ON THE HISTORY AND RELIABILITY OF THE FLEXURAL STRENGTH OF FRP REINFORCED CONCRETE MEMBERS IN ACI 440.1R

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Synopsis: The structural reliability of concrete flexural members reinforced with fiber reinforced polymer (FRP) reinforcement is investigated. Reliability indices based on the equations for flexure in ACI 440.1R-03, which uses the load factors from ACI 318-99 are presented. Choice of a resistance factor for flexure for ACI 440.1R-06, which uses the load factors from ACI318-02 is also presented. Flexural designs using either ACI 440.1R-03 or ACI 440.1R-06 provide sufficient reliability, with reliability indices between 3.5 and 4.8, with the older versions of ACI 440.1R yielding higher reliability. An analysis of curvature of the beams at failure showed that flexural members that fail by FRP reinforcement rupture have ductilities similar to those that fail by concrete crushing, indicating that FRP reinforcement fracture is not necessarily a more brittle failure mode than concrete crushing.

Keywords: Ductility, Flexure, FRP, Reliability

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INTRODUCTION

Historically, steel bars primarily have been used to reinforce concrete. Steel reinforcing bars have performed well in many applications, however, where the members are subjected to corrosive environments, primarily road salt or coastal salts, the steel bars corrode. Bridge decks, wastewater treatment plants, and parking garages are a few example structures that have had severe corrosion problems. The chloride ions from sodium and calcium hydroxide, found in de-icing salts in northern climates and sea water along coastal areas, create an excellent environment for corrosion. As steel corrodes, its volume expands and can produce large enough tensile stresses in the concrete to produce cracking and ultimately spalling of the concrete; in addition, the cross-sectional area of the steel decreases and the safety of the member can be compromised.

Several techniques have been developed to prevent corrosion or at least extend the life of steel. Zinc rebar coatings, cathodic protection, corrosion inhibiting admixtures in concrete, and increased concrete cover are a few methods of preventing or inhibiting corrosion (Matlock and Krauss 1990). Another common method, used frequently in bridge decks and parking garages, is to use epoxy coated steel bars (Race 1995). The epoxy coatings are often times chipped, scratched, and damaged in the field, which can lead to accelerated corrosion in the damaged areas (De Girorgi 1993). Premature corrosion of epoxy-coated bars has been discovered and has led to skepticism about their long-term performance (Ehsani et al. 1996).

Fiber reinforced polymer (FRP) reinforcing bars provide another solution to providing long-lasting reinforcement for concrete. The FRP bars are corrosion resistant throughout the entire depth of the bar. FRP bars are currently being used in bridge decks and other structural elements where corrosion can cause damage. In the United States of America, design of concrete structures reinforced with FRP reinforcing bars is largely based on documents produced by ACI Committee 440. Committee 440 produces a design guideline: ACI440.1R, first published in 2001 with updates in 2003 and 2006 (ACI Committee 440 2001, 2003, 2006), a materials specification: ACI 440.6 (ACI Committee 440 2008a) published in 2008 and a construction specification: ACI 440.5 (ACI Committee 440 2008b) published in 2008.

RESEARCH SIGNIFICANCE

As the field of FRP reinforced concrete matured, Committee 440 has updated the design guideline relying less on committee consensus and more on experimental evidence. When developing ACI440.1R-06, the committee determined that there were a sufficient number of well documented experimental results to perform a reliability analysis and calibrate the resistance factors for flexure in a manner similar to the calibration of the resistance factors for ACI 318. This paper presents the results of a determination of the flexural reliability of structures designed using ACI 440.1R-03 and presents the development of the resistance factors for flexure found in ACI440.1R-06.

