Recently Identified Aspects of Ductile Seismic Torsional Response of Reinforced Concrete Buildings

by T. Paulay

Synopsis: With few exceptions, code provisions relevant to torsional phenomena in buildings subjected to seismic effects, are based on elastic structural behaviour. The key parameter is stiffness eccentricity. The appropriateness of this approach to the design of systems expected to respond in a ductile manner is questioned. The degree of restraint with respect to system twist, strength eccentricity and the pattern of element yield displacements are considered to be more important parameters. For the purposes of seismic design, bi-linear force-displacement approximations of the elasto-plastic behaviour of reinforced concrete systems and their constituent elements, are considered to be adequate. Strategies aiming at the elimination of undesirable effects of torsional phenomena in ductile systems are addressed.

The findings of this study are based on a re-definition of some common terms of structural engineering, such stiffness, yield displacement and displacement ductility relationships. Contradictions with corresponding terms applicable to elastic systems are demonstrated. The introduction of these features, relevant to bilinear modelling of reinforced concrete elements, precedes the examination of the designer's options for the control of earthquake-induced displacement demands resulting in system translations and twist.

Keywords: codes; deflections; ductility; earthquake-resistant structures; reinforced concrete; stiffness; strength; structural design; torsion in buildings

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INTRODUCTION

Studies of the assessment of the structural performance of existing buildings with earthquake risk (1) triggered inquiries addressing the likely response of buildings as constructed, rather than their compliance with a particular code. A major perceived need was the estimation of torsion-induced displacements of elements of ductile systems (2,3). In the process several issues emerged with apparent conflict with ingredients of existing design practice. The description of progressively emerging fallacies, firmly entrenched in widely accepted routine seismic design techniques (4), is the subject of this presentation.

The perceived need to address earthquake-induced displacements in ductile systems motivated this study. It led to the introduction of unfamiliar, yet not necessarily new, principles. The freedom available to the designer, when strength is to be assigned to elements of a system, is a particularly interesting feature of the conclusions. Identification of structural behaviour, rationale and transparency of a viable design strategy, combined with simplicity of application, were central issues of this motivation.

DESIGN CRITERIA

The primary purpose of this study was to address means by which performance criteria, conforming with the appropriate limit state, may be rationally executed. The criteria considered were:

- Expected earthquake-induced deformations, including those due to system twist, should be limited to ensure that the displacement ductility capacity of any element of the system, $\mu_{\Delta imax}$, will not be exceeded.
- Maximum interstorey displacements, to be expected at locations remote from the centre of mass, should not exceed those considered acceptable for buildings, typically 2-2.5% of the storey height.
- More restrictive performance criteria may require displacements associated with a specific limit state to be less than those allowed by the displacement ductility capacity of constituent elements.

TERMINOLOGY USED

- In the study of earthquake-induced displacement of buildings, reference will be made to the structural system.
- A structural system comprises lateral force-resisting elements generally arranged in two orthogonal directions. Due to torsional effects, elements of the system may be subject to different storey displacements. Typical elements are bents of ductile frames or interconnected walls in the same plane.
- A lateral force-resisting element may comprise several components. Components will be subjected to identical displacements. Typical components are beams or columns or walls.

Examples are illustrated in Fig. 1. In this study only simple mass systems are considered.

TRADITIONAL CONCEPTS OF THE THEORY OF ELASTICITY

The requirements for static equilibrium and deformation compatibility in a statically indeterminate structure, are well established. These principles are still widely used when strength is assigned to lateral force-resisting elements of a system. The procedure is consistent with the now abandoned principle, that the resulting stresses should be well within the elastic domain of material behaviour.

With the introduction of equivalent lateral static seismic design forces and the acceptance of ductile response, the same technique continued to be widely used. It implied the notion that strength assigned proportionally to element stiffness will eventually result in the simultaneous onset of yielding in all elements.

As a corollary it is generally assumed that components with traditionally defined stiffness, based on flexural rigidity, EI, will have increased yield displacements as their strength is being increased. The flexural rigidity is the product of the modulus of elasticity of the material, E, such as concrete, and I is the second moment of the cross sectional area of a prismatic member. Its value may be adjusted in recognition of the effects of cracking (5).

