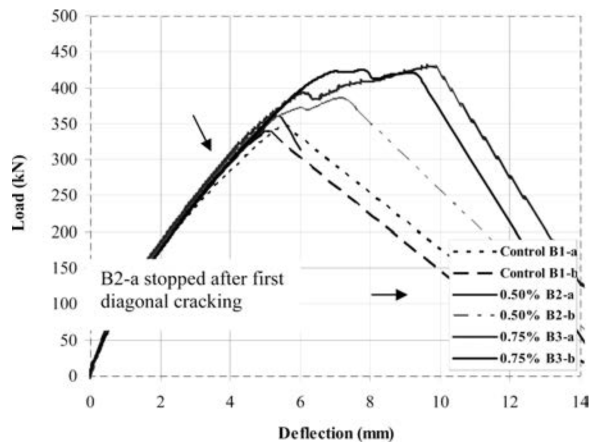

Fig. 4—Load versus midspan deflection response of the beams.

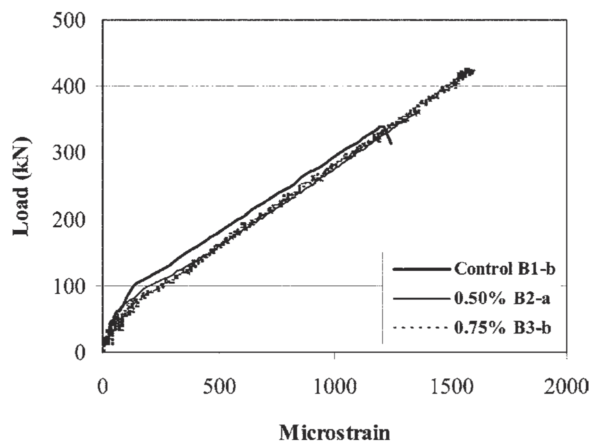

Fig. 5—Load versus strain response of the reinforcing steel bar at midspan.

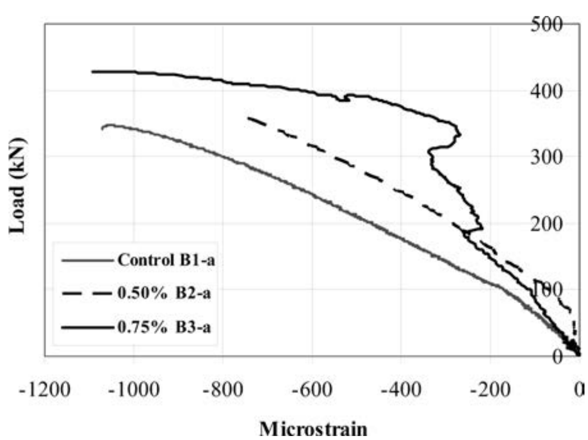

Fig. 6—Load versus strain response at the extreme compression fiber at midspan.

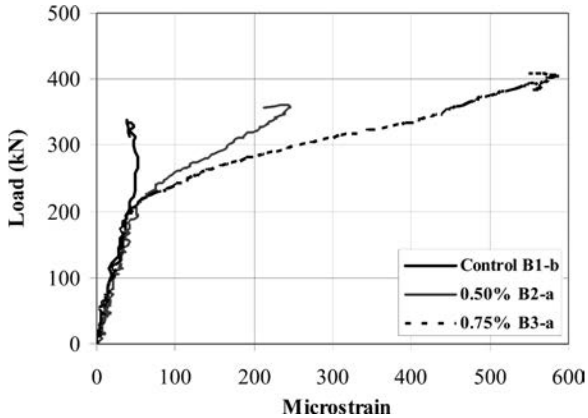

Fig. 7—Load versus strain diagram of embedded inclined gauge #4.