ACI 440.1R FLEXURAL STRENGTH EQUATIONS

One of the main differences between steel and FRP is that FRP is linear elastic up until failure, whereas steel yields. This difference means that unlike steel reinforced concrete, concrete crushing is not the only form of flexural failure for FRP reinforced concrete, it is also possible for the FRP reinforcement to fracture. Flexural capacity is determined by one of two different equations in ACI 440.1R depending on the expected governing mode of failure: concrete crushing or reinforcement fracture. The anticipated failure mode can be determined by comparing the FRP reinforcement ratio to the balanced reinforcement ratio. The balanced reinforcement ratio occurs when concrete crushing and FRP reinforcement rupture happen simultaneously and is given by

$$\rho_{fb}=0.85\beta_1\frac{f_c'}{f_{fu}}\frac{E_f\varepsilon_{cu}}{E_f\varepsilon_{cu}+f_{fu}}\ , \tag{1}$$

where f_c' is the concrete compressive strength, E_f is the modulus of elasticity of the FRP reinforcment, β_1 is the factor relating depth of the equivalent rectangular compressive stress block to the neutral axis depth, and f_{fu} is the design tensile strength of the FRP bar considering reductions for environmental conditions (i.e. $f_{fu} = C_E f_{fu}^*$, where f_{fu}^* is the guaranteed immediate tensile strength of the FRP bar and C_E is the environmental reduction factor).

When the FRP reinforcement ratio is greater than the balanced reinforcement ratio, concrete crushing is the predicted failure mode and the stress distribution in the concrete can be approximated with the ACI rectangular stress block. In this case, the stress in the FRP bar is less than f_{fu}, but can be found from strain compatibility, with the flexural strength given by

$$M_n = A_f f_f\left(d-\frac{1}{2}\frac{A_f f_f}{0.85 f_c' b}\right), \tag{2}$$

where

$$f_f = \sqrt{\frac{\left(E_f\varepsilon_{cu}\right)^2}{4}+\frac{0.85\beta_1 f_c' E_f\varepsilon_{cu}}{\rho_f}} - 0.5E_f\varepsilon_{cu} \le f_{fu},$$

and A_f is the FRP reinforcement area, d is the effective depth of the beam, b is the width of the beam, and ρ_f is the FRP reinforcement ratio.

When the FRP reinforcement ratio is smaller than the balanced reinforcement ratio, FRP reinforcement rupture is the predicted failure mode and the ACI rectangular stress block may not be applicable because the maximum concrete strain (0.003) may not be achieved. In this case, the stress in the FRP reinforcement at failure is f_{fu} and the resulting net tension and hence compression is known ($T = C = A_f f_{fu}$). However, the location of the centroid of the compression is unknown and hence the moment arm is unknown. ACI 440.1R recommends that the moment arm at balance be used as a lower bound estimate yielding a nominal moment capacity of

$$M_n = A_f f_{fu}\left(d - \frac{\beta_1 c_b}{2}\right), \tag{3}$$

where c_b is the location of the neutral axis at the balanced condition given by

$$c_b = \left(\frac{\varepsilon_{cu}}{\varepsilon_{cu}+\varepsilon_{fu}}\right) d, \tag{4}$$

where ε_{fu} is the strain corresponding to f_{fu}.

In versions of 440.1R prior to 2006, Eqn. (3) was multiplied by 0.8 and appeared as

$$M_n = 0.8 A_f f_{fu}\left(d - \frac{\beta_1 c_b}{2}\right), \tag{5}$$

The Committee explained that the 0.8 was added to the equation to better fit the data. With the reliability calibration of the equation for the 2006 edition, the Committee felt it was better to have this constant reflected in the calibrated resistance factor, so Eqn. (3) is the version of the equation found in 440.1R-06.

ACI 440.1R-03 RESISTANCE FACTORS

The ACI design guideline (ACI 440.1R) is based on load and resistance factor design (LRFD) using factored loads to determine the demand for the ultimate strength limit states and reducing the nominal capacity by a resistance factor so that for flexure,

$$\phi M_n \geq M_u, \tag{6}$$

where M_n is the nominal flexural capacity given by Eqns. (2), (3), or (5) depending on the expected failure mode and version of ACI 440.1R, ϕ is the resistance factor for flexure and M_u is the factored ultimate moment based on the load factors from ACI 318-99 (ACI Committee 318 1999) or ACI318-02 (ACI Committee 318 2002) for ACI 440.1R-03 and ACI 440.1R-06, respectively. As mentioned previously, the resistance factors in ACI 440.1R-03 were not based on a reliability analysis, but instead, were based on committee consensus. Because flexural members reinforced with FRP do not exhibit traditional ductile behavior, the Committee sought to implement conservative resistance factors to provide higher reserve strength.