Requirements for static equilibrium and deformation compatibility in statically indeterminate structures, are well established. Strength allocation based on these principles implies that the intended strength of all components of a lateral force-resisting element will occur at the same displacement. Subsequent adjustments of strength, relying of strength redistribution in a partially nonlinear system, is assumed to result in corresponding changes in yield displacements. Some fallacies relevant to the traditional modelling of elasto-plastic behaviour, widely used in seismic design practice, have been identified (6,7,8).

It is postulated that for the purposes of seismic design the simulation of the non-linear behaviour of a ductile system, can be adequately modelled with bi-linear force-displacement relationships. Implications in terms of component, element or system response, are briefly summarized. The findings enable displacement relationships, including those due to system twist, to be quantified in a simple form.

MODELLING OF DUCTILE COMPONENT BEHAVIOUR

The typical elasto-plastic behaviour of a reinforced concrete and steel component is shown by the full line curves in Fig.2. The relative unit strength shown corresponds to that developed when yield strain is first developed in the extreme tension fibre. The dashed line for the RC section illustrates the effects of crack developments during first loading. This is of no interest in seismic design based on ductile response.

The associated yield curvature of the critical section is readily determined (9). Even under reversing repeated loading, the stiffness of the component is adequately simulated by the slope of the force-displacement relationship up to the unit relative strength. Once the pattern of bending moments is known, the relevant displacement associated with the onset of yielding may also be determined. For a realistic estimate of this displacement the second moment of the section area, transformed into properties of the concrete, needs to be used. In routine design this and the strength at first yield, involve cumbersome computations, preferably to be avoided. Simpler approaches are suggested.

The evaluation of the nominal strength, V_{ni}, of the component, shown subsequently to be a choice of the designer, is an essential part of the design routine. As Fig.2 shows, a linear extension of the elastic response (Δ'_{yc}) allows the nominal or reference yield displacement, Δ_{yc}, to be determined. The dashed lines in Fig.2 show how the nonlinear response, for example of a RC component, can than be simulated. If desired, the designer may allow for post-yield stiffness of the component.

Yield Curvature

The first step in estimating the yield displacement, Δ'_{yi}, of a component is the evaluation of the yield curvature at the critical section of a potential plastic hinge. From the study of strain patterns it was found (10,11) that nominal curvatures linearly extrapolated, as in Fig. 2, from that at the onset of yielding at the extreme tension fibres of typical reinforced concrete sections subjected to flexure with moderate axial loads, are approximately constant. For the purposes of seismic design such estimates should be considered adequate. For example the yield curvature associated with bilinear modelling, as in Fig.2, of a typical rectangular wall section (10), is of the order of

$$\phi_y \approx 2\epsilon_y/\ell_w \tag{1}$$

where ϵ_y is the yield strain of the steel used and ℓ_w is the length of the wall (Fig.3(a)).

This simple relationship shows that yield deformations are sensitive to the grade of reinforcement used and are inversely proportional to the overall depth of a component. Yield curvatures for components with different lengths cannot be identical, as assumed when using conventional analyses based on elastic behaviour. An important feature of yield curvature is, that for the purposes of seismic design, it is not affected by the strength assigned to the component.

Yield Displacements

As previously stated, once the nominal yield curvature, ϕ_{yi}, at the critical section or sections of a component is established, the yield displacement, used in the elasto-plastic bi-linear modelling, is readily evaluated. For example, yield displacements of reinforced concrete structural walls, subjected to a particular pattern of lateral forces are found to be

$$\Delta_{yi} = C\phi_{yi}\,h_{wi}^2 \approx (2C\,h_{wi}^2\,\epsilon_y)/\ell_{wi} \propto 1/\ell_{wi} \tag{2}$$

where C is a coefficient which quantifies the pattern of lateral forces, and h_{wi} is the height of the wall. When wall heights are identical and the grade of reinforcement used is the same in one building, the bracketed term in eq.(2) is a constant. Therefore, in these common cases yield displacements of wall components, such as shown in Fig.3(a), are inversely proportional to the length of the walls, ℓ_{wi}.

