Failure Mechanism of Reinforced Concrete Under Cyclic Loading

by T. C. Hsu and M. Y. Mansour

Synopsis: The load-deformation response of R/C membrane elements (panels) subjected to reversed cyclic shear shows that the orientation of the steel bars with respect to the principal coordinate of the applied stresses has a strong effect on the "pinched shape" of the post-yield hysteretic loops. When the steel bars in a panel are oriented in the coordinate of the applied principal stresses, there is no "pinching effect," and the panel exhibits ductile behavior and high capacity of energy dissipation. Whereas, when the steel bars are oriented at an angle of 45° to the applied principal stresses, severe pinching effect is observed and the panel becomes more brittle.

This paper presents concisely a rational theory, called the *Cyclic Softened Membrane Model* (CSMM). This new rational theory is capable of predicting the entire history of the hysteretic loops (pre- and post-yield); can explain the mechanism behind the "pinching effect"; and can elucidate the failure mechanism that causes the deterioration of reinforced concrete structures under cyclic loading.

Keywords: cyclic loading; mechanism; pinching; reinforced concrete; shear; strain; steel; stress; softened truss models

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INTRODUCTION

Under earthquake condition, the behavior of wall-type reinforced concrete structures (such as shear walls) can each be visualized as assemblies of membrane elements subjected to in-plane cyclic stresses. The key to rational analysis of these structures is to study and thoroughly understand the behavior of isolated reinforced concrete elements, separated from the complications of whole-structure behavior. By studying panel elements' behavior under cyclic loading, rational analytical models are developed to predict their cyclic behavior. Then by incorporating these models into a finite element program, one can predict the behavior of whole structures under earthquake loading.

Structures located in high earthquake regions are designed to withstand not only moderate seismic loading within the elastic range, but must also be able to absorb the energy of high seismic loading in the inelastic range. Thus, it is necessary to evaluate the inelastic response of structures under high earthquake loading. When the shear force governs the response, as in the case of low-rise shear walls, the effect of shear on the panel response was thought to be responsible for the "pinching effect" in the hysteretic loops. The "pinching effect" which results in the degradation of stiffness and the reduction of energy dissipation capacity, was also surmized to be caused by the bond slips between concrete and steel bars.

Since 1997 a total of 15 panels had been subjected to reversed cyclic loading at the University of Houston using the Universal Panel Tester (Hsu, Belarbi and Pang, 1995), (1), equipped with a servo-control system that was capable of conducting strain-control tests (Hsu, Zhang and Gomez, 1995). Using this strain-control feature, the panels were loaded beyond the yield point up to 10 times the yield strain. It was found that the primary factor affecting the pinched behavior of hysteretic loops was the orientation of the steel bars with respect to the applied principal stresses. When the steel bars were oriented in the principal coordinate of the applied stresses (i.e. at a fixed angle α_2 of 90°), there was no pinching effect. When the steel bars orientation was $\alpha_2 = 45°$ or 68.2° to the principal coordinate, there was severe pinching effect.

Based on the testing of panels during the past 13 years, a series of theories were developed at UH to predict the monotonic behavior of cracked RC membrane elements. These theories include the rotating-angle softened truss models (RA-STM) (Hsu, 1993; Belarbi and Hsu, 1994, 1995; Pang and Hsu, 1995), (3,4,5,6), the fixed-angle softened truss model (FA-STM) (Pang and Hsu, 1996; Hsu and Zhang, 1997, Zhang and Hsu, 1998), (7,8,9), and the softened membrane model (SMM) (Zhu, 2000; Hsu & Zhu, 2001; Zhu and Hsu, 2002; Hsu and Zhu, 2002), (10,11,12,13). The SMM is an extension of the FA-STM with two improvements: First, SMM can predict the entire load-deformation history of panel behavior, including the post-peak descending branches, because the Hsu/Zhu ratios (or Poisson effect) are taken into account. Second, the complicated and empirical shear modulus of concrete in FA-STM is replaced by a simple and rational shear modulus in SMM. As a result, the solution algorithm of SMM is considerably simpler and the prediction more accurate than those of FA-STM.