ACI 318-99 specifies a resistance factor of 0.7 for steel reinforced concrete members that fail by concrete crushing prior to reinforcement yielding. The committee consensus was that because the reinforcing material is not the weak link in a concrete beam that fails by concrete crushing, it should not matter if the beam is reinforced with steel or FRP. Therefore, Committee 440 set the resistance factor for FRP reinforced concrete beams failing by concrete crushing to 0.70; the same resistance factor as steel reinforced concrete beams failing by concrete crushing in ACI 318-99. When developing ACI 440.1R-03, ACI Committee 440 believed that failure by concrete crushing would be more ductile than failure by FRP reinforcement rupture; therefore, the committee picked a smaller resistance factor, 0.5, for flexural failure by FRP reinforcement rupture.

To transition between the two flexural strength equations (Eqns. (2) and (5)) and associated two different resistance factors, ACI 440.1R-03 calls for an interpolation of the resistance factor when the reinforcement ratio is between 1 and 1.4 times the balanced reinforcement ratio,

$$\phi = \frac{\rho_f}{2\rho_{fb}} \quad \text{for} \quad \rho_{fb} < \rho_f < 1.4\rho_{fb}. \tag{7a}$$

This definition for the resistance factor in combination with the resistance factors for concrete crushing controlled failures

$$\phi = 0.7 \quad \text{for} \quad \rho_f \geq 1.4\rho_{fb} \tag{7b}$$

and FRP reinforcement rupture failures

$$\phi = 0.5 \quad \text{for} \quad \rho_f < \rho_{fb} \tag{7b}$$

define the resistance factors for all cases. The 0.8 factor in Eqn. (5) causes a discontinuity in the nominal flexural capacity (and hence the design flexural capacity) at the balanced reinforcement ratio. If a designer switches from the reinforcement rupture formula, Eqn. (5) to the concrete crushing formula, Eqn. (2) right at the balanced reinforcement ratio. Many designers used the minimum of Eqns. (2) and (5) when the reinforcement ratio was between the balanced ratio and 1.4 times the balanced ratio. ACI 440.1-06 avoids this problem with the removal of the 0.8 factor.

PROFESSIONAL FACTORS FOR FLEXURE

The accuracy of the nominal flexural capacity equations, Eqns. (2), (3) and (5), was obtained by comparing experimentally determined results found in the literature with the predictions from the appropriate equation. A literature search identified 181 tests of FRP reinforced concrete beams tested to flexural failure by 19 different researchers. A full description of the database can be found in Gulbrandsen (2005). In order to perform the comparison, the literature needed to document the measured material propertied (f_{fu}^* , E_f, and f_c') as well as the geometrical parameters (b, L, d, and nominal reinforcement size). Sixty-two flexural tests from 9 different research articles contained all of the necessary information (Nakono et al. 1993, Wang et al. 1998, Toutanji and Saafi 2000, Benmokrane et al. 1996, 1997, Li and Wang 2002, Zhao et al. 1997, Nawy and Neuwerth 1971, Pecce et al. 2000).

