Microcomputer-Aided Design of Reinforced and Prestressed Concrete Elements

by F. C. Filippou

Synopsis: The potential advantages of using electronic spreadsheets in advanced reinforced and prestressed concrete design are presented. The power of this tool is demonstrated through a series of examples which include the moment-curvature relation of reinforced and prestressed concrete beams, the axial load-uniaxial and biaxial bending moment interaction diagram of R/C sections and the tendon layout of simply supported beams prestressed with draped or straight tendons.

Keywords: axial loads, beams (supports), bending moments, microcomputers, prestressed concrete; reinforced concrete; structural design

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INTRODUCTION

The last ten years have seen a dramatic change in the way engineers perform their work. The most important factor contributing to this change was the introduction of microcomputers into the market in the late 70's. A number of related factors led to the rapid spread of microcomputers in engineering offices of any size:

(1) the low cost of microcomputers afforded small engineering offices access to the new technology; the large market has led to a technological race which further reduced prices and dramatically increased performance of the typical workstation

(2) the development of powerful software packages which are easy to learn encouraged non-specialists to use the new technology. The typical engineer cannot afford to spend a large portion of his time learning new tools which in a few years will become obsolete.

(3) microcomputers help the average engineer in a variety of tasks; these include proposal and report preparation, calculations, budget preparation, scheduling, drafting, presentation etc. The lack of software on mini-and mainframe computers makes many of these tasks difficult, if not impossible, to perform.

The widespread use of microcomputers by professional engineers and their appearance in large numbers on university campuses necessitates their introduction in graduate and undergraduate courses. This fact presents the instructor with the challenge of adjusting course content and presentation to take advantage of the new technology. The use of microcomputers as a learning tool is, however, not an easy task. The pitfalls of the project have led many educators to rejecting microcomputers as a learning tool altogether and adhering to the old and established ways of teaching engineering principles.

The work presented in this paper attempts to show the tremendous potential of the use of electronic spreadsheets in graduate and undergraduate courses of structural design. To demonstrate the potential of this tool attention will be focused on advanced applications which are suited for graduate level courses on behavior and design of structural elements.

ADVANTAGES OF ELECTRONIC SPREADSHEETS

Microcomputers appeared in the market in the late 70's. Electronic spreadsheet programs followed in their footsteps. Some observers of the microcomputer world argue that the first electronic spreadsheet program VisiCalc launched the entire microcomputer industry.

Electronic spreadsheets can find many applications in the area of structural engineering. The integration of a very powerful electronic calculator with database functions and easy-to-use graphics appears to be especially appealing in structural design. Engineering calculations are often organized in tabular fashion which is precisely the structure that lies at the heart of electronic spreadsheets. The power of these software packages is combined with great flexibility of use allowing the designer to readily develop procedures to fit his special needs. Modification of existing spreadsheets is extremely easy, since the spreadsheet structure is transparent to the user.

These characteristics make electronic spreadsheets an ideal tool in the instruction of structural design. While teaching tools are available in structural analysis [1], no such tools have been developed to date for structural design.

To be successful as a teaching tool a program should satisfy the following requirements:

(1) the amount of time the student spends learning the details of the program should be kept to a minimum; this amount of time should enable the student to actively interact with the program by modifying portions of it or developing new features and solution schemes;

(2) the program should be based on a language that is easy to learn and use; most engineering tasks are based on simple functions and solution schemes;

(3) the program organization should resemble that of the engineering notepad; this facilitates going back and forth between hand and computer calculations

Electronic spreadsheets satisfy all these requirements. Some of the most striking features are:

(1) ease of programming of complex formulas which include scientific functions and logical operators

(2) integration of calculation of complex formulas with easy-to-use graphics and basic database concepts; this corresponds to the manual "calculate and look-up" activity of design engineers; the integration of graphics into the spreadsheet allows the student to quickly check his calculations and at the same time enhances his understanding of the interrelationship between key variables of the particular design problem

(3) the spreadsheet is transparent to the user and can be modified with great ease; this satisfies the need for flexibility in the solution of design problems; at the same time it allows the student to follow the logical flow of the solution process in a spreadsheet that he did not develop himself