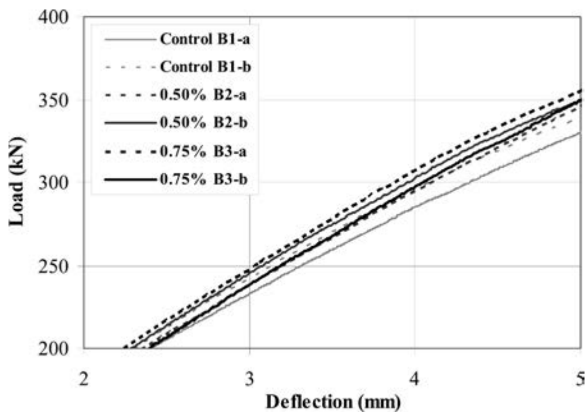

Fig. 8—A closer look at the load-deflection diagram after initiation of diagonal cracks.

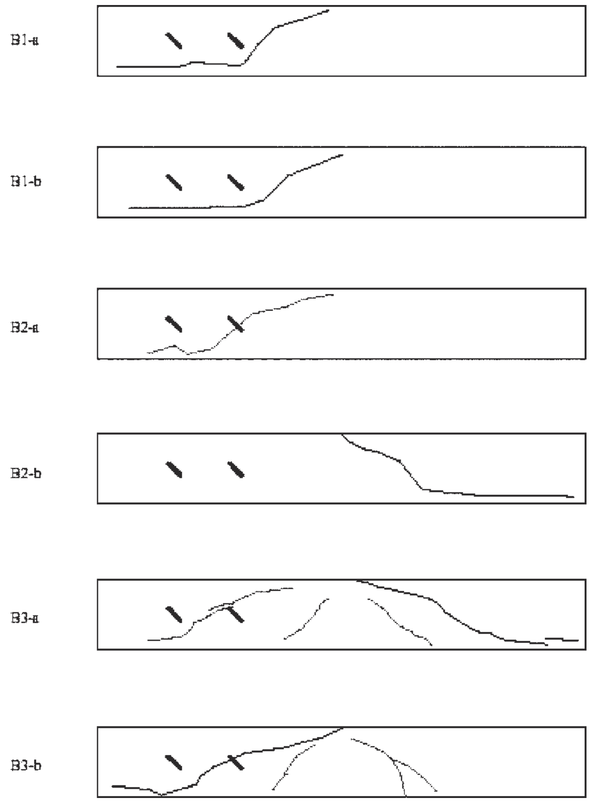

Fig. 9—Crack patterns for control and SNFRC beams. The cracks that lead to rupture are indicated with bold black lines.

Fig. 10—First shear crack (left: control beam and right: fiber-reinforced concrete beam (0.75%)).

Fig. 11—Cracking pattern of the fiber-reinforced concrete beam (0.75%) at failure.

SP-248—5

Deflection-Softening and Deflection-Hardening FRC Composites: Characterization and Modeling

by A.E. Naaman

Synopsis: The load-deflection response of fiber reinforced cement composites generally starts by an initial portion that is linear elastic up to a certain load at which it deviates from linearity; this is often identified as the onset of first cracking in the matrix. If the cement matrix is not reinforced, first cracking is followed by a sudden drop in the load-deflection curve, and failure occurs. The addition of fibers mostly influences the response of the composite after cracking. For all practical purposes, the load-deflection response of fiber reinforced cement composites after first cracking can be simply classified as either "deflection-softening" or "deflection-hardening." This paper describes first the different types of load-deflection curves observed in various experimental tests and illustrates the influence of some fiber reinforcing parameters with steel and polymeric fibers. Then, an analytical formulation is suggested to predict the value of the critical volume fraction of a given fiber to achieve deflection-hardening behavior. Several parameters influence the "deflection-hardening" portion of the curve and include the fiber content, fiber aspect ratio, and fiber to matrix bond.

Keywords: deflection; deflection hardening; deflection softening; fiber-reinforced concrete; strain hardening; strain softening; polypropylene fibers; steel fibers

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INTRODUCTION

The addition of fibers to concrete is meant to primarily change the nature of its tensile response, that is, to make it more ductile and if possible lead to higher strength. Compressive strength is much less sensitive to fiber addition than the tensile response. Since bending depends on both the tensile and compression properties of a material, the bending response of fiber reinforced concrete is mostly influenced by its tensile response as affected by fibers. It is therefore important to understand the tensile response before addressing the bending response.

