SP 161-1

Deflection of a Concrete Beam and Slab Roof

by Russell S. Fling

Synopsis: An 8,000 sq.ft. (740 sq.m) portion of an industrial building was load tested and vertical movements measured to an accuracy of 0.0043 inches (0.11 mm). Measured deflections were compared to those estimated before testing and to revised estimates after testing. Using simplified procedures and judicious estimates of design parameters, computed deflection normally should be within 40 percent of actual average deflection and the coefficient of variation should be less than 50 percent. With a complete and accurate selection of design parameters, the accuracy and statistical variability can be improved to 15 and 40 percent respectively.

Keywords: Beams (supports); cracking (fracturing); creep properties; deflection; flexural strength; load tests (structural); prestressing; reinforced concrete; slabs; statistical analysis; stiffness; structural analysis; torsion; volume/surface ratio

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INTRODUCTION

An 8,000 sq.ft. [740 sq.m] portion of a 500,000 sq.ft. [46,500 sq.m] industrial building built in 1942 and 1945 was load tested in June and July 1993 and measurements of vertical movements at ends and mid-point of each member (50 points), accurate to 0.0043 inches (0.11 mm), were taken at each of 40 load steps. Measured deflections were compared to those estimated before testing. After the load test, more careful estimates of deflection were computed. The purpose of the load test was to determine the safety of the structure because large shear cracks appeared in many beams and girders in zones where shear reinforcement was not provided. The structure did pass the load test. Details are given in another paper (1).

The building tested was one story, 30 ft [9m] high, 252 ft [77m] wide by 2,000 ft [610m] long with a reinforced concrete roof. Cast-in-place roof girders (24" by 60") [600 x 1800 mm] spanning 50 ft [15m] supported 40 ft [12m] beams (18" by 40") [555 x 1220 mm] spaced 16'-8" [5m] on centers. Six inch thick by 12 inch wide [150 x 300 mm], lightly prestressed, precast slabs rested on top of the beams. Deflections of four girders, 12 beams, and 12 slab panels were evaluated at each stage of loading. See the roof framing plan in Figure 1 which gives the general arrangement, member marks, and measuring points. Bays numbers indicate the loading sequence, i.e., Bay #1 was loaded first. The original design live loads were presumed to be 40 psf on the roof and 10 psf suspended below the roof.

Before load testing, estimates of deflections were made for the purpose of detecting incipient failure of members if this should occur. Later it was realized that the data accumulated in conduct of the test would be useful in evaluating the accuracy and variability of deflection calculated by the usual methods. Therefore, a second calculation of deflection was made using more accurate assumptions. There is an obvious risk that the assumptions after the test will be selected to fit the test results. However, assumptions listed here were reasonably foreseeable before the test. Both sets of calculations used the procedure outlined in the current ACI Code (2).

TEST PROCEDURES

The structure was loaded with water. Tanks similar to concrete formwork without form ties were built over each test member and lined with polyethylene film. Due to the physical dimensions of tank construction, tanks for beams and slabs were slightly shorter than the clear spans. Nevertheless, tanks were filled with the entire required uniformly distributed test loads with no allowance for the resulting increase in moment and shear in the members. In calculating required test weights of water, allowance was made for the weight of tanks.

Each 40 ft by 50 ft [12 x 15 m] bay was loaded in the sequence indicated on Figure 1 in five steps with deflection readings taken at the end of each step. First, full load was applied on beams and girders. Thereafter, one-quarter-load increments were applied on the slabs until the full load had been applied. A sixth reading was taken 24 hours later without the addition of more load. Twenty four hours after all bays were fully loaded, each bay was unloaded in one step, one day per bay, in the same sequence as they were loaded.

During the loading of bay #3, top bars in beam B12 delaminated, so the test was stopped, all bays unloaded, the beam repaired, then the test restarted and carried to completion. Loading of bays #1 and #2 are called Phase I and loading of bays #3 and #4 are called Phase II in this investigation.

To measure deflection, wires were hung from the ends and midspan of all members (midbay for slabs), connected to extension scales which in turn were connected to concrete blocks resting on the floor slab. Scales had an 80 lb. [36 kg] capacity graduated by 0.1 lb. [45g] increments and were set initially to read in the range of 30 to 50 lbs [13 to 22 kg]. Deflection of members reduced the scale reading. All scales (a total of 50) were read after application of each load increment, after the required waiting periods, and before loading started each day. Calibration indicated that a scale reading of 0.10 lb. [45g] equals an average movement of 0.0043 inches [0.11 mm] with a variability from one scale to another of up to 5 to 7 percent. The variability within one scale from pound to pound is unknown.