(4) the program does not have to be compiled and linked before it can be executed; execution rather takes place immediately; any syntax errors are flagged as they occur reducing the frustrations associated with debugging the program

(5) electronic spreadsheets are geared towards resources that all engineers will possess in the very near future; this fact makes their impact immediate; they illustrate to students the power of computers in a very direct way, since the organization and internal structure of electronic spreadsheets very closely resembles that of manual calculations

(6) electronic spreadsheets allow the user to investigate the effect of variation of key parameters on the solution of the problem under study; this can be done with a few keystrokes; this should be an important ingredient of instruction tools for structural design

(7) most electronic spreadsheets are very user-friendly with extensive on-line help facilities; this helps allay the fear of inexperienced users towards microcomputers.

In developing the spreadsheets presented in this paper it was decided to adopt Lotus 1-2-3, because of its features and its widespread use in the business world. Since most of today's successful electronic spreadsheet programs have features similar to those of Lotus 1-2-3, the same spreadsheets can be developed using other packages.

MOMENT-CURVATURE OF REINFORCED AND PRESTRESSED CONCRETE SECTIONS

General

The development of a procedure to derive the moment-curvature relation of reinforced (RC) and prestressed concrete (PC) sections is an important part of any advanced graduate course on reinforced or prestressed concrete design. Understanding the mechanisms that contribute to the characteristic nonlinear behavior of RC sections and being able to assess the influence of important parameters such as reinforcing ratio, amount of compression steel and mechanical properties of reinforcing steel and concrete on the ultimate moment capacity and the curvature ductility of the section is an important first step towards grasping the actual behavior of reinforced and prestressed concrete elements and the approximations involved in the procedures proposed in current design specifications. A thorough investigation of the moment-curvature relation becomes an indispensable tool in connection with the philosophy of capacity design.

In this section the basic procedure for determining the moment-curvature relation of R/C sections will be presented. It is proposed that students enrolled in a basic graduate course on reinforced concrete design be asked to develop a spreadsheet which incorporates this procedure. In this way they will become familiar with the fundamental concepts exactly as they would, if they were asked to solve the same problem by hand.

Following the introduction of the basic spreadsheet the enhancements which form part of spreadsheet **MOMCURV** will be described. These enhancements include the effect of confinement through stirrup-ties or hoops on the stress-strain relation of concrete, the buckling of reinforcing bars under compression, the effect of the slab and the modeling of the uncracked section behavior. Most importantly, the enhanced spreadsheet incorporates an automatic iteration procedure for finding the location of the neutral axis at each load step and culminates with the graphic representation of the entire moment-curvature relation of the section.

The spreadsheet **MOMCURV** determines the moment-curvature relation of rectangular, T and I-shaped reinforced and prestressed concrete sections. It is capable of accurately representing the load-deformation behavior of RC and PC sections for the entire load range from zero moment up to failure, which is typically initiated by crushing of the cover concrete and buckling of the compression steel. Fracture of the tensile reinforcing steel occurs only in very lightly reinforced sections and T-beams with very wide flanges. The spreadsheet follows the behavior of the section through the uncracked and cracked stages of response. While the uncracked section behavior is of limited value in the case of reinforced concrete sections, it is of great importance in the study of prestressed concrete sections which are typically designed to remain uncracked under service loads.

The spreadsheet **MOMCURV** can then be used for a number of studies of the effects of various parameters on the moment-curvature relation of RC and PC sections. Depending on time constraints in the course these studies can be conducted by the students or provided by the instructor as a set of lecture notes.

It it important to emphasize that the approach outlined above ensures that the student concentrates almost entirely on the concepts behind the derivation of the moment-curvature relation exactly as he would do in the case of hand calculations. His involvement in the refinements of spreadsheet **MOMCURV** or the numerical procedures which automate the calculations is limited to a minimum. Since he has been exposed to the basic concepts, he is, however, able to appreciate the details of the enhanced spreadsheet.