TENSILE RESPONSE: BRITTLENESS, MATERIAL DUCTILITY AND STRUCTURAL DUCTILITY

Although brittleness and ductility of a material or structural element can be characterized quantitatively by a property such as fracture toughness or ductility index, here only a qualitative definition is suggested. A brittle material such as a glass rod would have a linear elastic stress-strain response in tension up to failure, characterized by a sudden fracture with no resistance left after maximum stress (Fig. 1a). A ductile material may have different forms of stress versus elongation response after its elastic limit, such as described in Fig. 1b. Curve 1 of Fig. 1b is elastic perfectly plastic, curve 2 can be described as "stress-softening" after maximum stress, while curve 3 can be described as "stress-hardening" or "strain-hardening" a term often used to describe the stress-strain response of typical steels. When the "strain-hardening" property of a material can be maintained in large scale tensile specimens, then the material can be safely used in stand-alone structural elements which will show structural ductility, a desirable property in all structural applications.

For all practical purposes, the tensile stress-strain (or stress-elongation) response of fiber reinforced cement (FRC) composites can be classified as either "strain-softening" or "strain-hardening" [10 to 12, 15]. In the "strain-softening" case, localization occurs immediately after first cracking and, with increasing elongation, the stress after first cracking is smaller than that at first cracking (Fig. 2a). In the "strain-hardening" case, the stress after first cracking increases with strain and multiple cracking occurs up to the maximum post-cracking stress (Fig. 2b). At that point, localization occurs and the stress decreases with increasing elongation, similarly to the case of a strain-softening material.

Note that in describing the curve of Fig. 2a, the term "stress-softening" may be more appropriate than the term "strain-softening". Indeed, after first cracking, localization occurs, and the strain looses its meaning; the elongation is primarily controlled by the opening of a single crack; thus the term "stress-softening" after first cracking is more accurate. In the case of Fig. 2b, one can also used the term "stress-hardening" after first cracking; however, since this occurs at increasing strains (with multiple cracks and prior to localization) the term "strain-hardening" is used. To minimize confusion, the terms "strain-softening" and "strain-hardening" are used here as suggested in past publications [15]. Figures 3 and 4 show typical photographs of tensile FRC specimens that exhibited either strain-softening or strain-hardening and multiple cracking behavior under loading.

RESEARCH SIGNIFICANCE

Most stand-alone structural applications of fiber reinforced cement and concrete composites involve bending elements such as cladding panels, industrial slabs, and pavements. There is a dire need to provide a rational characterization of the bending response of these composites based on fundamental principles. This study contributes to this effort by suggesting a simple and general classification that ties with the tensile response of the composite and facilitates the use of a performance based specification.

TYPICAL LOAD-DEFLECTION RESPONSE OF FRC BEAMS

Load-deflection response curves of typical fiber reinforced cement composites are shown in Fig. 5. An almost linear response is observed up to first cracking also termed "Bend Over Point – BOP" or "Limit of Proportionality – LOP." Following this point, the curve can soften in different ways (Figs. 5a and 5b), or harden (Fig. 5c). In either case another point of interest corresponds to the point of maximum load after first cracking. From the load at this point, the Modulus of Rupture (MOR) is calculated as the equivalent elastic bending stress at nominal resistance. Note that for a plain matrix without fibers, the MOR point is the same as the BOP or LOP and thus matches the definition of MOR by ACI for plain concrete. The shape of the curves in Figs 5a and 5b is described as "deflection-softening" while that of Fig. 5c is described as "deflection-hardening" and relates to the segment of the curve between the LOP and the MOR. Most importantly, deflection-softening behavior leads to only one flexural crack and localization similarly to the case of a "strain-softening" material in tension (Fig. 2a), while "deflection-hardening" implies the occurrence of multiple cracks similarly to a "strain-hardening" material in tension (Fig. 2b). Photos of bending specimens that showed either deflection-softening or deflection-hardening behavior are shown in Figs. 6 and 7.

