cantilever beam	(-) M	$= qL^2/2$
simple beam	(+) M	$= qL^2/8$
hinged-fixed beam	(-) M	$= qL^2/8$
fixed-fixed beam	(-) M	$= qL^2/12$

Q = force due to differential shrinkage between the precast beam and the slab. See Term (8) of Eq. (2)

t = subscript denoting time

u = subscript denoting ultimate value

y_{cs} = distance from centroid of gross section to the extreme fiber in tension

Δ = maximum deflection of the beam. For simply supported beams, this refers to the center of the span.

ϵ_{sh} = shrinkage strain in inches/inch.

φ_{sh} = curvature due to shrinkage warping

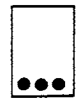
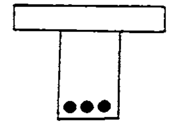
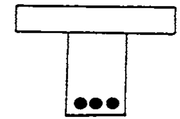
φ_{ss} = curvature due to shrinkage warping of precast beam up to slab casting

γ_s = ratio of shrinkage at slab casting to shrinkage at ultimate (referred to 7-day initial reading). The following values apply:

	for 'beam in position' @ 7 days*		
for the time between 'beam in position' and slab casting.	2 weeks	γ_s = 0.29	$(1-\gamma_s)$ = 0.71
	3 weeks	= 0.38	= 0.62
	1 month	= 0.46	= 0.54
	2 months	= 0.63	= 0.37
	3 months	= 0.72	= 0.28

*The differentials are to be used when the beam is 'in position' at an age other than 7 days.

TABLE 1 DETAILS OF LABORATORY SPECIMENS*

Beam No.	F1	F2[1]	F3[1]
Beam[2]			
Beam size	6" x 8"	6" x 8"	6" x 8"
Bar size and no.	3 - #4	3 - #4	3 - #4
$A_s (in^2)$	0.60	0.60	0.60
$\rho = A_s/bd$	0.01667	0.01667	0.01667

*multiply in. by 2.54 to obtain cm.

[1] the slabs were 21" x 3" in dimension and were made of normal weight concrete

[2] all the beams had a simply supported span of 15'-0", and were made of lightweight concrete.

Table 2. Measured and Computed Midspan Deflection for three Laboratory Reinforced Concrete Beams*

Beam name and comp. initial defl. (in).	Time between[2] 'slab in position' and slab cast-days	Defl. just before slab is cast			Comp. defl. by Eqs. (1), (2) with exp. para @180d.			Comp. Ult.[3] Deflection		
		meas (in)	comp. (in)	ratio[1]	meas (in)	comp (in)	ratio[1]	Eqs. (1), (2) with exp. para (in)	Eqs. (1), (2) with gen. para (in)	Eqs. (3), (4) with gen. para (in)
F1 0.07	--	--	--	--	0.34	0.30	0.89	0.38	0.47	0.55
F2 0.07	7 days	0.13	0.14	1.08	0.32	0.28	0.88	0.30	0.45	0.58
F3 0.07	51 days	0.25	0.24	0.96	0.45	0.40	0.89	0.43	0.59	0.58

[1] all ratios are computed/measured

[2] all beams were in position at beam age of 21 days

[3] terms (1) - (3) for beam F1 are 0.07", 0.09" and 0.22" respectively as per Eq. (1)
terms (1) - (8) for beam F2 are 0.07", 0.02", 0.01", 0.02", 0.04", 0.10", 0.02" and 0.02" as per Eq. (2)
terms (1) - (8) for beam F3 are 0.07", 0.05", 0.01", 0.11", 0.02", 0.10", 0.02" and 0.05" as per Eq. (2)

*multiply in. by 2.54 to obtain cm.