Basic procedure

The procedure for establishing the moment-curvature relation of RC sections is well known and is described in a number of graduate textbooks of reinforced concrete design [2]. It will be presented below for the case of rectangular RC sections which are subjected to an axial load P. In the case of girders with no axial load P is set equal to zero. It is assumed that the section is cracked and that a single material model is used to describe the stress-strain relation of concrete (Fig. 1). The basic assumptions of the bending theory of reinforced concrete sections such as plane sections remaining plane after deformation and the complete compatibility of strains between reinforcing steel and surrounding concrete at the same distance from the neutral axis are adopted here.

The procedure consists of the following steps:

(1) At a given curvature ϕ the depth of the neutral axis c measured from the top of the section is assumed.

(2) The compression zone is subdivided into n layers of typically equal height Δx (Fig. 2).

$$\Delta x = \frac{c}{n} \quad (1)$$

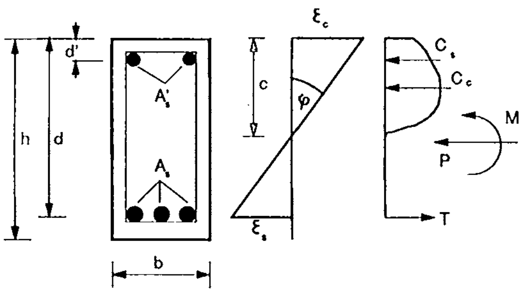

Fig. 1 **Distribution of stresses and strains in a rectangular R/C section subjected to bending moment M and axial load P**

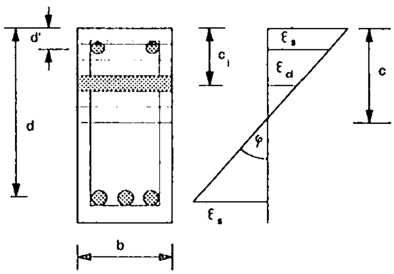

Fig. 2 Subdivision of compression zone in concrete layers

(3) Based on the curvature ϕ and the assumed neutral axis depth c the strain at the center of gravity of each concrete layer is established

$$\varepsilon_{ci} = \phi \cdot (c - c_i) \tag{2}$$

where c_i is the distance of the center of gravity of concrete layer i from the top of the section.

(4) The strain in each layer of reinforcing steel is established in exactly the same way. For the bottom layer of reinforcing steel this yields

$$\varepsilon_s = \phi \cdot (d - c) \tag{3}$$

For the top layer we have

$$\varepsilon'_s = \phi \cdot (c - d') \tag{4}$$

(5) The stress in each concrete layer is established based on the strain determined in (3) and the concrete stress-strain relation

$$f_{ci} = f(\varepsilon_{ci}) \tag{5}$$

(6) The stress in each reinforcing steel layer is established based on the strain determined in (4) and the steel stress-strain relation. For the bottom steel this yields

$$f_s = g(\varepsilon_s) \tag{6}$$

For the top steel we have similarly

$$f'_s = g(\varepsilon'_s) \tag{7}$$

(7) The concrete compressive force is calculated by summing up the contribution of all concrete layers

$$C_c = \sum_i f_{ci} \cdot \Delta x \cdot b \tag{8}$$

where b is the width of each layer and is here assumed constant

(8) The force in the reinforcing steel layers is calculated by multiplying the steel stress in the corresponding layer by the steel area. For the bottom steel

$$T = f_s \cdot A_s \tag{9}$$

and for the top steel

$$C_s = f'_s \cdot A'_s \tag{10}$$

(9) The summation of all forces from steps (7) and (8) should equal the externally applied axial force P. In the first iteration this is unlikely to be the case. The difference establishes a residual or axial force unbalance P_{unb} which is used in an iterative scheme based on the bisection method to achieve convergence to the correct neutral axis depth c.

$$C_c + C_s - T - P = P_{unb} \tag{11}$$

In Eq. (11) all terms represent the absolute value of the corresponding force with the exception of the externally applied axial load P which is considered positive if compressive and negative if tensile.