The 62 tests obtained from the literature covered three different reinforcing materials: glass, aramid and carbon FRP. The tensile strength of the FRP reinforcement varied not only with fiber material, but also with bar size, with smaller bars having apparently larger strengths. Over the three materials, reinforcement strengths from 500 to 2070 MPa were included in the tests. The moduli of elasticity of the reinforcement ranged from 41 GPa (glass) to 150 GPa (carbon). Nominal bar diameters in the database ranged from 3 to 19 mm. The bars used in the experiments had a variety of surface finishes from spirally wrapped to braided to sand coated. The depth of the beams in the database varied from 145 mm to 510 mm, with the majority of beams in the 150 to 255 mm range. The width of the beams in the database varied from 90 to 500 mm, with the majority of the widths between 100 and 200 mm. The depth to width ratios for the beams in the database varied between 0.29 and 2.5, with the majority of the beams falling between 1 and 1.5. The depth to bar diameter ratios of the beams in the database varied between 9.9 and 54.2, with about 40 percent lying between 20 and 30. Ratios of the FRP reinforcement ratio to the balanced ratio ranged from 0.73 to 2 for beams that failed by FRP reinforcement rupture and between 0.93 to 16.36 for beams that failed by concrete crushing. It is important to note that reinforcement rupture failures occurred at reinforcement ratios up to 2 times the balanced reinforcement ratio. A total of 12 beams in this study with $\rho_f \geq 1.4\ \rho_{fb}$ failed by reinforcement rupture. The concrete compressive strengths for the beams in the database varied between 22.8 and 75.8 MPa.

The bar size distribution was biased, with the distribution for tests that failed by reinforcement rupture; approximately eighty five percent of the tests that ended in FRP reinforcement rupture used FRP reinforcement that was smaller than or equal to a No. 3 bar. The bar size distribution for the concrete crushing failures was more evenly distributed; the bar size used most frequently was a No. 5 bar and was used in approximately thirty five percent of the tests.

The professional factors (i.e. the mean value of the test-to-predicted ratio) were determined independently for beams that failed by FRP reinforcement rupture and beams that failed by concrete crushing. The mean value for the ratio of the test-to predicted flexural strength for all of the beams that failed by FRP reinforcement rupture was 1.11 based on Eqn. (5) and 0.89 based on Eqn. (3). The coefficient of variation was 16% for both equations. The mean value for the ratio of the test-to-predicted flexural strength for all the beams that failed by concrete crushing was 1.19 based on Eqn. (2) with a coefficient of variation of 16%.

RELIABILITY ANALYSIS

Load and resistance factor design has its basis in the underlying assumption that loads and resistances are random variables (Nowak and Collins 2000). Statistics on these random variables can be used to predict the reliability of structural members. The reliability of a member is determined by the probability of the load effect exceeding the resistance of the member. As the probability of the load effect exceeding the resistance decreases, the reliability increases. Because there is always some chance of failure, the resistance cannot always exceed the load effect. The basic equation to avoid failure or achieve safety is given by

$$R \geq Q \tag{8}$$

where R is the resistance and Q is the load effect (Nowak and Collins). The resistance depends on the material properties and fabrication procedures. The predicted load effect depends on load models and analysis assumptions. The structure is safe (desired performance) if the resistance exceeds the load effect. Conversely, the limit state is exceeded and failure occurs when the resistance is less than the load effect. The limit state function $g(R,Q)=R-Q$ can be expressed in terms of reduced variables

$$g\left(Z_R, Z_Q\right) = \mu_R + Z_R\sigma_R - \mu_Q - Z_Q\sigma_Q, \tag{9}$$

where Z_R and Z_Q are termed the reduced variables, μ_R and μ_Q are the means of the resistance and load effect, respectively, and σ_R and σ_Q are the standard deviations of the resistance and load effect, respectively (i.e. $R = \mu_R + Z_R$ and $Q = \mu_Q + Z_Q$). For any specific value of $g(Z_R, Z_Q)$, Eqn. (9) represents a straight line in the space of reduced variables Z_R and Z_Q. The line of interest for reliability analysis is the line that corresponds to $g(Z_R, Z_Q) = 0$ because it separates the domains representing safety and failure. The reliability index can be defined as the shortest distance from the origin of reduced variables to the line $g(Z_R, Z_Q) = 0$ (Nowak and Collins 2000) . Using geometry illustrated in Fig. 1, the shortest distance between $g(Z_R, Z_Q) = 0$ and the origin is

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}}, \tag{10}$$

where β is the inverse of the coefficient of variation of the function $g(Z_R, Z_Q) = 0$ when R and Q are uncorrelated. For normally distributed random variables R and Q, it can be shown that the reliability index is related to the probability of failure by

$$P_f = \varphi(-\beta), \tag{11}$$

where φ represents the value of the cumulative distribution function (CDF) of a normal random variable (Nowak and Collins 2000).