The simple relationship (eq.(2)) can be conveniently used in seismic design whenever relative values, for example for the estimation of displacement ductilities, are sufficient. Equation (2) is fundamental in establishing displacement relationships between components, elements and the entire system (Fig.1). Implications of these relationships have been previously reviewed (7,8,12). The important conclusion to be drawn is the fact that yield displacements are functions of geometric and material properties and are independent of the nominal strength of components.

A RE-DEFINITION OF STIFFNESS

When seismic design is based on bi-linear modelling of ductile behaviour, as shown in Fig.2, the stiffness of the component is simply

$$k_i = V_{ni}/\Delta_{yi} \tag{3}$$

where V_{ni} is the nominal strength assigned to the component and Δ_{yi} is a geometry-dependent predetermined property, as defined in eq.(2). It is seen that, contrary to traditional assumptions which are extensively used in seismic design, stiffness is proportional to strength assigned to the component by the designer. An example to be presented in Fig.3, where the bi-liner force-displacement behaviour of components with different lengths compared, will offer further explanations.

ASSIGNMENT OF STRENGTHS

As the bi-linear modelling in Fig. 2 implies, components will develop their nominal strength, V_{ni}, when their nominal yield displacement is imposed. Therefore, components with different yield displacements can never yield simultaneously. Correspondingly, components of an element, such as seen in Fig.1, will develop their nominal strength in a given sequence until all components have yielded, dictated by the attainment of their yield displacements. From this it may be concluded that, within rational limits, strength to components may be assigned arbitrarily. In this and subsequent examples it is assumed that the total nominal strength, V_E, is unity.

The principle offers great possibilities to the designer to exploit this freedom of choice in nominal strengths. Thereby more rational and economic solutions may be achieved. It is a fundamental tool aiding the mitigation or even elimination of the detrimental effects of torsional phenomena in ductile systems.

STRENGTH, STIFFNESS AND DISPLACEMENT RELATIONSHIPS

The relevance of these relationships, so important in seismic design, will be illustrated with an example presented in Fig.3. The relative lengths of four interconnected rectangular reinforced concrete cantilever walls with identical width, are such that the traditionally defined second moment of sectional area of the components bear the ratio of 1, 2, 4 and 8, respectively, to each other. Conventional design will assign lateral strength, V_{ni}, in these proportions.

The bilinear simulation of the components and the element, based on this traditional distribution of strengths, is shown in Fig.3(b). The relative strength of component (4) with the greatest length, is thus 8/15 = 0.53, while its relative yield displacement is according to eq.(2) $\Delta_{y4} = ½ = 0.5$. Therefore, its relative stiffness (eq.(3)) is $k_4 = 0.53/0.5 = 1.06$. The superposition of bilinear component responses results in the total response of the element. This can be again simulated by simple bi-linear modelling. No post-yield stiffness was assumed in this example. The superposition relevant to this traditional strength distribution, shown in Fig.3(b), allows the total translational stiffness of the 4-component element to be defined as:

$$k_e = \Sigma k_i = \Sigma(V_{ni}/\Delta_{yi}) \tag{4}$$

This in turn enables the nominal yield displacement of the element to be quantified as

$$\Delta_y = \Sigma V_{ni} / \Sigma k_i = V_E / k_e \tag{5}$$

The element nominal yield displacement, so derived and shown as 0.58 displacement units in Fig.3(b), is the weighted average of the component yield displacements. It is seen that when this displacement is imposed, some components will have yielded while some others would not. The purpose of the element yield displacement, given by eq.(5), is to allow the element displacement ductility capacity to be defined.

It is evident that, when the displacement ductility capacity of the components is specified, for example, as $\mu_{\Delta imax} = 5$, the element displacement at the ultimate limit state must be limited to $\Delta_u \leq 5\Delta_{yimin}$. In the example the smallest yield displacement is that of component (4), i.e., 0.5 displacement units. Therefore, the displacement ductility demand on this element, controlled by component (4),should be limited to $\mu_\Delta \leq 5 \times 0.5/0.58 \approx 4.3$.