(10) Once the correct neutral axis location is established, the moment resisted by the section at the specific curvature can be determined by summing up the contribution of all concrete and reinforcing steel layers about a convenient reference point. The contribution of the externally applied axial load P should also be accounted for. Taking moments about the bottom reinforcing steel layer yields

$$M = \sum_i f_{ci} \cdot b \cdot \Delta x \cdot (d - c_i) + C_s \cdot (d - d') - P \cdot (h - y_p) \tag{12}$$

where y_p is the distance of the plastic centroid from the bottom fiber of the section. The applied axial load P is assumed positive, if it is compressive, and all other terms involve the absolute value of the corresponding forces.

Implementation in Lotus 1-2-3

The basic procedure outlined above is relatively simple to implement in an electronic spreadsheet. Following the concept of tabular organization which lies at the heart of electronic spreadsheets the student must clearly identify the basic steps of the process, since each step has to be contained in a single cell of the spreadsheet. In addition, he has to translate the entire procedure into a tidy and readable spreadsheet. The spreadsheet is subdivided into two basic parts: the first part contains the input data of the problem while the second part is made up of the cells which calculate the forces and moments associated with all steel and concrete layers. The data of all concrete layers which make up the compression zone of the section are represented in a series of columns containing the layer number, the distance of the center of gravity from the top fiber, the strain, the stress, the force and the moment contribution of each concrete layer.

An interesting aspect of the implementation of the basic procedure is the description of the material models for reinforcing steel and concrete. These will be described in more detail later.

The automatic iteration process which establishes the neutral axis depth at each load step presents a serious challenge. This can only be accomplished with the aid of the macro command language of Lotus 1-2-3. It is not expected that the students will be interested in devoting time to such a task in a course concerned with the behavior and design of structural elements. The automatic iteration process should, therefore, be provided by the instructor.

Since the purpose of the first homework assignment related to the moment-curvature relation of R/C sections is to stress the fundamental concepts, the students should only be asked to determine a few characteristic points of the M-ϕ diagram by trial and error. There is, therefore, no need for the automatic iteration procedure at this stage. The trial and error approach has the added advantage of providing insight into the nonlinear behavior of R/C sections. It is this ability of electronic spreadsheets to operate in different modes from complete automation down to an entirely interactive mode that makes them great tools of computer-aided instruction.

Moment-curvature of rectangular, T- and I-shaped RC and PC sections

The basic model for establishing the moment-curvature relation of RC sections described in the previous section is satisfactory from the educational standpoint, since it conveys the basic concepts behind the problem. It has, however, severe limitations with regard to the actual load-deformation behavior of typical RC sections.

First and foremost, typical RC sections include various amounts of transverse reinforcement which significantly alters the stress-strain behavior of confined concrete in the strain softening range past the maximum strength. This aspect is very important in the study of the ultimate moment capacity and curvature ductility of plastic hinge regions in beams and columns.

Secondly, most reinforced concrete floor girders are cast monolithically with the slab resulting in T-shaped sections in the positive moment region of the girder span.

With respect to prestressed (PC) and partially prestressed concrete (PPC) sections the following additional points need to be taken into consideration:

(1) The precracking behavior of the section becomes very important, since most prestressed concrete girders are designed to remain uncracked under service loads.

(2) The initial strain due to prestressing should be accounted for.

(3) Prestress losses due to creep, shrinkage, relaxation of prestressing steel and elastic shortening at transfer of prestress should be accounted for.

(4) Commonly used shapes include I-shaped girders in precast prestressed construction and box girders in post-tensioned bridge construction.

All these important factors have been taken into account in the development of spreadsheet **MOMCURV**. The spreadsheet **MOMCURV** determines the moment-curvature relation of rectangular, T- and I-shaped reinforced and prestressed concrete sections. The spreadsheet follows the behavior of the section through the uncracked and cracked stages of response up to failure which is usually initiated by crushing of the concrete cover and buckling of the compression steel. It takes into account the effect of transverse steel in confining the concrete enclosed by stirrup-ties or hoops. It includes accurate models describing the monotonic stress-strain relation of reinforcing and prestressing steel and accounts for the inelastic buckling failure of reinforcing bars under high compressive strains. More details of **MOMCURV** are presented in [3].

Material models

Reinforcing steel-- The material model describing the monotonic stress-strain behavior of reinforcing steel consists of a linear elastic portion, a plastic yield plateau and a strain hardening portion up to the fracture strain ε_u. A parabolic variation of stress with strain is assumed between the strain at the onset of strain hardening ε_h and the strain at fracture (Fig. 3).