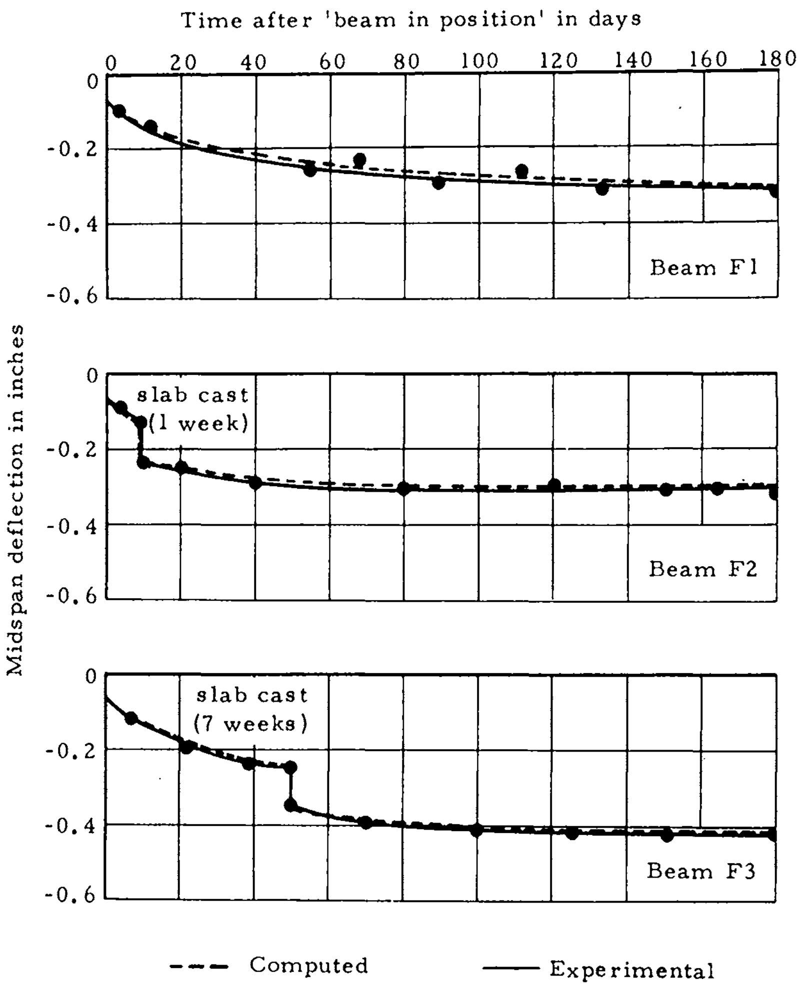


Fig. 1– Computed and experimental midspan deflection of beams of Group F (one non-composite and two composite beams

SP 43-17

Time-Dependent Deflections Of Composite Prestressed Concrete Beams

By
V. Jagannadha Rao and W. H. Dilger

Synopsis : Experiments and a method of analysis for the time - dependent deflection of composite prestressed concrete beams are described. Six different prestressed beams with and without additional non-prestressed steel were tested for the instantaneous and time-dependent strains and deflections for about 150 days after the application of the prestress. Good agreement is observed between the analytical results of the "varying stiffness method" and experimental values. The variation of time-dependent camber with different influencing parameters is discussed.

Keywords: beams (supports); composite construction (concrete and steel); creep properties; deflection; modulus of elasticity; prestressed concrete; reinforcing steels; shrinkage; stiffness methods; structural analysis.

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INTRODUCTION

The use of prestressed concrete beams combined with in-situ plain or reinforced concrete, is increasing in this decade in view of its many advantages as summarised by Okada et.al. (1), and Connoly (2). But the ACI Committee report (3) mentions that there are only a few tests for the evaluation of the long-time deflections of the composite prestressed concrete beams. Hence it recommends any rational method for the determination of the time-dependent deflection such as the one described by the subcommittee 2 of ACI Committee 209 (4). But as this method is only approximate, the authors carried out this investigation.

The principal objectives of this investigation were :

(1) to develop a new and rational method of analysis called "The Varying Stiffness Method" for the time-dependent deflection of the beams,

(2) to compare the analytical results with the experimental values, and

(3) to study the influence of untensioned steel in web, of deck reinforcement and of superimposed load on the time-dependent deflection.

METHOD OF ANALYSIS

In this method, the total time is divided into a number of intervals, and the changes of strain at the centroid of web and curvature of a section are determined in each interval. The changes of forces are assumed to act at the middle of the interval and the changes of strains are determined at the end of the interval. The section is assumed to consist of different elements of areas, like the areas of web and deck sections, areas of prestressing and reinforcing steels. The following assumptions (Ref. 5) are made in this method of analysis.

1. Stress-strain relationship is linear upto 40 percent of the ultimate strength for concrete.

2. Creep strains are proportional to the sustained stress upto 40 percent of the concrete strength.

3. The shrinkage strains are uniform over the depth of the section.

4. Shrinkage, creep and elastic strains are additive independent phenomena.

5. The relaxation loss in prestressing steel during any time interval is proportional to the force in the prestressing steel at the begining of the time - interval.

6. The modulus of elasticity of concrete is assumed to change with strength of concrete as per the equation given by ACI Committee 209 (4).

