

Prediction of FRP Debonding Using the Global-Energy-Balance Approach

Chris J. Burgoyne¹, Mithila Achintha², Garfield X. Guan³

¹ *Dept of Engineering, University of Cambridge, Cambridge, UK, cjb@eng.cam.ac.uk*

² *Dept of Eng. Science, University of Oxford, Oxford, UK, mithila.achintha@eng.ox.ac.uk*

³ *Dept of Engineering, University of Cambridge, Cambridge, UK, xgg20@cam.ac.uk*

SYNOPSIS:

A major research program was carried out to analyze the mechanism of FRP debonding from concrete beams using global-energy-balance approach (GEBA). The key findings are that the fracture process zone is small so there is no R-curve to consider, failure is dominated by Mode I behavior, and the theory agrees well with tests. The analyses developed in the study provide an essential tool that will enable fracture mechanics to be used to determine the load at which FRP plates will debond from concrete beams. This obviates the need for finite element (FE) analyses in situations where reliable details of the interface geometry and crack-tip stress fields are not attainable for an accurate analysis. This paper presents an overview of the GEBA analyses that is described in detail elsewhere, and explains the slightly unconventional assumptions made in the analyses, such as the revised moment-curvature model, the location of an effective centroid, the separate consideration of the FRP and the RC beam for the purposes of the analysis, the use of Mode I fracture energies and the absence of an R-curve in the fracture mechanics analysis.

Keywords: Branson's model, external strengthening, fracture mechanics, FRP, interface debonding, mixed-mode crack

Chris J. Burgoyne is Head of the Structures Group at the University of Cambridge. He has carried out extensive research into the use of advanced fibers for use in a variety of Civil, Structural and Marine applications. He has published extensively and was a founder member of ACI 440.

Mithila Achintha is a post-doctoral research assistant in the Department of Engineering Science, University of Oxford, UK. He received his BSc(Eng) from the University of Moratuwa, Sri Lanka and PhD from University of Cambridge, UK in 2003 and 2010 respectively. His research interests include the application of fiber-reinforced polymers in civil engineering and fracture mechanics of concrete.

Garfield X. Guan is a PhD student at the University of Cambridge. He received his B.Eng. degree in Civil Engineering from the University of Hong Kong. He is currently studying the application of concrete fracture mechanics in RC structure retrofitting, and he has wider research interests in RC structures and seismic engineering.

INTRODUCTION

A comprehensive study of the debonding of FRP plates from concrete beams using the global-energy-balance approach (GEBA) has been undertaken; many of the concepts used have been described in elsewhere^{1,2,3,4}. This paper summarizes the overall logic but space does not permit a detailed discussion of the individual elements.

A high stress may cause a crack to form near the interface between the concrete and the FRP, but that crack will only propagate if more energy is thereby released than it takes to form the new fracture surfaces. It is thus a fracture mechanics problem, not a stress-analysis problem. The Global Energy Balance Approach to the study of debonding of FRP plates from concrete beams is a very simple concept, but requires understanding of some complex mechanics. When a crack forms, the beam loses some of its stiffness so the load does more work; most of which is stored by an increase in strain energy within the beam. So to calculate the energy that is released these two quantities have to be calculated with reasonable accuracy.

This paper addresses a number of issues that are important in the analysis of debonding of FRP plates from concrete beams. The evaluation of energy states in cracked concrete beams using the stress-strain (σ - ϵ) behavior over the whole beam is very complex, so in the present model a simpler integration of moment-curvature (M - κ) is used, but even determining the curvature is complex. Branson's model⁵ was conceived only for beams with conventional steel reinforcement, and only up to the point where the steel yields. When external FRP is added there is an additional layer of reinforcement, with different bond characteristics so it is incorrect to incorporate the FRP as a second layer of steel reinforcement in the Branson's model. The model also has to be applicable after the steel has yielded. The present paper shows how these issues have been addressed.

The debonding analyses rely on knowing the fracture energy of concrete (G_C), a parameter that is easy to define but less easy to determine, and one that is rarely assessed in experimental studies, even if it is the most important concrete parameter when studying debonding. Unlike glass, the fracture process zone (FPZ) in concrete is large, typically over 300 mm (1 ft) long, and may have a width of several times the aggregate size⁶. Conventional fracture analysis would require modelling the whole of this zone, but a debonding fracture occurs in a narrow zone of the concrete cover that has FRP on one side and the steel rebar on the other (Fig. 1a). Propagation of a short crack is unlikely to allow the FPZ to develop fully. The present GEBA analyses rely on the fact that fracture energy is not affected by the length of the debonding crack; this is discussed.

Premature FRP debonding hampers efficient use of externally bonded FRP plates in flexural strengthening of reinforced concrete (RC) beams, and uncertainty about the governing mechanisms means that there is no reliable theory that can be applied by designers. The earlier work of the present authors has shown that the area near the plate end, and zones where widening of flexural cracks causes interface flaws, are those most susceptible to the initiation of debonding (Fig. 1b); the two modes are referred to as "plate-end" (PE) and "intermediate-crack-induced" (IC) debonding respectively³. PE debonding initiates from the vicinity of the plate end and propagates towards the mid-span of the beam, whereas IC debonding initiates at a high-moment zone and propagates towards a low-moment zone (Fig. 1b). It has also been shown that the present GEBA model can be used to analyze debonding of steel plates provided that the plates remain within elastic limits.

