to-member, site-to-site, and plant-to-plant variability.

Care must be taken in this uncertainty assessment, because portions of this uncertainty V_M are effectively the result of code simplifications rather than inherent or "unexplainable" dispersion. For example, because the code does not chose to specify the mean yield stress f_y as a function of bar diameter, this V_M must "cover" the well documented systematic increase in yield strength with decreasing bar diameter. If a sample of bars are to be used to estimate the mean (f_y) and COV (V_M) of yield stress, their estimated values will depend on p_i, the fraction of bars of diameter i contained in the sample. In the extreme, for example, if half the bars are very small diameter (with high mean stress) and half are very large diameter (with low mean stress), the composite COV will be very large. In fact, the composite mean and COV are simply related to the p_i's and to the means (m_i) and COV's (V_i) of the different diameters[15]:

$$f_y = \sum_i m_i p_i \tag{8}$$

$$V_M^2 f_y^2 = \sum_i V_i^2 m_i^2 p_i + \sum_i (m_i - f_y)^2 p_i \tag{9}$$

Alternatively, these relationships can be used to determine f_y and V_M after adequate testing has estimated each m_i and V_i, and once a committee has established the values of the p_i.

For a numerical example, consider 5 diameter groups and the results for Grade 60 steel reported in Reference 18 for sample sizes of 24 or more. 19 sources are represented, but other sampling procedures are not reported. For yield stress (strictly for the stress at 0.35% strain):

Diameter (1/8 in. units)	#3-#5	#6-#8	#9-#11	#14	#18
m_i, ksi	67.2	64.9	65.0	64.9	63.0
V_i	0.071	0.060	0.060	0.076	0.063

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Assuming for simplicity all the p_i's equal to 1/5:

$$f_y = (0.2)(67.9 + 64.9 + 65.0 + 64.9 + 63.0) = 65.1 \text{ ksi } (4750 \text{ kgf/cm}^2)$$

$$V_M = \frac{1}{65.1}\sqrt{18.68 + 1.84} = 0.070$$

For these data and these p_i values V_M is not much larger than the "average COV" ($\frac{\sqrt{18.68}}{65.1} = 0.66$), because most of the deviations of the individual means from the composite mean are not large.

It should be clear, however, that these p_i's should not necessarily be chosen to be equal; a much larger fraction of the beams to which the code is applied use #8 bars rather than #18 bars. The choice of these p_i values will be discussed more thoroughly subsequently.

Notice that, if the code factors were made bar-diameter dependent, the (generally smaller) values V_i could replace V_M (Eq. 9), and φ would be larger for a given design. The single value V_M (Eq. 9) must cover the incomplete information associated with the present code's procedure, which does not require the user to identify the bar diameter.

Similarly, as will be discussed more generally below, the estimation of V_M should, in principle, be based on samples with representative proportions of specimens from different heats, different mills, different storage and exposure conditions, different loading rates, etc. Also, being representative of all the bars in a given cross-section, V_M should, in principle, depend on the number of bars[29].

In addition, V_M should in theory account for the fact that the information used to estimate the two parameters mean yield stress and COV is incomplete (i.e., samples are of finite size). This "statistical" uncertainty can be treated in a variety of ways, but Bayesian techniques, which treat the mean yield stress as a random variable[15], are most appropriate in these circumstances. A simple approach of this kind is as follows. Assume $M = m_M + \epsilon_M$, in which m_M is the

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(uncertain) mean of M and ϵ_M (= $M-m_M$) is the mean-zero random deviation of M from its mean. If m_M where known with certainty, V'_M, the COV of M would be associated with ϵ_M only. The mean of m_M (and hence of M) is the present best estimate of m_M; call it $\bar{m}$. The "total" COV of M is $\sqrt{V_{mM}^2 + \hat{V}_M^2}$, in which $\hat{V}_M$ is then the present best estimate of V'_M, and V_{mM} is the COV expressing the present (statistical) uncertainty in m_M. The latter will depend on the sample size used to estimate m_M; it can be reduced by further sampling. If the sample size (n) is relatively large, V_{mM} can be assumed to be $\hat{V}_M/\sqrt{n}$. The implication is that statistical uncertainty can be ignored in the total COV if n is more than 5 to 10. If the sample size is negligible, the so-called prior[15] V_{mM} can be estimated by answering the question: in what range $\bar{m}(1 \pm x)$ is m_M as likely to lie as not? Then, $V_{mM} = 3x/2$ (because, generally, $m(1 \pm 2/3V)$ contains about 50% of the probability mass of most typical distributions). As data becomes available, the reciprocal of the new V_{mM}^2 is the sum of the reciprocals of this prior V_{mM}^2 and $\hat{V}_M^2/n$. The total COV, $\sqrt{V_{mM}^2 + \hat{V}_M^2}$, which reflects both the variability from beam to beam (V'_M) and the uncertainty (V_{mM}) in the mean yield stress, should in principle be used in the code. Presumably statistical uncertainty in V'_M can be assumed to be only a second-order correction.