The shrinkage and creep strains are obtained using the ACI equations as given in equations 1 and 2 along with the experimentally determined constants for ultimate shrinkage and ultimate creep coefficient.

$$\epsilon_{sh}(k) = \epsilon_{shu} \left\{ t_e(k) - 7.0 \right\} / \left\{ 35.0 + t_e(k) - 7.0 \right\} \ldots \quad (1)$$

$$\epsilon_{cc}(k,j) = \frac{f_c}{E_c(j)} \, \phi \, (k,j) \quad (2)$$

Where

$$\phi\,(k,j) = \frac{\left\{ t_e(k) - t_m(j) \right\}^{0.6} \; 1.25\, t_m^{-0.118}(j)}{10.0 + \left\{ t_e(k) - t_m(j) \right\}^{0.6}} \cdot \phi_{cu}$$

The constitutive equation used for the relationship between the time-dependent strains and stresses is a modified version of the superposition principle:

$$\epsilon_{ct}(k) = \frac{f_c(1)}{E_c(1)} \left\{ 1.0 + \phi\,(k,1) \right\} + \sum_{i=2,k} \frac{f_c(i)}{E_c(i)} \left\{ 1 + R(k)\, \phi\,(k,i) \right\} + \epsilon_{sh}(k) \quad (3)$$

Where R(k) is the creeprecovery factor

$$R(k) = 0.6 + \frac{t_e(k) - t_e(i)}{40.0 + 3.2 \left\{ t_e(k) - t_m(j) \right\}}$$

The changes in the strains at the centroids of the different elements of the cross section are expressed in terms of the unknown strain vector - strain at the centroid of the web and curvature - using the compatibility conditions as given in (6) in any interval.

The changes in the forces of these elements are obtained in terms of the strain vector of the interval using their stiffnesses from the respective constitutive equations. The strain vector of the interval is determined after substituting the changes of forces into the equations of equilibrium. The total strains of any interval are obtained by adding the above changes to the total values of the previous interval. The same method is repeated at different sections, and the curvature diagram along the span is obtained. Camber at the midspan is calculated in every interval using the curvature diagram and conjugate beam method.

In brief the procedure is similar to the stiffness method of analysis, except that the stiffness of the composite section is varying from interval to interval. Hence the name "varying stiffness method" is suggested for this approach.

Elastic Analysis at Transfer of Prestress

During the first interval, the force vector - normal forces in the concrete, prestressing steel and reinforcing steel, if any , and moment in the concrete section - is due to the transfer of the prestressing force. The changes of the forces at the end of the first interval after transfer of prestress (age (2)) are given by the equations 4 to 7 in terms of the unknown strain vector

$$\Delta P_{c1}(2) = E_{c1}(2)\, A_{c1}\, \Delta\epsilon_{c1}(2) \tag{4}$$

$$\Delta M_{c1}(2) = E_{c1}(2)\, I_{c1} \Delta\psi_{c1}(2) \tag{5}$$

$$\Delta P_{s}(2) = A_{ps}\, E_{ps} \left\{\Delta\epsilon_{c1}(2) + e\Delta\psi_{c1}(2)\right\} + P_{i} \tag{6}$$

$$\Delta P_{s1}(2) = E_{s1}\, A_{s1} \left\{\Delta\epsilon_{c1}(2) + y_{s1}\Delta\psi_{c1}(2)\right\} \tag{7}$$

These forces are sustituted into the equations of equilibrium as shown in Equation B 1 of Appendix B. Equation B 2 shows the solution for the strain vector. The instantaneous curvature diagram is obtained by repeating the procedure at 8 sections in the span and hence the camber is evaluated.

Time-Dependent Analysis Before Composite Action

During the subsequent intervals of age till the time of casting the deck slab, there is only internal redistribution of forces at a section consistent with compatibility and equilibrium conditions. The total change of strain in any interval is composed of three components :

(1) cumulative strain due to the known forces of the previous intervals,

(2) strain due to the unknown forces of the present interval,

(3) strain due to shrinkage.