In order to predict the behavior of membrane elements under cyclic loading, SMM for monotonic loading was extended for application to cyclic loading by adding the constitutive models of materials (concrete and embedded steel bars) in the unloading and reloading regions (Mansour, 2001; Mansour, Lee and Hsu, 2001; Mansour, Hsu and Lee, 2001), (14,15,16). This paper presents a new model, called the "Cyclic Softened Membrane Model (CSMM)," that is capable of predicting the entire cyclic history of load-deformation relationship, including the post-yield hysteretic loops and the pinching effect.

Applying the CSMM to predict the behavior of two panels CA3 ($\alpha_2 = 45°$) and CE3 ($\alpha_2 = 90°$), this paper discusses rationally the presence and absence of the pinching effect in the hysteretic loops. The comparison of the behavior of these two panels not only reveals the mechanism inherent in the pinching phenomenon, it also elucidates the failure mechanism of reinforced concrete composites under cyclic loading.

TESTS OF R/C SHEAR PANELS CA3 AND CE3

The two test panels, CA3 and CE3, have a size of 1398 mm x 1398 mm x 178 mm, and the steel bars are placed at angles α_2 of 45 and 90 degrees, respectively, as shown in Fig. 1, to form orthogonal steel grids. The reinforcing ratios of panels CA3 and CE3 are 1.7% and 1.2%, respectively, in each direction. The material properties for these two panels are summarized in Table 1.

The panels were subjected to reversed cyclic stresses in the horizontal and vertical directions using the Universal Panel Tester. When these two principal applied stresses were equal in magnitude and opposite in direction, a state of pure shear stress τ_{45° was created at the 45° direction to the applied principal stresses. The testing facility was equipped with a servo-control system capable of switching from load-control mode to strain-control mode as the yielding load was approached. In the strain-control mode, the shear strain (i.e. the algebraic sum of the horizontal and vertical strains), which followed a specified strain history, was used as an input signal to control the horizontal principal stress. The horizontal principal stress was, in turn, used to control the vertical principal stress such that they were always equal in magnitude and opposite in direction.

The hysteretic loops of the two panels, CA3 and CE3, are shown in Fig. 2 (a) and (b), respectively. In these figures, the vertical and horizontal axes represent the shear stress τ_{45° and the shear strain γ_{45° at 45° degrees to the principal coordinate of applied stresses. In Fig. 2 (a), the hysteretic loops of panel CA3 displayed a highly pinched shape that are generally associated with shear dominated behavior. The envelope curve of this panel also exhibited a distinct descending branch indicating a severe strength degradation of the panel with increasing shear strain magnitude. In contrast, no pinching effect was observed in Fig. 2 (b) for the hysteretic loops of panel CE3, with its steel grid parallel to the applied principal stresses. The envelope curve of panel CE3 did not have a descending branch and the strength deterioration was not noticeable.

CYCLIC SOFTENED MEMBRANE MODEL (CSMM)

In the basic concept of CSMM, the cracks are smeared throughout the reinforced concrete elements, and the reinforcing bars are uniformly distributed in two orthogonal directions (ℓ and t). The behavior of the panels is, therefore, formulated in terms of smeared (average) stresses and smeared (average) strains, and the continuum mechanics can be applied. The equilibrium equations, compatibility equations, and the material constitutive models are summarized in this paper:

Equilibrium Equations:

$$\sigma_\ell = \sigma_V^c \cos^2\alpha_2 + \sigma_H^c \sin^2\alpha_2 + \tau_{VH}^c 2\sin\alpha_2\cos\alpha_2 + \rho_\ell f_\ell \quad (1)$$

$$\sigma_t = \sigma_V^c \sin^2\alpha_2 + \sigma_H^c \cos^2\alpha_2 - \tau_{VH}^c 2\sin\alpha_2\cos\alpha_2 + \rho_t f_t \quad (2)$$

$$\tau_{\ell t} = (-\sigma_V^c + \sigma_H^c)\sin\alpha_2\cos\alpha_2 + \tau_{VH}^c(\cos^2\alpha_2 - \sin^2\alpha_2) \quad (3)$$