The quantity of water was measured by two meters as well as by measuring the depth of water in each tank. Temperature was almost constant at about 90° F [32° C] so that no adjustments were made for density of water at ambient temperature nor for the length of wires connected to the scales.

Raw data from scale readings at 50 points for 40 load steps were entered into a spreadsheet program and converted to reference elevations and measured deflections. The program also computed the expected deflection for each member at each step, ratios of measured to calculated deflection, and statistical analyses.

SLAB DEFLECTIONS

Assumptions used in calculating deflection before the load test (first calculation) and after the load test (second calculation) are summarized in Table 1 and discussed below, listed here in approximate decreasing order of effect on the result.

1. For the first calculation, prestressing was not considered. For the second calculation, the cracking moment was increased by the amount of the prestressing moment, using an assumed initial stress of 20,000 psi [140 MPa] with a consequent reduction in the amount of cracking and increase in stiffness. While low levels of prestress are commonly ignored in strength calculations because creep in concrete reduces or even eliminates the initial prestress in the steel, it seems evident that creep does not reduce the cracking moment.

2. In the first calculation, the deflection under test load was calculated by taking the difference between the initial deflection under dead load only using an assumed concrete strength of 3,500 psi [24 MPa] when the slabs were removed from the forms and the final deflection computed for dead and test loads with strength of 9240 psi [64 MPa]. For the second set of calculations, deflection was calculated for each step using only the test load and the stiffness at the end of each load step. The first load step was over 20 lbs per sq.ft. [960 Pa] and probably exceeded the maximum live load experienced by the structure at any time before the load test. During the load test, as the slabs became increasingly more cracked and less stiff, the measured deflection increased correspondingly. Of course, evaluation of the stiffness at each load step had to consider the pre-test load and loading history.

3. In the second calculation, the effective depth of the reinforcement was increased to 5-1/16 inches [13 mm] which still allows for a 3/4 inch [19 mm] concrete cover on the bottom. This change increased both the prestressing moment and the cracked moment of inertia.

4. A concrete strength of 9,240 psi [64 MPa], the same as assumed for the cast-in-place concrete, was used in the first calculation. Concrete strength was reduced to 6,500 psi [45 MPa] in the second calculation because the lower value is more consistent with the probable current strength. In any case, this factor had a small effect on the computed deflection.

5. In the second calculation, the test load used was reduced 4 percent because this represents the weight of test equipment applied before deflection measurements were started. On the other hand, moments were increased 5.5 percent because the full test load was applied to less than the full length of the span. The net effect of these two adjustments was small.

For both the first and second calculation, a modulus of rupture of $7.5(f_c')^{\wedge}0.5$ was used, and a span length equal to the clear span with no restraining moments was assumed (simple span).

BEAM and GIRDER DEFLECTIONS

Assumptions used in calculating deflection before the load test (first calculation) and after the load test (second calculation) are summarized in Table 2 and discussed below, listed here in approximate order of effect on the result.

1. For simplicity, no column stiffness was assumed in the first calculation. In the second calculation, a pseudo-column supporting the ends of each member was used. The pseudo-column included the stiffness of the real column as well as all columns in the same line and parallel, unloaded beams (for the beams) and parallel, unloaded girders (for the girders). The increase in stiffness over the real column was 24 percent for the girders G3-G5, 42 percent for girders G4-G6, and an average of 690 percent for beams. Moments were computed only for G3-G5 and proportioned for G4-G6.

   Inclusion of parallel flexural members and adjacent columns is justified by the torsional resistance of the massive members, especially the girders. It was evaluated by assuming that the pseudo-column stiffness K_{pc} is the sum of the stiffnesses of the adjacent real members (columns, beams, or girders) reduced by the ratio of [rotation due to flexural stiffness of the real member] to [total rotation due to flexural stiffness of the real member plus rotation due to twist of the girder or beam] as shown in the following expression and Figure 2, assuming a constant modulus of elasticity, E_c, of the concrete.

$$K_{pc} = \sum K_b * \frac{1/K_b}{1/K_b + 1/K_t} = \sum \frac{1}{1/K_b + 1/K_t}$$

where K = unit rotation = flexural stiffness = 4I/L

K_{pc} = K of the pseudo-column

K_b = K of the column, beam or girder resisting the torsional moment

K_t = the unit rotation due to twist of the torsional member
= $0.1X^3Y/L_t$
I = gross concrete moment of inertia
L = center to center span length
L_t = length of torsional member from point where torsional moment is applied to where it is resisted
X = shortest cross-sectional dimension of the torsional member
Y = longest cross-sectional dimension of the torsional member

To reduce the volume of computations, an average value for the pseudo-column stiffness was used to calculate moments for all beams. The range in individual pseudo-column stiffnesses, compared to the average, was from +45 percent for beams nearest the center of the building to -19 percent for beams nearest the edge. The variation in computed moments was much smaller than these values.