The results of an entirely different assignment of component strengths, the aim of which will be referred to subsequently, is illustrated in Fig.3(c). With different strengths, component stiffness are also different. The reduced total stiffness in this case, defined by eq.(4), resulted in a small increase of the nominal element yield displacement to 0.66 units and a corresponding reduction of the element displacement ductility capacity to 3.79.

A similar procedure can be used when the corresponding properties of the entire building system, comprising a number of parallel elements is determined.

TORSIONAL PHENOMENA IN DUCTILE SYSTEMS

The Origin of Torsional Actions

The purpose of considering the effects of torque and consequent twist on a system, is to estimate displacements imposed on elements, additional to that which would occur at the centre of the mass. The study is based on the usual assumption that lateral force resisting elements are interconnected by infinitely rigid floor diaphragms.

Under the action of a base shear force, the static torque, resulting from stiffness or strength eccentricities, may be readily determined. The possibility of resisting such a torque during the elastic or ductile response needs to be studied first. Whenever diaphragm rotation, i.e., system twist, occurs, a dynamically induced torque, due to the rotary inertia of the distributed mass, will also be introduced. This is difficult to predict. However, the conditions under which a static and/or a dynamic torque can develop may be identified. This is subsequently illustrated.

Eccentricities

The cause of torsional phenomena is eccentricity. To define this, familiar locations within the plan of a building need to be identified. With reference to Fig.4 the centre of the distributed mass is shown as CM. For the assessment of elastic response the centre of element stiffness, commonly referred to as the centre of rigidity, CR, is of importance. After the elements have entered the inelastic domain of response and the base shear capacity, V_E, of the system is developed, the associated centre of stiffness becomes meaningless. At this stage the important location is the centre of resistance, i.e., that of the nominal strengths of all the yielding elements, CV. Simple equilibrium criteria enables the location, CV, of the corresponding resultant forces, generated by displacements in either of the principal directions, larger than that causing yielding in the element with the smallest length, to be readily determined. A typical example is shown in Fig. 4.

At certain instants of a seismic event, some elements may be subjected to displacements which are less than the relevant yield displacement. At such a stage the full base shear capacity, V_E, will not be developed. Moreover, maximum displacements of the yielding elements are not likely to approach their displacement capacity, unless an extremely large angle of twist is imposed by the earthquake. As a general rule, the maximum displacement ductility demand on any element, which is of major interest to the designer, can be expected to occur with the ductile response of all elements, when the full base shear capacity, V_E, is developed.

Torsional phenomena will arise whenever stiffness or strength eccentricities with respect to the base shear force, V_{Ey}, acting at the centre of mass, CM, denoted as e_{rx} and e_{vx}, respectively, exist. Both can be readily determined. The definition by eq.(3) of the strength-dependent element stiffness implies that the two types of eccentricities are related to each other. This feature is not recognised in current codified design procedures. The key ingredient of the design strategy, relevant to torsional phenomena, is the ability of the astute designer to assign strengths to lateral force-resisting elements in such a way as to obtain a suitable location for the centre of strength, CV.

In the study of the influence of stiffness (e_{rx}) and strength (e_{vx}) eccentricities, the following four classes of structures are distinguished: (a) $e_{rx} = e_{vx} = 0$, (b) $e_{vx} = 0$ but $e_{rx} \neq 0$, (c) $e_{rx} = 0$ but $e_{vx} \neq 0$, (d) $e_{rx} \neq 0$ and $e_{vx} \neq 0$. Case (a) is the condition for, what is commonly referred to as, a torsionally balanced system. In these torsional phenomena do not arise, unless rotary motions are introduced at the foundations. Case (d) represents the majority of real structures. If desired, the designer may readily achieve conditions (b) or (c) above.

Mass eccentricity, e_{mx}, with respect to the geometric centre of the plan, may also exist. This, however, will affect only the rotary inertia of the mass, to be taken into account in the evaluation of dynamic response, a subject absent in current design practice and beyond the scope of this presentation.

Degree of Torsional Restraint

It has been suggested (13) that, as part of routine structural design, when uni-directional seismic attack in the principal directions of the building are considered separately, two types of torsional mechanisms should be distinguished. Special features of the behaviour of each of these are briefly reviewed subsequently.