Compatibility Equations:

$$\varepsilon_\ell = \varepsilon_V \cos^2\alpha_2 + \varepsilon_H \sin^2\alpha_2 + \frac{\gamma_{VH}}{2} 2\sin\alpha_2\cos\alpha_2 \quad (4)$$

$$\varepsilon_t = \varepsilon_V \sin^2\alpha_2 + \varepsilon_H \cos^2\alpha_2 - \frac{\gamma_{VH}}{2} 2\sin\alpha_2\cos\alpha_2 \quad (5)$$

$$\frac{\gamma_{\ell t}}{2} = (-\varepsilon_V + \varepsilon_H)\sin\alpha_2\cos\alpha_2 + \frac{\gamma_{VH}}{2}(\cos^2\alpha_2 - \sin^2\alpha_2) \tag{6}$$

where the symbols are given in the list of Notations.

The constitutive laws of steel bars and concrete are summarized in the following sections.

Constitutive Relationships of Steel Bars

The proposed cyclic stress-strain relationship of steel bar embedded in concrete is shown by the solid curves in Fig. 3. This solid curves can be divided into two groups: the backbone envelope curves and the unloading and reloading curves. The figure also gives the monotonic stress-strain curves of bare steel bars as shown by the dotted lines.

Backbone Envelope Curves (Stage 1, Stage 2T and Stage 2C) - The monotonic tensile stress-strain curve of embedded steel bars proposed by Belarbi and Hsu (8) can be used to approximate the backbone envelope curve of the cyclic tensile stress-strain curves of steel bars. This monotonic, bilinear stress-strain relationship, which was adopted for Stage 1 and Stage 2T, is expressed as:

(Stage 1) $$f_s = E_s\varepsilon_s \qquad (\varepsilon_s \le \varepsilon_n) \tag{7a}$$

(Stage 2T) $$f_s = f_y\left[(0.91 - 2B) + (0.02 + 0.25B\frac{\varepsilon_s}{\varepsilon_y})\right] \qquad (\varepsilon_s > \varepsilon_n) \tag{7b}$$

where f_s and ε_s are the average stress and strain of mild steel bars, respectively; f_y and ε_y are the yield stress and strain of bare mild steel bars, respectively; E_s is the modulus of elasticity of steel bars; and $\varepsilon_n = \varepsilon_y(0.93 - 2B)$. The parameter B is given by $B = (f_{cr}/f_y)^{1.5}/\rho$, where ρ is the reinforcement steel ratio and ≥ 0.5%. f_{cr} is the cracking strength of concrete given by $f_{cr} = 0.31\sqrt{f'_c(MPa)}$.

If the steel stress progresses in the compression region, the average maximum stress f_s is limited to the compressive yield stress $-f_y$, Eq. (7c), as indicated in Stage 2C.

(Stage 2C) $$f_s = -f_y \qquad (f_s \le -f_y) \tag{7c}$$

Unloading and Reloading Curves (Stage 3 and Stage 4) - The cyclic stress-strain relationship for steel bars subjected to reversed cyclic loading must include the unloading and the reloading branches. In this analysis, the unloading and reloading stress-strain relationships are expressed by the Ramberg-Osgood type of equations:

(Stage 3 and Sage 4) $$\varepsilon_s - \varepsilon_i = \frac{f_s - f_i}{E_s}\left[1 + A^{-R}\left|\frac{f_s - f_i}{f_y}\right|^{R-1}\right] \qquad (7d)$$

where f_s and ε_s are the smeared stress and smeared strain of an embedded steel bar; f_i and ε_i are the smeared stress and smeared strain of steel bars at the load reversal point.