Note that after the point marked MOR (Fig. 5), deflection softening starts anyway. Such behavior corresponds essentially to the localization of a pseudo-plastic hinge where all the deformation (or rotational bending) is localized. Examples of an actual load-deflection response for steel fiber reinforced concrete composites are shown in Fig. 8. These were carried out according to ASTM C1018 standard testing, that is using small beams of cross section 4x4 in (102x102 mm) tested over a 12 in (300 mm) span (see also Fig. 6). Note that instead of the load for the vertical axis, the equivalent elastic bending stress is shown to allow comparison with other tests. The curve for 0.4% fiber content by volume can be described as "deflection-softening" while the two others for fiber contents of 0.8% and 1% can be described as "deflection-hardening." Figure 9 illustrates the load deflection curves from another set of small beams using synthetic polypropylene (PP) fibers. The specimens were 75 mm wide, 37.5 mm thick and 500 mm long. They were tested in third point loading over a span of 425 mm. The curve at 0.5% fiber content can be described as "deflection-softening" while the one at 1.5% fiber content can be considered "deflection-hardening." Since PP fibers have an elastic modulus significantly smaller than steel fibers, the slope of the hardening portion between first cracking and the MOR point is significantly larger for the specimens with steel fibers (Fig. 8) than for the specimens with PP fibers (Fig. 9). Additional examples of a bending response with PVA and Spectra fibers can be found in [3].

CORRELATION BETWEEN TENSILE AND BENDING RESPONSE

The terminology "deflection-softening" and "deflection-hardening" is convenient and reflects a similar description of the tensile response of fiber reinforced cement composites as shown in Fig. 10. Indeed all fiber reinforced cement composites can be generally and simply classified as either "strain-softening" or "strain-hardening" in tension. Such a description can provide the basis for a performance specification for the composite. The general relationship that ties tensile and bending behavior together is also illustrated in Fig. 10 [12,15]. Note that while the tensile response represents a fundamental property of the composite, the bending response (while not fundamental in nature) is related to the most common applications of fiber reinforced cement and concrete composites. It is noted that all tension "strain-hardening" FRC composites are expected to be "deflection-hardening" while some tension "strain-softening" can lead to "deflection-hardening." Additional information on the classification of Fig. 10 can be found in Ref. [15]. The mechanical condition for a tension strain-softening material to lead to a deflection-hardening behavior is expanded upon further below.

SUMMARY OF KEY RESULTS FROM BENDING MODEL

Several numerical models have been developed to predict the bending response (moment-curvature or moment-deflection) of fiber reinforced cement composites [2, 4, 18]; they generally require a computerized analysis and extensive input data. However, for design purposes the prediction of nominal bending resistance and related curvature may be sufficient.

In the derivations described next, the following notation is used: σ_{cc} = the cracking strength of the composite in direct tension; σ_{pc} = maximum postcracking strength of the composite in direct tension (Fig. 2); $\bar{\sigma}_{pc}$ = average post-cracking stress following cracking over a small elongation or deflection increment (see Refs. 5 to 10 for details).

A comprehensive model to predict the nominal bending resistance of fiber reinforced cement composites was developed by Naaman [9, 10]. A closed-form solution was derived providing the value of neutral axis of bending at nominal resistance, and the nominal bending moment.

To illustrate typical results the following case is considered: 1) rectangular stress block in compression at ultimate is same as recommended by ACI, 2) the tensile response is perfectly plastic with uniform post-cracking stress $\bar{\sigma}_{pc} = \sigma_{pc}$ (Fig. 11). For this case the following solution is obtained for the nominal bending moment resistance of a rectangular section of width *b* and depth *h.*

The neutral axis at nominal bending resistance is given by:

$$\frac{c}{h} = \frac{\bar{\sigma}_{pc}}{0.85\beta_1 f_c' + \bar{\sigma}_{pc}} \qquad (1)$$

The nominal moment resistance is given by:

$$M_n = \text{ Force } \times \text{ Lever arm} = \bar{\sigma}_{pc}(h-c)b \times \frac{h}{2}[1+\frac{c}{h}(1-\beta_1)] \qquad (2)$$

which can be put in the following form:

$$M_n = \left(\frac{bh^2}{6}\right)\bar{\sigma}_{pc} \times 3\left(1-\frac{c}{h}\right)\left(1+\frac{c}{h}(1-\beta_1)\right) \qquad (3)$$

where β_1 is the depth ratio for the rectangular compression stress block as defined by ACI. Similar solutions for other cases of stress block profiles in compression and tension are given in [9, 10, 16]. The following results were obtained from their analysis.