Torsionally unrestrained systems - When only one element, transverse to the direction of the base shear, V_{Ey}, and concurrent with CM, is present, as shown in Fig.5(a), no torque can be introduced to the system after the translatory elements have entered the inelastic domain. This feature is not recognized in existing code provisions (14). Therefore, in terms of rotary motions, the ductile system is unrestrained. Element (3) is effective only in resisting earthquake-induced forces in the x direction.

Figure 5(b) shows a similar example where the sole two-component transverse element is eccentric with respect to CM. In terms of a static base shear, V_{Ey}, this ductile system is also torsionally unrestrained. However, during dynamic response system twist will introduce displacements in the x direction to the transverse components (3) and (4) and to CM. The acceleration of the mass in the x direction will thus introduce an inertia force at CM. An equal and opposite force will then be developed in the transverse element, leading to a dynamically induced torque.

To illustrate relationships between stiffness and strength eccentricities, the simple specific model comprising rectangular cantilever walls, as shown in Figs 5(a) and 6, will be considered. The two elements (1) and (2) are required to sustain a base shear in the y direction, V_{Ey}. Equilibrium criteria suggest that the nominal strength of element (1) should be twice that of element (2). However, this condition will not be achieved in real buildings because compliance with existing code requirements, and inevitable variance of the strengths of the elements, as constructed, will lead to some strength eccentricity. For example the nominal strength of element (2) may be exactly as intended, i.e., $V_{n2} = V_{Ey}/3$. However, the reinforcement of element (1) is such that its nominal strength turned out to be greater than that required, i.e., $V_{n1} = \lambda_1 (2V_{Ey}/3)$, where $\lambda_1 \geq 1.0$. Similarly situations may be considered when inevitably element (2) may have some excess strength, quantified by the parameter λ_2.

Figure 6 shows how strength eccentricities, expressed as the ratio e_{vx}/D, vary with increasing excess strength of one or the other of the two elements. With increased values of λ_i, the total base shear capacity of the system will also increase. This is shown by the dashed straight lines in Fig.6.

In a previous section it was demonstrated by eq.(3), that the stiffness of an element with given sectional dimensions is proportional to its nominal strength. Hence the stiffness eccentricities corresponding with the excess strength of the elements of the model in Fig.5(a) are readily determined. These are shown, again in terms of e_{rx}/D, in Fig.6.

The purpose of presenting this example is to illustrate that:

- Stiffness and strength eccentricities are interdependent.
- For a wide range of the variation of the excess strength of elements, differences between the respective eccentricities, i.e., $|e_{rx} - e_{vx}|$, remain essentially constant for systems with a given geometry.
- The designer, having full control over strength eccentricity, thereby also controls stiffness eccentricity. According to current design practice the latter is only a geometry-dependent property of the system, beyond the control of the designer.
- Classes of systems characterized by the relations of the two types of eccentricities, previously presented as (a) to (d), are readily identified. Because in this example structure (Fig. 5(a)) the geometry of the two elements is different, a torsionally balanced condition can never be achieved.
- In terms of dynamic response, a particularly favourable situation arises in this structure when $\lambda_2 \approx 1.2$, i.e., when, contrary to indications of statics, the nominal strength of element (2) is made 60%, rather than 50%, of that of element (1). In this case CR and CV are approximately equidistant on opposite sides of CM. In attempting to reduce the adverse effects of torsional phenomena, the designer can readily approach this condition.

<u>Torsionally restrained systems</u> - When transverse elements in at least two planes are provided, as in Fig. 4, a torque, for example due to strength eccentricity, can be resisted after all translatory elements have yielded. Thereby rotational deformations of the diaphragm, i.e., twist, are restrained. This is particularly the case when the transverse elements remain elastic while they resist the torque generated. Designers are aware that such systems are preferable. Both models, shown in Fig. 5, are torsionally restrained in terms of a base shear, V_{Ex}.

Specific examples will be used to illustrate how the designer can influence effects of torsional phenomena on inelastic element displacement demands.

To facilitate meaningful comparisons, three alternatives of an example structure are presented. For these, important design quantities, such as eccentricities, total translatory system stiffness, system yield displacement and the attainable maximum