The value of V_F reflecting fabrication (e.g., forming and bar placement) uncertainty is assumed to influence R primarily through the depth of steel, d. The value used here was $V_F = 0.04$. It is probably a function of nominal beam depth[6] but the Phase 1 no-added-complexity rule precludes this refinement. The accurate choice of smaller COV values in the presence of much larger ones is not critical; as long as the true value of V_F is no more than about 40% of V_R different from its assumed value, the error in V_R is less than 10%. (Here 40% of V_R is about 0.06.)

The value of V_P was chosen equal to 0.11, based primarily on the value of

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the COV of the ratios of observed to predicted beam capacities reported by Sexsmith[14]. As discussed above these are capacities predicted by the implied code procedure when materials properties and dimensions are measured values (i.e., V_P represents the uncertainty conditional on no uncertainty in M and F). V_P represents uncertainty in the total algorithm, including the use of standard cylinder strengths instead of in-beam concrete strengths. Turkstra[22] has shown that both the bias and the COV of this procedure are functions of the $q = A_s f_y / f_c bd$. (He and Allen[6] have produced through regression analysis equations that both remove this dependence and lower V_P, at the expense of greater complexity.) Therefore, as with V_M, in principle, V_P should be based on a carefully designed representative sample, here a sample of test beams. To properly cover known or unknown systematic errors in the prediction algorithm, all widths, depths, aspect ratios, steel ratios, etc., should be selected in relation to the proportions used in the practice to which the code will be applied. Again, V_P will thereby cover systematic as well as "random" uncertainties implicit in the adopted code prediction procedure.

Putting together the component uncertainties,

$V_R = \sqrt{V_M^2 + V_F^2 + V_P^2} = \sqrt{0.12^2 + 0.04^2 + 0.11^2} = 0.167$, and

$1 - kV_R = 1 - (2)(0.167) = 0.66.$

Load Parameters

Loads are not the primary focus of this study, but at least rough values of means and COV's are needed to complete the specifications. In this first phase the principle of no-change from existing codes will be carried to the extreme.

For this loading combination, the dead load of interest is the value at the instant that the peak live load effect occurs[11]. The value m_D should be an unbiased estimate of the dead load effects. V_D and V_A will be estimated together as $\sqrt{V_D^2 + V_A^2} = 0.04$, because their influence is relatively small. Any value less

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than about 40% of V_{SD} will not influence $\gamma_D D^*$ significantly. Data on material density variability to improve the estimate of V_D should be readily obtainable, but its small influence on design may not justify even this expense. Since most dead load is fixed in location and static, V_A, the uncertainty in modeling dead load is small.

The value of V_E, the uncertainty in load-effect prediction under given applied dead and live load, was assumed to be 0.1. The basis is judgment and "calibration(1)". The question to be asked and answered by the committee who must, without data, estimate V_E is "with precisely known values of the dead and live loads, with what confidence does the structural analysis procedure predict the load effects?" If, for example, the answer is: "the true value will be within ± 7% of the predicted value, in about 50% of all cases to which the code is applied," then the implied estimate of V_E is about (3/2)(7%) or 10% (for reasons discussed above). It is clear that load tests on instrumented structures could provide data to estimate V_E. But V_E should also consider the fact that when predicting the load effect of the peak live load, the code procedure does not include explicitly factors such as creep, settlement, stresses due to simultaneous wind and temperature "loads," etc. The present use of an algorithm based on elastic analysis to predict ultimate load effects on cross-sections causes some difficulty in V_E interpretations, because for indeterminate members this procedure may produce less reliable estimates of individual cross-section moments than of the total static moment resisted by the beam. Under ductile collapse conditions, however, this total is the more significant load effect. It is difficult to resolve this problem until the code algorithm is made more explicit and without increasing code complexity (e.g., different values of V_E for determinate and indeterminate beams).

With these values, $V_{SD} = \sqrt{V_E^2 + V_A^2 + V_D^2} = \sqrt{0.1^2 + 0.04^2} = 0.11$, and $(1 + k_S V_{SD}) = 1 + (2)(0.11) = 1.22$. Note that V_E, uncertainty in translating

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load to load effect is the major influence causing the nominal dead load effect D^* to be 22% greater than m_D.