The solution was shown to have very little sensitivity to the shape of the concrete stress block in compression (rectangular, parabolic, or triangular) but was mostly influenced as expected by the tensile response of the composite after cracking. It showed that for typical fiber reinforced cement composites, where the compressive strength is significantly higher than the direct tensile strength (before and after cracking) the depth of neutral axis of bending at nominal bending resistance is generally very small in comparison to the depth of the member.

If the average post-cracking stress of the composite in tension ($\bar{\sigma}_{pc}$) is assumed constant (i.e., leading to a rectangular stress block in the cracked tension zone) then the modulus of rupture in bending ranged from 2.5 to 3 times the average tensile post-cracking stress. This implies that for an elastic perfectly plastic stress-strain response in tension, the modulus of rupture will be 2.5 to 3 times the tensile strength (cracking or post-cracking). This range increases for a strain-hardening composite in tension. In a poor case scenario where the stress block in tension is triangular with zero resistance at the extreme tension fiber, the modulus of rupture becomes equal to the maximum post-cracking stress in tension (σ_{pc} or $\bar{\sigma}_{pc}$); such a scenario is unlikely unless the fibers are very small compared to the expected maximum crack opening at nominal bending resistance, and to the size of the member.

CONDITION FOR DEFLECTION-HARDENING RESPONSE

In order for the load-deflection curve to show a deflection hardening response after cracking (Fig. 5c), with possible multiple cracking, it is necessary for the nominal bending resistance (taken at the MOR point, Fig. 5) to exceed the cracking moment calculated from the LOP or BOP point (Fig. 5); this can be expressed as:

$$M_n \geq M_{cr} \qquad (4)$$

or $$\frac{M_n}{M_{cr}} \geq 1 \tag{5}$$

where M_n is the nominal moment resistance and M_{cr} is the cracking moment.

The above condition can be similarly expressed in terms of stresses as:

$$\frac{MOR}{\sigma_{cc}} \geq 1 \tag{6}$$

where MOR is the modulus of rupture or equivalent elastic bending stress at nominal bending resistance and σ_{cc} is the stress at first percolation cracking of the composite in tension (Fig. 2). The following general result was obtained in [9, 10].

> General Result: *In a fiber reinforced cement composite where fiber pull-out prevails after cracking, and the length of the fiber is larger than the expected crack opening, deflection hardening will occur when the average post-cracking strength in tension satisfies the following relation:*
>
> $$\bar{\sigma}_{pc} \geq k\sigma_{cc} \tag{7}$$
>
> *where k is a coefficient exceeding 1/3.*

Note that for $k = 1$ the material will qualify as "strain-hardening" and all "strain-hardening" FRC composites are expected to lead to a "deflection-hardening" response (Fig. 10). The real interest of Eq. (7) is that "deflection-hardening" can be achieved with "strain-softening" composites (Figs. 2a and 6).

Several examples were covered in [9, 10]. It was observed that in order to achieve a deflection hardening response in a fiber reinforced concrete where the fiber length is significantly larger than the expected crack mouth opening in bending, the value of $\bar{\sigma}_{pc}$ can be taken equal to σ_{pc} and a rounded value of k = 0.4 can be used for practical design with commonly used steel fibers.