Beam B4 framing directly into columns deflected 87 percent of the four adjacent beams while beam B8 deflected 101 percent of the four adjacent beams. The average of 94 percent is within the range of variation of beam deflections but might indicate that beams framing directly into columns deflect less than those that do not.

In some cases, the torsional moment induced into the girders probably exceeds their nominal torsional strength as computed by Section 11.6.1 of ACI 318 Code (2) and increased by a factor of four as suggested by the corresponding Section in the Commentary. However, using uncracked torsional stiffness was simpler. Using torsional stiffness increased the accuracy of calculated deflections in the subject structure. Without this refinement, the similarity in deflection behavior between adjacent beams would have been difficult to explain.

2. Past experience indicates that members with a large volume/surface ratio have a higher flexural strength than given in the ACI Code Section 9.5.2.3, resulting in increased cracking moment and stiffness, and reduction in deflection. In this project, beams have a V/S ratio of 6.2 inches [160 mm] and girders have a V/S ratio of 8.6 inches [220 mm] compared to V/S ratios of 3 to 4 inches [75 to 100 mm] for typical concrete structural members. The same value was used for both before-test and after-test calculations.

3. For simplicity in the first calculation, assuming that the beams and girders had been previously loaded to the load-test level, the beam stiffness at each load stage was assumed to be proportional to the

stiffness at the final load stage. After the load test, it was obvious that the members had never been loaded more than the current code design live load of 25 psf [1200 Pa] and certainly not to the project design live load of 50 psf [2400 Pa]. The load-deflection curves in Figures 3 through 9 clearly indicate an increasing rate of deflection with an increase in load. Therefore, an attempt was made to calculate deflection based on the flexural stiffness determined by the amount of cracking produced by the load at that stage. Even with the help of a computer this proved to be a massive undertaking. Hence, the stiffness at each load stage was assumed to be proportional to the stiffness at each load stage for beams B2 and B3 which were believed to be representative. Adjustment were made for members receiving only one-half the test load such as beams B1 and B5. This assumption is reasonably accurate for some members but introduces error for others.

Stiffness at the haunch end of girders is much larger than at midspan so that the stiffnesses at the ends and the midspan were averaged for the girders in accordance with Eq. 2.15 of the ACI 435 Report (3). The average stiffness was 9 to 28 percent more than midspan stiffness only, depending on span and loading conditions. The stiffnesses at ends and midspan for beams was similar, so the stiffness at midspan only was used in the calculations.

4. In the second calculation, the haunch was used in computing moments as well as stiffnesses, as mentioned above. As a result, maximum positive moments increased about one percent. No attempt was made to evaluate the effect on deflections of the distribution of maximum, or near maximum, moments along the length of the span. The shape of the moment curve could be significant.

5. A series of impact hammer tests before the load test indicated a compressive strength of 9240 psi [64 Mpa] in the beams and girders. The average compressive strength of nine cored cylinders taken after the load test was 2900 psi [20 MPa] and this value was increased to 3410 psi [24 Mpa] by dividing 2900 by 85 percent as suggested by ACI318 Commentary Section R5.6.3.4 (4).

6. For the first calculation, the full test load in the tributary area was applied to the full span. For the second calculation, since construction of the tanks prevented loading beams near their ends, only water load (not tank weight) was applied to a portion of the span corresponding to the actual conditions. The net affect of these two modifications was very small (about one percent). Also, the concentrated load applied by the beams to the girders was adjusted to account for the shear transfer in the beams due to unbalanced moments.

The actual cracked beam stiffness was not used for moment distribution at each load stage because of the computational effort. Instead, the customary assumption of moment distribution based on the stiffness of uncracked concrete cross-section was used. Preliminary analysis indicated cracked stiffness may affect distribution of moments as much as five percent or more.

RESULTS

The maximum calculated and measured deflection of members is summarized in Table 3 and shown on Figures 3 through 9. In these figures, loading occurred in steps 8 through 19 for Phase I, steps 20 through 31 for Phase II, and unloading occurred in steps 32 through 38. In all cases, the before-test calculated deflection is shown.

In all cases, the values in Table 3 are for the load giving the maximum deflection. Thus, when both interior and exterior spans are loaded, the actual deflection would be the combination of the downward and upward deflections.

Creep Deflection

In the 16 to 24 hours after each bay was loaded, during which there was no additional load, most members continued to deflect. The average increase in deflection for beams was 4.4 percent and for slabs was 6.4 percent. Because simple-span slabs are not affected by loading of adjacent members, their creep deflection for longer periods could also be computed. Their average creep per day for periods of 1.5 to 5 days was 5.6 percent. On average, deflection of girders did not increase over the first 24 hours.