The coefficients A and R in Eq. (7d) were determined from the reversed cyclic loading tests at the University of Houston (Mansour, Lee and Hsu, 2000), (15), to best fit the test results: $A = 1.9k_p^{-0.1}$, $R = 10k_p^{-0.2}$. The parameter in the coefficients A and R is the plastic strain ratio k_p which is defined as the ratio $\varepsilon_p/\varepsilon_n = (\varepsilon_i - \varepsilon_n)/\varepsilon_n$. In this expression ε_p is the plastic strain, and ε_n is the initial yield strain.

Constitutive Relationships of Concrete

The cyclic stress-strain curves of concrete in the CSMM are shown in Fig. 4. The curves are divided into three groups: the compressive backbone envelope curves, the tensile backbone envelope curves, and the unloading and reloading curves. In addition, a rational shear modulus of concrete is adopted in CSMM. This shear modulus is simply a function of the stress-strain relationships of concrete in compression and in tension.

Compressive Backbone Envelope Curves (C1 and C2) - As pointed out by Mansour, Lee and Hsu (2001), (15), the backbone envelope curves for the cyclic compressive stress-strain curves of concrete can be expressed by a small modification of the monotonic compressive stress-strain curve of concrete proposed by Belarbi and Hsu (1995), (5):

(Stage C1) $$\sigma_c = D(\zeta f_c' - f_{cT4}')\left[2\left(\frac{\varepsilon_c}{\zeta\varepsilon_o}\right) - \left(\frac{\varepsilon_c}{\zeta\varepsilon_o}\right)^2\right] + f_{cT4}' \qquad \varepsilon_o \le \varepsilon_c < 0 \quad (8a)$$

(Stage C2) $$\sigma_c = D\zeta f_c'\left[1 - \left(\frac{\varepsilon_c/\varepsilon_o - 1}{4/\zeta - 1}\right)^2\right] \qquad \varepsilon_c < \varepsilon_o \quad (8b)$$

where σ_c is the smeared stress of concrete; ε_c is the smeared strain of concrete; $f_c^{'}$ and ε_o are the maximum concrete compressive cylinder strength and the peak cylinder compressive strain, respectively ; $f_{cT4}^{'}$ is the stress at point TD (Fig. 4) between Stage C1 and Stage T4.

The softening coefficient ζ in Eqs. (8a) and (8b) was given by Zhang and Hsu (1998), (9):

$$\zeta = \frac{5.8}{\sqrt{f_c^{'}(MPa)}}\frac{1}{\sqrt{1+k\varepsilon_1}} \leq 0.9 \tag{8c}$$

where ε_1 is the tensile strains, either ε_H or ε_V in cyclic loading; k is the loading coefficient taken as $400/\eta$ for proportional loading, where $\eta = 1$ for the case of pure shear and equal volume of steel in the ℓ and t directions.

The damage coefficient, D, takes into account the effect of concrete damage in one direction on the concrete strength in the perpendicular direction:

$$D = 1 - 0.4\frac{\varepsilon_{mc}}{\varepsilon_o} \tag{8d}$$

where ε_{mc} is the maximum compression strain occurring in the immediate opposite direction and perpendicular to the compression strain being considered. ε_o is the strain corresponding to the peak cylinder strength. The damage coefficient, which is not considered in Mansour, Hsu and Lee (2001), (16), and Mansour, Lee and Hsu (2001), (15), does take care of the descending branches of the backbone envelope curves, as shown in Fig. 2 (a) for panel CA3.