To accurately represent the behavior of reinforcing steel in compression the effect of buckling of the bars under compression should be taken into account. The inclusion of this effect in instructional software serves the purpose of demonstrating to the student the important role that transverse reinforcement plays in not only confining the concrete, but also restraining the compression reinforcement against buckling. The onset of buckling of the compression steel cannot be represented with a great degree of accuracy, since it depends on a number of factors that are difficult to assess. In the spreadsheet **MOMCURV** it is assumed that the effective length of the reinforcing bars which are laterally supported by stirrups is equal to the stirrup spacing s. Then the critical buckling stress f_{cr} is given by the following relation:

$$f_{cr} = \frac{E_t \cdot \pi^2}{(s/r)^2} \tag{13}$$

where E_t is the tangent modulus of the steel stress-strain relation. To simplify the calculation and because of the inherent uncertainty of the problem an average strain hardening modulus is assumed in calculating the buckling stress. Thus

$$E_t = \frac{(f_u - f_y)}{(\varepsilon_u - \varepsilon_h)} \tag{14}$$

r in Eq. (13) is the radius of gyration of the reinforcing bar and is equal to 0.25 times the bar diameter.

Prestressing steel-- The stress-strain relation of prestressing steel does not exhibit a well-defined yield point as do most reinforcing steels. As a result, the stress-strain relation presented in the previous section for mild reinforcing steel cannot adequately represent the behavior of prestressing steel. Based on previous studies and the recommendations of Naaman in [4] the model proposed by Menegotto and Pinto was adopted in the spreadsheet **MOMCURV**. The model has the great advantage that the stress is an explicit function of the strain. Since the strain in the prestressing steel is known for a given curvature and an assumed depth of the neutral axis, the stress can be directly established and no iterations are needed. More details are presented in [3].

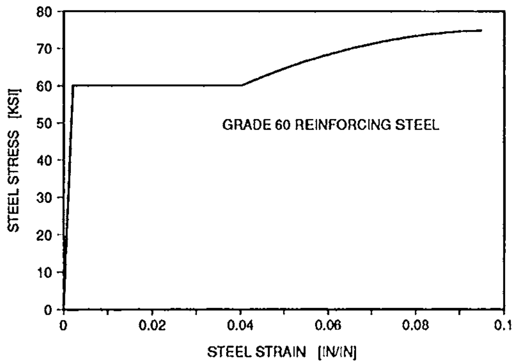

Fig. 3 Material model for reinforcing steel

Concrete-- Many models representing the monotonic stress-strain behavior of plain concrete in uniaxial compression have been proposed in the past [2]. Relatively few models have been proposed for confined concrete. The latter models attempt to account for the effect of longitudinal and transverse reinforcement on the stress-strain relation of concrete which lies within the stirrup-ties or hoops. The model originally proposed by Kent and Park [2] and later modified by Scott, Priestley and Park [5] includes only the effect of transverse reinforcement on confinement. The confinement provided by longitudinal reinforcement in conjunction with transverse reinforcement is not taken into account.

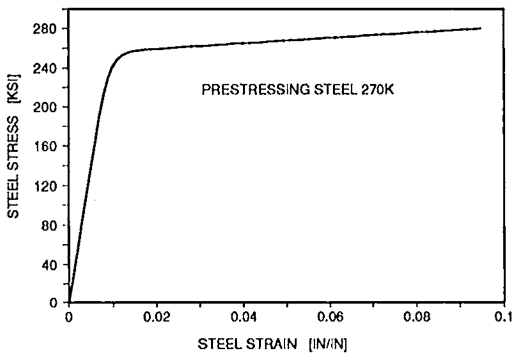

Fig. 4 Material model for prestressing steel

The accurate representation of the effect of confinement on the monotonic stress-strain relation of concrete is of great importance in the study of the load-deformation response of potential plastic hinge regions in beams and columns. By incorporating a confined concrete model in the moment-curvature spreadsheet realistic estimates of the actual moment capacity and curvature ductility of the section can be made.