Column Shortening

Column shortening was measured as a matter of interest, but it had no affect on beam, girder or slab deflections. Data were quite erratic, perhaps because beams along lines (A) and (51) may have gained support from walls under them and because gages rested on the floor over column footings. No attempt was made to estimate footing settlement caused by the test loads.

Ratio of Measured to Computed Deflections

The ratio of measured to computed deflections was calculated for each member and each load increment during the loading stages. The average ratio of measured to calculated deflection is given in Table 4. Due to the large quantity of deflection readings it was possible to evaluate the data by statistical means using Eq. 5 of ACI 214 (5) and the results are also given in Table 4. A statistical analysis was performed for Phases I and II separately because there appeared to be an unexplained, slight difference in deflection behavior between them. For unexplained reasons, deflection of slab S10 was consistently about two-thirds that of the other slabs. Therefore, all other slabs were analyzed without slab S10 and the results are also shown in Table.

Statistical analysis for the first calculation used data for all steps including both loading and unloading, upward and downward, except for loading steps where calculated or measured deflection was 0.008 inches [0.2 mm] or less. This value was twice the reading accuracy of the gages. Thus the error introduced by the reading accuracy level ranged from 1 to 2 percent for readings at maximum deflection to 22 percent (+/-0.002 in./0.009 in.) for readings at the minimum reading evaluated This error was in addition to error introduced by inaccuracies in the scales themselves, as noted above.

For the second calculation, only downward deflections in the loading cycle were included. Upward deflections and deflections on unloading or 24 hours after full load was applied were not included. Limiting the data in this manner may have improved the accuracy for the second cycle.

Slabs

For slabs, the calculated deflection was quite close to the measured average deflection and the variability was very low. See Figure 3 for slab load-deflection curves. Possible reasons the computed slab deflections were more accurate than the beam computed deflections include the following.

- The simple span almost totally eliminated the uncertainty of moments in the slabs, whereas the beams were continuous with subjectively indeterminate moments.
- The slabs were keyed together so that the measured deflection represented not just the one slab to which the equipment was attached but the average of a large number of slab units.
- Concrete in the precast plant was probably of more uniform quality and strength than that produced in the field.

Beams and Girders

The accuracy of calculation of beam and girder deflections was improved by more careful selection of assumptions. An important factor limiting the accuracy of deflection calculations was that the beams and girders deflected unpredictably. For example, interior beams B6 and B7 deflected more than exterior beams B2 and B3, contrary to normal expectations. This behavior might be explained by one of the following construction or service life conditions.

- Some columns could have settled and caused higher moment in the beams and girders than those calculated. Indeed, there was indication that some columns in the area had settled but the data were too incomplete to make a careful analysis of moment redistribution.

- Decentering procedures of the contractor or other construction aberration may have caused higher moments in certain beams and girders that those calculated or caused then to be more thoroughly cracked. At this late date, it would be difficult if not impossible to determine what procedures were used and how they might have affected the structure.

- Some bays could have been overloaded many years earlier, cracking these beams, and resulting in a lower stiffness. However, it seems unlikely that only certain bays would have been overloaded and not adjacent bays or that beams and girders were overloaded but not the slabs.

See Figures 4, 5, 6, and 7 for beam load-deflection curves and Figures 8 and 9 for girder load-deflection curves.

CONCLUSIONS and RECOMMENDATIONS

For reliably accurate deflection calculations, it is essential that an accurate estimate of the distribution of moments in a member be determined. Moments must be computed using actual member stiffnesses. A complete and careful selection of design parameters will improve the accuracy and statistical variability of deflection computations. The possibility of framework action caused by torsional stiffness of supporting flexural members should be considered. With these precautions, an engineer should be able to compute deflections within 15 percent of the actual average value, with a coefficient of variability of 40 percent or less. In this investigation, after the load test, calculated deflection was within 8 percent except for beams and girders in Phase II which had unexplained, irrational deflection behavior, and the coefficient of variation was always under 40 percent.

Availability of accurate design parameters such as loading history, concrete compressive and flexural strength and modulus of elasticity, etc. and inability to measure deflection accurately will prevent reaching this level of accuracy in many, or even most cases.

Using simplified procedures and judicious estimates of design parameters, computed deflection normally should be within 40 percent of actual average deflection and the coefficient of variation should be less than 50 percent unless unknown factors affect deflection dramatically. In this investigation, before the test, calculated deflection was within 40 percent for beams and girders and 52 percent for slabs even though important factors were not considered such as column stiffnesses and prestressing in the slabs. Had