Hypothesis. Not shown is the doubled value (22 kcf) at the edges of the mat for the variable Winkler cases. For the Chemistry Laboratory in particular (Fig.26), the match between the Winkler assumptions and RSC results is quite poor;

4. the results of the recently suggested beam-column analogy (which incorporates the Pasternak subgrade model) are slightly to significantly better than Winkler's Hypothesis. Thus, the beam-column analogy shows promise as an improved analytical technique compared to Winkler's Hypothesis, at least on an interim basis; and,

5. of the subgrade models considered, the RSC consistently provided the best agreement between observed and calculated settlements. This is consistent with conclusions based on previously published theoretical work (9).

Structural Effects

Consideration of structural effects, particularly mat-superstructure interaction, are also important in mat analysis. This conclusion is consistent with numerous others (4,20,24). For example, it is believed that even better agreement between observed settlements and calculated results could have been achieved if the superstructure were modeled more accurately than using the simple single-story frame model in the *SSIH* program used for this study.

This is especially true for the Chemistry Building where the hogging pattern of settlement would have transferred more load to Column Line C near the center of the mat, thus flattening the calculated settlement profile more as was observed. Although other structural effects such as change in concrete modulus with time and cracked section behavior did not appear to be major variables in the mats considered, these are known phenomenon that are relatively simple to consider in routine design.

RECOMMENDATIONS

Subgrade Models

The conclusions reached in this paper support previous recommendations (9,12) that implementation of improved subgrade models in routine mat foundation design practice is both highly desirable from consideration of computational accuracy and feasible from practical considerations. The overall recommendation is that a single, reasonably accurate subgrade model be used to calculate all parameters of interest in the design of a mat foundation (moments, settlements, etc.). Within this context, the following specific recommendations are made:

1. use of the Conventional Method of Static Equilibrium ("rigid method") should be discontinued;

2. use the traditional form of Winkler's Hypothesis with $k_W(x)$ assumed constant ($=k_{WO}$) should be discontinued;

3. use the general form of Winkler's Hypothesis with a variable $k_W(x)$ (the pseudo-coupled concept) can produce acceptable results provided the reference analysis used to produce the values of $k_W(x)$ matches the problem of interest in terms of geometry, loading, and mat stiffness. Thus the simple methods of doubling the Winkler coefficient of subgrade reaction at the edges or using a generic variation based on an elastic solution should not be used. Unfortunately, the choices of reasonably accurate pseudo-coupled methods are limited. The Discrete Area Method is conceptually sound and produces good results consistently (24), but requires close coordination between structural and geotechnical engineers. In the writer's opinion, this is too cumbersome for routine practice, especially on smaller projects, and will likely continue to limit its use. This is supported by the fact that this technique has been around for at least 20 years, yet the number of engineers using it in practice appears to be very small;

4. as an interim general-purpose model, the beam-column analogy should be used as it incorporates the Pasternak subgrade model which is the simplest model that inherently incorporates "spring coupling." However, a boundary condition involving $w'(x)$ (the first

derivative of $w(x)$) at the edges of the mat must be dealt with. This issue is discussed in (16). Based on study of this model to date, it is recommended that continuity of $w'(x)$ be assumed. This can be achieved by specifying a zero-column-tension boundary condition at the edges of the mat. It is also recommended that zero horizontal deformation boundary conditions be imposed at each edge of the mat. This is to prevent calculation of fictitious horizontal deformations of the mat of very large magnitude; and,

5. the preceding recommendation should be considered only interim until such time that subgrade models that are consistently more-accurate, such as the Reissner Simplified Continuum, are implemented in structural analysis software available to practicing engineers. Research is currently in progress in this regard.

Structural Analysis

Other details relative to the structural analysis should also be given careful consideration:

1. superstructure interaction effects are, in general, important, and should be included even for relatively modest structures such as described in this paper. Engineers should recognize that the superstructure, mat, and soil subgrade are a single, interactive unit that should be analyzed together. Given the computational capabilities available to engineers, there is no reason why this cannot be a reality on every project; and,

2. attention should be given to considering in mat analysis well-known behavioral aspects of concrete in the mat such as modulus reduction with time and cracked section behavior. These issues are now considered routinely for superstructure concrete, so there is no reason why the mat concrete should not receive similar attention.

Although not evaluated as part of this study, others have evaluated the use of "thick" elements for the mat (in which shear effects are considered) versus the usual "thin" elements in which only simple-beam effects ("plane sections remain plane") are modeled (17). The inclusion of shear in the flexural behavior of a beam effectively makes the beam more flexible. This tends to increase differential settlement and reduce bending moments. The conclusion reached in (17) was that shear effects may be important in some cases. Therefore, it would appear to be prudent to model a mat using "thick" elements if the computer software package used has this capability.

COMMENTARY

The basic recommendation made in this paper, to use a more-accurate subgrade model in mat-analysis practice, is not new. There is a demonstrated need for this as well as practical solutions to this need. However, experience indicates that objections will still be raised to this recommendation. Consequently, specific comments are offered in advance to address these anticipated objections:

1. all recommended subgrade models, including those involving Winkler's Hypothesis, rely on a knowledge of Young's Modulus of the subgrade to calculate the appropriate model parameters. It has been argued that this is essentially impossible to do reliably for soil, so any potential benefit of using an inherently more-accurate subgrade model is lost in the inaccuracy of the soil modulus. Thus there is no reason to change the status quo. While Young's Modulus for soil has always been difficult to estimate accurately, and the calculated results (especially settlement) are sensitive to the value chosen, estimation of Young's Modulus is not a hopeless task. Tremendous advances in soil-testing technology, especially using in-situ testing, have been made in recent years. There is every indication that the reliability and accuracy of these methods will continue to improve in the future as the database of theoretical, calibration-chamber, and case-history knowledge expands. It should, however, be understood that the Young's Modulus of soil will never be a parameter that can be picked reliably from a table or chart in a textbook or handbook as with other engineering materials. It will always require the judgement of an experienced geotechnical engineer who evaluates, on a site-specific basis, the stress history of the subgrade soils and its relation to the stress increase resulting from the proposed mat. In summary, reasonably accurate estimates of Young's Modulus of soil can be made now, and the accuracy will continue to improve in the future. However, it requires the input of an experienced geotechnical engineer; and,

2. a premise stated at the beginning of this paper is that an acceptable subgrade model should be able to produce accurate estimates of at least the primary parameters of interest (moments and settlement) from a single analysis. A counter argument is that there is nothing wrong with performing separate moment and settlement calculations. Corollaries of this argument are a) Winkler's Hypothesis is simple to use and provides an acceptable estimate of moments, so its use should not be abandoned, and b) the moments calculated using the rigid method or Winkler's Hypothesis are conservative and result in "safe" designs, so these methods should still be used in practice. The response to this has several parts:

i. in general, separation of load and deformation analyses is neither necessary nor acceptable in modern structural analysis. As discussed previously, the reasons for separating moment and settlement calculations for mats derives only from a time when it