Tensile Backbone Envelope Curves (T1 and T2) - The envelope curves for the cyclic tensile stress-strain curves of concrete can be expressed by the monotonic tensile stress-strain curve of concrete proposed by Belarbi and Hsu (1994), (4):

(Stage T1) $$\sigma_c = E_c\varepsilon_c \qquad 0 \leq \varepsilon_c \leq \varepsilon_{cr} \tag{8e}$$

(Stage T2) $$\sigma_c = f_{cr}\left(\frac{\varepsilon_{cr}}{\varepsilon_c}\right)^{0.4} \qquad \varepsilon_c > \varepsilon_{cr} \tag{8f}$$

where E_c is the modulus of elasticity of concrete taken as $3875\sqrt{f_c^{'}(MPa)}$; f_{cr} is the cracking stress of concrete taken as $0.31\sqrt{f_c^{'}(MPa)}$; and ε_{cr} is the cracking strain of concrete taken as 0.00008.

Unloading and Reloading Curves - A linear expression is proposed for the unloading and reloading curves as follows:

$$\sigma_c = \sigma_{ci} + E_{cc}(\varepsilon_{ci} - \varepsilon_c) \tag{8g}$$

where σ_{ci} and ε_{ci} are concrete stress and strain at the load reversal point "i" or at the point where the stages change; E_{cc} is the slope of the linear expression and is taken to be:

$$E_{cc} = \frac{\sigma_{ci} - \sigma_{ci+1}}{\varepsilon_{ci} - \varepsilon_{ci+1}} \tag{8h}$$

where σ_{ci+1} and ε_{ci+1} are the concrete stress and strain at the end of the stage under consideration. The points for ε_{ci}, σ_{ci}, ε_{ci+1} and σ_{ci+1} are specified in Fig. 4. Notice that the concrete stress of point TD is $1.5f_{cr} + 0.8f_{cT2}^{'}$, rather than $f_{cr} + 0.8f_{cT2}^{'}$ given in Mansour, Hsu and Lee (2001), (16) and Mansour, Lee and Hsu (2001), (15). This slight modification is taken to improve the agreement of crack closing strains between the CSMM predictions and the experimental results for all the test panels.

Constitutive Relationship of Concrete in Shear - Zhu, Hsu and Lee (2001), (17), showed that a rational relationship exists between the shear stress and the shear strain of concrete. This rational relationship in the (H,V) system of axes is given by the following expression:

$$\tau_{VH}^{c} = \frac{\sigma_H^c - \sigma_V^c}{2(\varepsilon_H - \varepsilon_V)}\gamma_{VH} \tag{9}$$

Poisson Effect – the Poisson effect of cracked reinforced concrete subjected to monotonic loading is characterized by two Hsu/Zhu ratios based on the smeared crack concept (Zhu, 2000; Hsu and Zhu, 2001), (10,11). The Hsu/Zhu ratio υ_{CT} (compression strain caused by tensile strain) was found to be zero for the entire post-cracking range. The Hsu/Zhu ratio υ_{TC} (tensile strain caused by compression strain), however, was found to be a function of the maximum steel strain and varied from 0.2 to 1.9. Before the Hsu/Zhu ratios of cracked reinforced concrete subjected to cyclic loading can be determined, an average value of $\upsilon_{TC} = 1.0$ is assumed in this study.

COMPARISON OF CSMM PREDICTIONS WITH EXPERIMENTS

Pinching Effect

The CSMM-predicted hysteretic loops of the two panels CA3 and CD3 are plotted in Fig. 2 (a) and (b), together with the experimental curves. It can be seen that the CSMM is capable of predicting the pinched shape of the hysteretic loops of panels CA3 as well as the fully-rounded hysteretic loops of panel CE3.

Panel CA3 ($\alpha_2 = 45°$) - The first cycle of the hysteretic loops beyond yielding for panel CA3 is plotted in Fig. 5(a). Four points A, B, C, and D are chosen in Fig. 5(a) to illustrate the *presence* of the pinched shape. Point A is at the maximum shear strain of the first cycle beyond yielding. Point B is at the stage where the shear stress is zero after unloading. Point C with a very low shear stress is taken in the negative shear strain region at the end of the low-stress pinching zone just before the sudden increase in stiffness. Point D is at the maximum negative shear strain of the first negative cycle. The three segments of curves from point A to point D in Fig. 5(a) clearly define the pinched shape of the hysteretic loops.