Equations 8, 9 and 10 give these three components.

$$\Delta \epsilon^{P}_{c1}(k) = \sum_{i=2,k-1} \frac{P_{c1}(i)}{E_{c1}(i)\, A_{c1}} \left\{ \phi_1(k,i) - \phi_1(k-1,i) \right\} R(k) \tag{8}$$

$$\Delta \epsilon_{c1}(k) = \frac{\Delta P_{c1}(k)}{E_{c1}(k)\, A_{c1}} \cdot \left\{ 1 + R(k) \cdot \phi_1(k,k) \right\} = \frac{P_{c1}(k)}{E_{v1}(k)\, A_{c1}} \tag{9}$$

$$\Delta \epsilon_{sh1}(k) = \epsilon_{sh1}(k) - \epsilon_{sh1}(k-1) \tag{10}$$

The iterative relation between the total strains of the consecutive intervals is given by

$$\epsilon_{ct1}(k) = \epsilon_{ct1}(k-1) + \Delta \epsilon^{P}_{c1}(k) + \Delta \epsilon_{c1}(k) + \Delta \epsilon_{sh1}(k) \tag{11}$$

The following equations 12 to 15 show the changes of forces in various elements expressed in terms of the unknown strain vector $\Delta \epsilon_{c1}(k)$, and $\psi_{c1}(k)$

$$\Delta P_{c1}(k) = E_{v1}(k)\, A_{c1}\, \Delta \epsilon_{c1}(k) \tag{12}$$

$$\Delta M_{c1}(k) = E_{v1}(k)\, I_{c1}\, \Delta \psi_{c1}(k) \tag{13}$$

$$\Delta P_s(k) = E_{ps}\, A_{ps} \left[\Delta \epsilon^{P}_{c1}(k) + \Delta \epsilon_{c1}(k) + \Delta \epsilon_{sh1}(k) + e \left\{ \Delta \psi^{P}_{c1}(k) + \Delta \psi_{c1}(k) \right\} \right] + \Delta P_r(k) \tag{14}$$

$$\Delta P_{s1}(k) = E_{s1}\, A_{s1} \left[\Delta \epsilon^{P}_{c1}(k) + \Delta \epsilon_{c1}(k) + \Delta \epsilon_{sh1}(k) + y_{s1} \left\{ \Delta \psi^{P}_{c1}(k) + \Delta \psi_{c1}(k) \right\} \right] \tag{15}$$

Equation B 3 gives the equilibrium equations involving the above forces, and the matrix equation B 4 shows the strain vector after solving the equilibrium equations. The time-dependent deflection is determined as explained in the previous section.

Analysis After Composite Action

Composite action is assumed to commence one day after casting the deck slab. In the subsequent time-intervals, only an internal redistribution of forces takes place due to creep and shrinkage of web concrete, differential shrinkage, differential creep and relaxation. Differential shrinkage and differential creep are defined as the differences in free shrinkage and free creep strains of the deck and web at the interface. The changes of forces are expressed in terms of the unknown strain vector as explained earlier and are shown in equations B 5 to B 11. These changes of forces are to be in equilibrium; and the matrix equation B 12 shows the equations of equilibrium. However, during the instantaneous applications of the dead load due to deck and the sustained superimposed dead load; the bending moments at the section due to the above are also taken into account while framing the equations of equilibrium. The solution for the strain vector is shown in matrix equation B 13. Then the time-dependent deflection is determined as before. The total analysis of the 6 beams is done on CDC 6400 computer, and a flow chart is provided in appendix C.

EXPERIMENTAL INVESTIGATION

The details of the beams of this investigation are shown in Figure 1. The rectangular web section is pretensioned and later a rectangular flange section is added to form a T-section.

Three beams were tested under superimposed dead load (two point loading) and three more similar beams were tested under no superimposed dead load. Table I gives the details of the two concretes which were used for the web and deck sections of the beams. Two dyform 7 - wire prestressing strands, having an ultimate tensile force of 43 kip (191 kN), were used with constant eccentricity to prestress the beams. A total pretensioning force of 65.8 kip (292 kN) was applied to the strands by a double acting hydraulic jack in two stages; and the force was measured by a load cell. The web concrete was poured one day after tensioning the steel. The top surface of the concrete was roughened to provide composite action with the deck slab to be cast after wards. Shear connectors were also used to aid for the composite action.

Five 8 in. (20 cm) gage lines were marked in the middle portion on either side of the beams. A demec gauge was used to measure the strains and dial gauges were used to measure the camber. At the age of 7 days the strands were slowly cut, using the oxyacetylene flame. The measurements were taken before and after the release of prestress. The difference between the two sets was taken as the elastic strain due to prestress and dead load of the beam. The beam was then simply supported and the time-dependent measurements were continued.

The deck section was cast 41 days after the casting of the web section, using externally supported form-work. The form-work was removed after 7 days, and