CRITICAL VOLUME FRACTION OF FIBERS

Strain-Hardening in Tension

The critical volume fraction of fiber to achieve strain hardening behavior in tension has been discussed in past publications [6, 9, 10, 14, 15]. One simple form is given by the following equation which was based on the condition that the post-cracking strength of the composite in tension must exceed its cracking strength, that is, $\sigma_{pc} \geq \sigma_{cc}$ (Fig. 2):

$$(V_f)_{\text{critical-tension}} = \frac{1}{1 + \dfrac{\bar{\tau}}{\sigma_{mu}} \dfrac{L}{d} (\lambda - \alpha)} \tag{8}$$

where:

d	=	diameter of fiber
L	=	length of fiber
L/d	=	aspect ratio of fiber
σ_{mu}	=	tensile strength of the matrix
$\bar{\tau}$	=	average bond strength at the fiber matrix interface

The coefficients λ and α are each the product of several other coefficients explained in [6, 16] and used in the example below. Equation 8 is illustrated in Fig. 12.

Figure 12 illustrates the variation of the critical volume fraction of fibers needed to achieve a strain-hardening response (as per Eq. 8) versus the fiber aspect ratio (*L/d*) at different values of the ratio of average bond strength to tensile strength of the matrix. The values of the coefficients assumed are given in the figure and are the same as used in the example below (also in Ref. 6).

Deflection-Hardening in Bending

In a derivation similar to the one leading to the critical volume fraction of fibers to ensure strain-hardening behavior, the author has developed a similar expression leading to the critical volume fraction of fibers for which deflection-hardening is achieved [9, 10]. Deflection-hardening implies that the maximum equivalent elastic bending stress (or modulus of rupture, MOR) after first cracking is larger than the stress at first cracking in bending (LOP or BOP) (Eq. 6), and that multiple cracking would generally occur after first cracking. This condition leads to the constraint on the tensile response expressed by Eq. (6). Details are given in [9, 10]. The following solution was obtained for the critical volume fraction of fibers to achieve deflection-hardening behavior:

$$(V_f)_{\text{critical-bending}} = \frac{k}{k + \frac{\tau}{\sigma_{mu}} \frac{L}{d} (\lambda - k\alpha)} \tag{9}$$

where the notation is same as above, and k is a coefficient $\geq 1/3$. A value of $k = 0.4$ is recommended for practical applications of steel fiber reinforced concrete when general fiber pull-out prevails at failure. For $k = 1$ Eq. (9) reverts to Eq. (8).

Equation (9) is graphically illustrated in Fig. 13. The following coefficients were used: $\lambda = 0.3$, $\alpha = 0.05$ and $\Omega = \lambda - k\alpha = 0.28$. Similar figures can be developed for different numerical values of these coefficients.

Figure 13 is similar to Fig. 12 except that it illustrates the conditions for deflection-hardening fiber reinforced cement composites instead of strain-hardening ones. In comparing Fig. 13 to Fig. 12, it can be observed that deflection-hardening behavior requires significantly lower values of fiber content than the strain-hardening response in tension. Note that for a typical steel fiber with an aspect ratio of 50 to 100 and reasonable bond strength ($\tau / \sigma \approx 1$ to 2), deflection hardening can be easily achieved with volume fractions of steel fibers in the range of 0.7% to 1.5%. Examples of FRC materials where deflection hardening occurs at volume fraction of fibers ranging between 0.7% and 1.5% abound in the technical literature, and support the range of results obtained from the above analysis. The following example provides a numerical illustration.

Numerical Example of Critical Volume Fraction of Fibers

Assume the following coefficients for an FRC composite with randomly oriented and distributed steel fibers of circular section. The description of these coefficients is given elsewhere and would take too much space to explain [6, 9, 10, 16].

$$\alpha = \alpha_1 \times \alpha_2 \times \alpha_3 = \Pi \alpha_i$$

$$\alpha_1 = 0.1,\ \alpha_2 = 0.5,\ \alpha_3 = 1$$

$$\lambda = \lambda_1 \lambda_2 \lambda_3 \lambda_5$$

$$\lambda_1 = 0.25,\ \lambda_3 = 0.75,\ \lambda_4 = 1,\ \lambda_5 = 0.8$$

$$\lambda_2 = 4\alpha_2 \lambda_4 = 4 \times 0.5 \times 1 = 2$$

$$\frac{L_f}{d} = 100,\ \frac{\tau}{\sigma_{mu}} = 1$$

Thus:

$$\alpha = \alpha_1 \alpha_2 \alpha_3 = 0.1 \times 0.5 \times 1 = 0.05$$

$$\lambda = \lambda_1 \lambda_2 \lambda_3 \lambda_5 = 0.25 \times 2 \times 0.75 \times 0.8 = 0.30$$