The live load parameters m_L, V_B, and V_L are particularly difficult to assess directly at this time. In keeping with the Phase 1 philosophy for a concrete code, live load results were sought which would involve no basic changes. Therefore V_B and V_L were pooled as $V_L' = \sqrt{V_L^2 + V_B^2}$; then m_L and V_L' selected together to give results similar to present practice. Many combinations of m_L and V_L' versus floor area, A, will produce useable results(1). Here, a particular function of V_L' versus A was selected first, namely(1), $V_L' = 13/\sqrt{A}$ $(150 \le A \le 900\ ft^2)(V_L' = 4/\sqrt{A},\ 50 \le A \le 300 m^2)$. V_L' equals 1.06 at $A = 150\ ft^2$ and 0.43 at $A = 900\ ft^2$. These two values are assumed also for values of A outside the indicated range. How V_L' is split between V_L and V_B is not hypothesized here but it is reasonable to anticipate that V_B would be larger for smaller areas where load effects will be more sensitive to non-uniform spatial distribution and to local dynamic effects. Secondly, m_L was selected to give designs comparable to the 1963 ACI code when dead load is zero. This requires that

$$m_L = \frac{[1 - 0.0008\ (A - 150)]\ 1.8\ L_C}{1 + \beta\alpha_S\sqrt{V_L^2 + V_B^2 + V_E^2}}$$

in which L_C is the present code load. The results at $A = 150\ ft^2 (50\ m^2)$ and $900\ ft^2 (300\ m^2)$ are $m_L = 0.47 L_C$ and $m_L = 0.33\ L_C$ respectively. Although inconsistent with available live load survey data this reduction in the mean live load with area may be quite reasonable for (as yet unmeasured) peak lifetime live loads.

Combining m_L, V_E, and V_L' will produce $L^* = m_L(1 + k_S V_{SL})$. It is $1.46 L_C$ at $A = 150\ ft^2$ and $0.62 L_C$ at $900\ ft^2$, and it is accurately approximated by a straight line for intermediate values of A. Common US codes call for $0.4\ L_C$ at $900\ ft^2$; L^* is therefore about 50% greater than present design loads at all values of A.

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Recall that L* represents a nominal live load effect, with V_{SL} accounting for uncertainty in structural analysis (V_E is negligible compared to V'_L in this case) and in load modeling, as well as in environmental live load itself. It is important that a committee recommending loading specifications include V_B as well as V_L in their report, since it is coupled with their recommended procedure. Vickery (28) has, for example, presented V_B values for modern gust-load procedures. A more complicated live load model, such as one that includes an area-dependent concentrated load as well as an area-dependent uniform load, can probably more accurately represent peak live load conditions. Procedures(17) with a multitude of load-effect-type factors (e.g., a larger load for slab corner moments than slab center moments) could be "better" still. They could be permitted lower V_B values even though the loading environment uncertainty is unchanged.

Other Parameters

To determine φ, γ_D, and γ_L values must be selected for α_R, α_S, α_D, and α_L. Selected values should be chosen with regard to the range of the ratios of uncertainty measures. The values used here were discussed above. For the value of $\beta = 3.55$ (to be discussed below), the values of φ and γ_D are as given above (0.83 and 0.96). The value of γ_L depends mildly on A through V_{SL}; γ_L is computed to be 1.20 for A = 150 ft^2 and 1.14 for A = 900 ft^2. The Phase 1 philosophy says this must be a single number; $\gamma_L = 1.17$ suggests itself. That γ_D is less than unity is not inconsistent with this procedure, but a higher value, such as unity, might be desirable for non-technical reasons. It can be adjusted by altering k_S (or k_S for dead load only). The product $\gamma_D D^*$ will remain unchanged.

The final step was the selection of β. This was chosen with the aid of a simple computer program which produced required steel areas both by the 1963 ACI code and by this scheme for a wide range of spans, dean loads, and tributary areas. The dead load/live load ratio is the critical variable. The value of β was incremented until inspection revealed what was judged to be satisfactory

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agreement over the range of variables. The selected value was $\beta = 3.55$. Any value from approximately 3.3 to 3.8 gave acceptable results; this calibration procedure is fortunately not a delicate one. Other parameters such as V_E and V_P were also varied within reasonable ranges. The effects were predictable: larger values of V_P, for example, were coupled with lower suggestions for β values; and larger V_E values, for a given β, caused insignificant differences in the required steel areas unless the dead load/live load ratio was unusually high. "Calibration" to the 1971 ACI code would produce a somewhat lower β value. The topic of code calibration or optimization is a new subject now under development[16,6].