Panel CE3 ($\alpha_2 = 90°$) - The first cycle of the hysteretic loops beyond yielding for panel CE3 is plotted in Fig. 5(b). Four points A, B, C, and D are chosen in Fig. 5(b) to illustrate the *absence* of the pinched shape. Points A, B and D correspond to the same three points in Fig. 5(a). However, point C with a high shear stress is taken in the negative shear strain region when the compression steel reaches yielding. The three segments of curves from point A to point D in Fig. 5(b) clearly show the absence of pinching.

Stresses and Strains Represented by Mohr Circles

The smeared strains, the applied stresses, the smeared concrete stresses, and the smeared steel stresses at the four points (A, B, C and D) chosen in Fig. 5 (a) and (b) are represented by Mohr circles as shown in Figs. 6 and 7 for panels CA3 and CE3, respectively. Using the Mohr circles has two advantages: First, Mohr circles represent the entire stress or strain state in an element, i.e. stresses or strains in all directions. Second, Mohr circles are the best means to help illustrate the mechanism behind the pinching effect and the failure mechanism of reinforced concrete elements under cyclic loading. To better visualize what is happening in the two panels, CA3 and CE3, subjected to reversed cyclic loading, we will examine the stresses and the strains as the applied shear stresses change from point A to point D.

Panel CA3 ($\alpha_2 = 45°$) - At point A in the first post-yield cycle, Fig. 6, the maximum applied shear stress, τ_{45°, of 6.84 MPa produces a large shear strain, γ_{45°, of 0.00828 (twice the number shown in the Mohr circle because the vertical axis represents $\gamma/2$). To resist this applied shear stress, the concrete is subjected to a maximum vertical compressive stress, σ_V^c, of 13.64 MPa, and a maximum smeared steel stresses, ρf_s, of 6.80 MPa in both the longitudinal and transverse directions.

When the panel is unloaded from point A to point B, the applied shear stress of 6.84 MPa is reduced to virtually zero. From equilibrium, the compressive stress in the concrete and the tensile stress in the steel are also approaching zero. Correspondingly, the shear strain is reduced from 0.00828 at point A to a value of 0.00288 at point B. This unloading process produces an almost linear shear stiffness due to the normal relaxation of steel and concrete. These nearly proportional reductions of stresses in the concrete and steel are also related to the closing of cracks in the vertical direction. The horizontal strain, ε_H, decreases from 0.00762 at point A to 0.00278 at point B, but remains in tension with significant size of crack width. The compressive strain in the vertical direction, ε_V, decrease from –0.00067 at point A to -0.00011 at point B.

When the positive shear strain of 0.00288 at point B is reversed to become a negative shear strain of –0.00268 at point C, the vertical strain increases to a tensile strain of 0.00273, while the horizontal strain further decreases to a small tensile value of 0.000039 (not in compression). This large change of shear strain through the origin, however, is not accompanied by a corresponding change in the applied shear stress, meaning that the shear stiffness in the BC region is very small. This is because at point C the vertical cracks are not fully closed, and the concrete compressive stress cannot be developed. Without forming an effective set of concrete compressive struts, the stresses in the steel bars also cannot be developed. Hence, both the concrete stress and steel stress remain small at point C.

When the negative shear strain reaches –0.00862 at point D, the vertical cracks are fully closed (ε_H = –0.00067) and the concrete compressive struts are fully formed. The concrete struts can now resist a compressive stress of 13.50 MPa, and the smeared steel in both the longitudinal and transverse directions are resisting a stress of 6.59 MPa. Correspondingly, the element is resisting an applied shear stress of 6.91 MPa. In other words, in the reloading CD range the shear stiffness is restored to its normal magnitude.

The pinched shape of the hysteretic loop of panel CA3, Fig. 2 (a), is formed by a small shear stiffness in the BC region, sandwiched between two large shear stiffnesses in the AB and the CD regions.