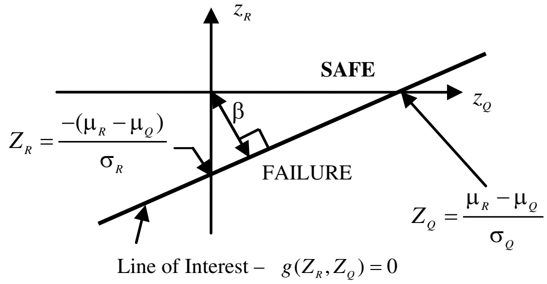

Figure 1 Reliability Index

If both R and Q are continuous random variables, then each has a probability density function (PDF). Moreover, the quantity R-Q is also a random variable with its own PDF. Figure 2 displays the PDF's of all three random variables, safety margin, load effect, and the resistance. The reliability index (β) depends only on the means and standard deviations of the random variables. Therefore, β is called a second moment measure of structural safety because only the first two moments (mean and variance) are required to calculate β. If the random variables are all normally distributed and uncorrelated, and the limit state function is linear, Eqns. (10) and (11) are exact. Otherwise, the equations provide an estimate of the reliability index and the probability of failure (Novak and Collins 2000).

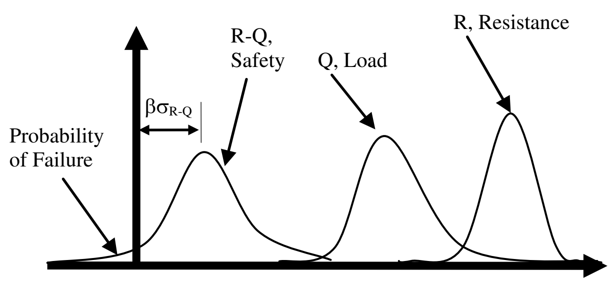

Figure 2 *PDF's of the Safety Margin, Load Effect, and Resistance*

There are two typical procedures used to determine the reliability index: 1) the Hasofer-Lind method (Hasofer and Lind 1974) to determine the design point (i.e. the point on the line $g(Z_R, Z_Q)$) that minimizes the distance from the line $g(Z_R, Z_Q)$ to the origin or 2) Monte Carlo simulation to determine the probability of failure of a given design. Using Monte Carlo simulation, the results from previous testing and the known distributions for the material and geometrical properties can be used to establish the probability density function characteristics to generate samples of numerical data which can reduce the uncertainty required to obtain the desired reliability estimate, and correspondingly, the probability of failure (Nowak and Collins 2000). Estimating relatively small probabilities of failure using Monte Carlo simulation while limiting the uncertainty in the estimate, requires a large number of simulations to be conducted. The required sample size depends on the desired coefficient of variation and the relative magnitude of the probability to be estimated (Nowak and Collins 2000). For the types of probabilities typical for structural design a large number of simulations (5,000,000) need to be performed for each trial design to determine an estimate of the reliability of that design. Whether using the Hasofer-Lind method or Monte Carlo simulation, the means and coefficients of variation for all of the random variables (i.e. material properties, geometrical properties, and loads) must be known.

Determination of Statistical Parameters

The means and coefficients of variation for the area of GFRP reinforcement, design GFRP tensile strength, f_{fu}^* (varied according to bar size), and the design GFRP modulus of elasticity were calculated from reports of over 400 bar tests provided by FRP manufacturers. Nominal (design) values for each of these parameters were obtained from the FRP bar manufacturers. The means and coefficients of variation for the width of the beams, depth of the beams, concrete compressive strengths, dead load moments, and live load moments were taken from Nowak and Szerszen (2003a) and are the same values as those used to calibrate the resistance factors in ACI 318-02. Biases for all of these parameters were calculated by taking the mean of the tested quantity divided by the nominal value of that quantity for each parameter. The values of the statistical parameters used in the reliability study are shown in Table 1.