For a strain-hardening response in tension:

$$\Omega = \lambda - \alpha = 0.30 - 0.05 = 0.25$$

$$(V_f)_{\text{critical-tension}} = \frac{1}{1+\dfrac{\tau}{\sigma_{mu}}\dfrac{L_f}{d}(\lambda-\alpha)} = \frac{1}{1+1\times 100(0.3-0.05)} = 0.0385 = 3.85\%$$ The corresponding point

can be located in Fig. 12 for $L_f/d = 100$ and $\tau/\sigma_{mu} = 1$

For a deflection-hardening response in bending, assuming $k = 0.4$:

$$\lambda - k\alpha = 0.30 - 0.4\times 0.05 = 0.28$$

$$(V_f)_{\text{critical-bending}} = \frac{k}{k+\dfrac{\tau}{\sigma_{mu}}\dfrac{L_f}{d}(\lambda-k\alpha)} = \frac{0.40}{0.40+1\times 100(0.28)} = 0.0141 = 1.41\%$$

This illustrates that deflection-hardening occurs at a volume fraction of fibers significantly smaller than that needed for strain-hardening in tension. Note that if the ratio of fiber bond strength to matrix tensile strength (τ/σ_{mu}) is doubled, the above volume values of critical volume fractions will be almost halved to $(V_f)_{\text{critical-tension}} = 1.96\%$ and $(V_f)_{\text{critical-bending}} = 0.71\%$. Such values of bond strength have been observed with the Torex twisted steel fibers [7] which were shown to maintain a high bond up to large slips.

Note that the values of α and λ used above are close to experimental values observed from tests on steel fiber reinforced concrete with deformed fibers.

Importance of Proper Mixing and Compaction

Equations (8 and 9) infer that the larger the volume fraction of fibers is, the better are the chances to achieve a strain-hardening or deflection-hardening response. However there is a practical limit beyond which proper mixing of the fibers is not possible, using standard mixing procedures, and a deterioration in mechanical properties may ensue, due to poor fiber dispersion, fiber balling, air entrapment and insufficient bonding at the fiber-matrix interface. Thus, optimization of composite performance should involve the manipulation of not only the fundamental composite parameters (matrix and fiber parameters), but also variables related to the production process, the rheology of the fresh mix, the properties of the hardening composite and the final application of the material.

CONCLUDING REMARKS ON COMPUTATION OF DEFLECTIONS

The moment versus curvature response of a given flexural member can be easily obtained from the tensile and compressive stress-strain curves of its material assuming linear strain distribution under bending and using a simple numerical iteration. In some instances where the stress-strain responses are simplified, closed form solutions for moment curvature can be developed [16]. Since, for concrete, the compressive strength is typically an order of magnitude higher than the tensile strength, the bending response is primarily controlled by the tensile response of the composite (the weakest link). For the typical case of FRC composites, the compressive response can be simulated by an elastic plastic curve up to a given maximum strain (say 0.003 or more depending on fiber parameters). The elastic modulus of the composite can be estimated with reasonable accuracy [1]. The bending response is not very sensitive to reasonable variations in compressive response, that is, by using for instance a parabolic stress profile instead of a rectangular stress profile. Special care, however, should be taken to select a realistic tensile stress-strain or stress-elongation response. This will influence the moment-curvature obtained from modeling bending, and therefore also influence the deflection calculated for a given flexural member by integrating the curvature.

The stress-elongation response in tension of strain-hardening FRC composites can be taken approximately as shown in Fig. 14. However, a multipoint curve can also be selected for better accuracy. Examples are given in Ref. [15] and also address stress-softening composites and the descending branch of a strain-hardening composite after crack localization.