The purpose of this illustration has been to provide concrete situations from which broader generalizations can be drawn.

GENERAL OBSERVATIONS

The standard methods used to predict member capacity or applied load effect should be considered as procedures or algorithms. The entire set of approximations and standard assumptions represent a prediction process. When carefully defined, it is reproducible by all users and, importantly, by committees or research workers who seek to compare the predictions with measured observations. Statistics of these comparisons can be used to correct the procedure empirically for any bias (i.e., average error), and to evaluate the COV of the prediction procedure. By this algorithm definition it is not necessary when computing V_R, for example, to be concerned with attempting to correct the given prediction procedure to allow for in-place material strengths versus standard quality control test strengths, for neglect of tensile strength of the concrete, for uncertainty in the neutral axis location (i.e., for uncertainty in the 1.7 factor in the moment prediction equation), etc. It is only necessary that the same assumptions be used for the predictions which are compared to observations of specimens.

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As will be discussed below, however, it is important that the sample of specimens used in such comparisons be properly representative of all combinations of variables found in practice. This "algorithm" philosophy will be further illustrated below.

Simplified formulas for COV's, such as $V_R = \sqrt{V_M^2 + V_F^2 + V_P^2}$, are to be encouraged, but they may not always be strictly applicable. For example, the COV of a function $R = g(f_y, d, f_c, A_s, b, \Delta)$ of assumed random quantifies f_y, d, f_c, A_s, b, and Δ (the ratio of observed resistance to prediction) should involve at least[1,10] the COV's of all the random quantities. This can be done approximately by

$$V_R^2 = \frac{1}{m_R^2} \sum_i \left(\frac{\partial g}{\partial x_i}\right)^2 V_i^2 m_i^2 \tag{10}$$

in which $(\partial g/\partial x_i)$ is the derivative of $g(\cdot)$ with respect to the i^{th} independent variable, evaluated at the means. In the ultimate moment equation discussed above, preliminary studies suggested that only f_y, d, and Δ contribute significantly to V_R and then in a manner approximately as if they appeared in a product form $R \propto f_y\ d\ \Delta$. This conclusion may very well not hold for certain conditions such as shallow or heavily reinforced beams[6]. In such situations, other forms for V_R may be more appropriate, as will be discussed in illustrations to follow. For these cases, the logical conclusion may well be that V_R (and hence ϕ) should be dependent on design variables such as the width-to-depth ratio or the reinforcement ratio. If, however, it is imperative (as in Phase 1) to maintain simply a single value of V_R, the committee should seek reasonable approximations (as discussed in the illustrations to follow). In principle, they should select a value in relation to the fraction of beams which will fall into the different V_R categories. Similar conclusions hold for other COV's.

This need to produce single values for COV's or simplified COV relationships is central to practical code development. Any engineering phenomenon can be viewed at finer and finer levels of detail, revealing "explanations" or

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systematic trends in what appeared at a cruder level to be "randomness[19]." For example, as discussed above, some of the dispersion in observed yield stresses in a sample of reinforcing bars will be due to systematic differences attributable to the bar diameters, the storage conditions, the producers' procedures, the heat of steel, etc.[20,21] The question is how to account for such factors when either (a) they are unknown to the designer at the time of design, or (b) the profession which must allow these factors choses not to complicate its code by making nominal values or "safety" factors a function of systematic effects of these relatively small magnitudes. In either case the lack of information produces a very real (but not unexplainable or inherent or "objective") uncertainty which the code must allow for.

This total uncertainty can be quantified by appropriately weighted sampling or by re-combining through Eq. 8 and 9 the results of a set of separate evaluations of the moments when the systematic factors are held fixed at different values. The index i in Eq. 8 and 9 is over all (mutually exclusive, collectively exhaustive) combinations of systematic factor levels which are known or are suspected to influence V (or m). This set should include all factors whose systematic influence will not be accounted for in the code procedure. For example, this would include, in principle, all combinations of mills and bar diameters when assessing the mean and COV of steel yield stress. The index may include one set of combinations when considering the mean and another when considering the COV. For example, if in a subsequent code revision the mean yield stress were bar diameter dependent, but V_R were not, a weighted set of diameters would be needed in selecting a common V_R, but not in selecting (each) m_Y. Similarly, for ultimate moment, a set of different beam depths should be used to select V_R, but not m_R, _if_ it is believed that the procedure's prediction equation adequately reflects the influence of depth on m_R for all design situations. If the equation is not this nearly

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