Table 1: Bias and Coefficient of Variation for Parameters used in Reliability Analysis

Parameter	**Bias**	**COV**
Eq. 3	0.89	0.16
Eq. 5	1.11	0.16
Eq. 2	1.19	0.16
A_f	1.00	0.03
f_{fu}^* (# 3)	1.18	0.12
f_{fu}^* (# 5)	1.20	0.08
f_{fu}^* (# 6)	1.22	0.07
f_{fu}^* (# 7)	1.12	0.05
f_{fu}^* (# 8)	1.06	0.04
f_{fu}^* (# 9)	1.13	0.05
B	1.01	0.04
d	0.99	0.04
E_f	1.04	0.08
f'_c	1.24	0.10
Dead Load	1.05	0.10
Live Load	1.00	0.18

Three of the limit state equation variables, the environmental service factor, C_E, the material property to define the location of the neutral axis from the depth of the compression block, β_1, and the ultimate compressive concrete strain, ε_{cu}, were considered to be deterministic. The latter two variables were considered to be deterministic also in the calibration of ACI 318-02. The environmental service factor is a non-calibrated coefficient that accounts for long-term exposure based on limited quantifiable data; therefore, the reliability analysis was conducted such that C_E was deterministic.

Design Space for Reliability Indicies

Reliability indices were determined using the two methods discussed above (Hasofer-Lind and Monte Carlo simulation) for twenty different beam designs using the design equations of ACI 440.1R-03 to determine the reliability of flexural members designed using that guideline. Additionally, another twenty beam designs were used to determine calibrated resistance factors for the new load factors and design equations appearing in ACI 440.1R-06. In each case, the FRP reinforced concrete design beams reinforced with only tensile reinforcement were designed according to either ACI 440.1R-03 or ACI 440.1R-06. For the designs based on ACI 440.1R-03, when the FRP reinforcement ratio, ρ_f, was between the balanced reinforcement ratio, ρ_{fb}, and 1.4 ρ_{fb}, the minimum of Eqns. (2) and (5) was used. For the designs based on ACI 440.1R-06, trial resistance factors or 0.5, 0.55 and 0.6 were each evaluated, with the goal of determining resistance factors that would yield a reliability index between 3.5 and 4 for the new load factors and equations in ACI 440.1R-06.

The beams were designed with common design criteria such as, keeping the width to height ratio between one-third and one, using a live to dead load ratio of one, two, or three, setting beam lengths from 3 meters to 9 meters, and using a concrete compressive strength of 28 MPa. All beams were designed with one layer of tensile reinforcement in the bottom of the beam, an environmental service factor of 0.80, two or more FRP reinforcement bars, an ultimate concrete compressive strain of 0.003, and a superimposed dead load from approximately 1.5 kN/m up to approximately 4.4 kN/m varying according to the length of the design beam. The three, six, and nine meter long beams were designed with approximately 1.5, 3, and 4.4 kN/m uniform dead loads, respectively. However, these values were adjusted to achieve the desired live to dead load ratio. Beams were designed with several sizes of GFRP reinforcing bars with a modulus of elasticity of 39.4 GPa including No. 3 bars, No. 5 bars, No. 6 bars , No. 7 bars, No. 8 bars and No.9 bars. The FRP bar design tensile strength f_{fu}^* varied according to bar size. Table 2 shows the FRP design strength (f_{fu}^*) for bar sizes from No. 3 up to No. 9 bar. Table 3 lists the geometrical properties for all 20 beam designs. The geometrical and material properties of the beams were not changed between the ACI440.1R-03 and ACI440.1R-06 analyses, instead, the applied loads were adjusted for the two cases to satisfy exactly M_u equal to ϕM_n. The environmental service factor (C_E) was chosen as 0.8 for GFRP bars not exposed to earth and weather from Table 7.1 in ACI 440.1R-03. The concrete compressive design strength was not varied in the beam designs. FRP reinforced concrete is most economical at lower concrete compressive strengths. By limiting the concrete compressive strength to 28 MPa, it was possible to get a fairly even distribution between FRP reinforcement rupture and concrete compression failures in the beams designed. Serviceability limit