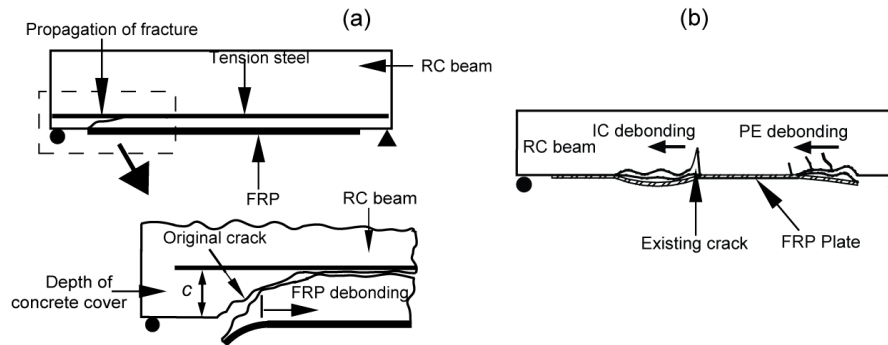


Figure 1. – (a) Debonding propagates in the concrete just above the interface
(b) PE and IC debonding

Manufacturers have now developed adhesives that are sufficiently tough that, if used correctly, debonding usually takes place in the concrete beam just above the interface (Fig. 1a). It has been observed that FRP debonding initiates from the propagation of a dominant crack in the vicinity of the interface, and hence, fracture-mechanics-based analyses have often been used in the literature to determine failure loads. However these analyses, (for example Günes⁷) were often based on the pioneering theories of Hutchinson and Suo⁸, which were intended for the analysis of interface debonding in thin-layered elastic materials. Because of the long FPZ associated with fractures in concrete, a reliable solution for FRP debonding cannot be obtained from the theories of linear-elastic-fracture-mechanics (LEFM). Furthermore, the non-linear FE models, such as the J-integral method, that would be needed to simulate the debonding of FRPs, require far more detail of the interface properties than will ever be available, even to the analyst of laboratory experiments, and certainly not to designers. Most analyses reported in the literature have only been calibrated against individual researchers' own, usually limited, test data, and none of these analyses has received a wide acceptance.

There is a need for a more physically-based fracture mechanics model that represents energy balance requirements, rather than an unreliable analysis of crack-tip stress field. The model has to be based on governing parameters that can be reliably determined and should be able to analyze all modes of debonding in beams with a wide range of dimensions. The earlier work of the present authors has shown that the energy states in beams can be determined to an accuracy good enough for models of RC beams, and the incorporation of these energy estimates in the GEBA-based debonding model correlated well with test data reported in the literature³. The work obviates the need for unreliable FE analyses which have often been used in the studies reported in the literature.

The "current state" of a system will be at a position of minimum total potential energy. The GEBA model determines that debonding will occur if the energy available for a potential small extension of an existing interface crack exceeds the energy needed to form the new fracture surfaces formed during this crack propagation. If the energy release rate (G_R) associated with an existing crack exceeds the fracture energy of concrete (G_C) the crack will propagate. How the initial crack developed up to the current state is immaterial and it is sufficient to assume that flaws of the relevant size are *likely* to exist in critical locations. The model can be used to determine the shortest crack that triggers failure at a given load and also the failure load of a beam with an existing crack of known length³.

However, determinations of both G_R and G_C , either theoretically or experimentally, are complex even in a research context; the present study has developed appropriate methods to calculate both the parameters to an accuracy that is reliable enough to be used in the analysis of FRP debonding. The G_R associated with a given crack can be determined by considering the energy changes that take place in the system during a potential unit extension of the crack. However, this analysis is not trivial; integration of $M-\kappa$ relationships determined on the assumptions that the section is uncracked (i.e. concrete in the tension zone is fully effective) significantly underestimates the energy state of the beam, while if the beam is assumed to be fully-cracked (i.e. no tensile contribution from concrete) the energy state is significantly overestimated. Various tension-stiffening models exist, although these were primarily developed to calculate the deflections of RC beams. Branson's model⁵ indirectly incorporates the effects of tension-

stiffening into the stiffness of cracked RC beam sections by defining an effective stiffness in the deflection analysis of conventional RC beams (i.e. with steel reinforcement only) and the model has been widely verified in the literature. In the present work the model has been modified to take account of several factors so that it can be used for the energy analysis of strengthened beams².

Fracture propagates in the concrete substrate, so it is necessary to know G_C of the concretes from which the beams are made. In different modes of fracture (i.e. opening; shear; or a combination of both), different stress–displacement fields will develop in the vicinity of the crack, so it is necessary to determine G_C corresponds to the correct fracture mode. The present study has shown that the interface of a strengthened beam, which is primarily carrying shear, actually fails in tension. The relative vertical displacements between the two crack faces of the original shear crack and the difference in the curvatures between the RC beam and the FRP at the plate end introduce significant peeling (tensile) stresses in the vicinity of the interface flaw that causes PE debonding (Fig. 2a). During IC debonding, the force in the FRP (F_p) acts with an eccentricity with respect to the tip of the interface crack and induces significant tension at the crack tip (Fig. 2b). The fracture propagates locally by opening (i.e. as a Mode I crack)⁴. The work has also shown that the results of shear–lap experiments, which have often been reported in the literature in studies of FRP debonding, only provide an estimate for the shear mode fracture energy which is not relevant for the debonding analyses. The incorporation of the Mode I fracture energy of concrete (G_{CI}) in the debonding analyses provided results that match with test data reported in the literature³. The present paper explains why particular values for G_{CI} have been chosen in that study.

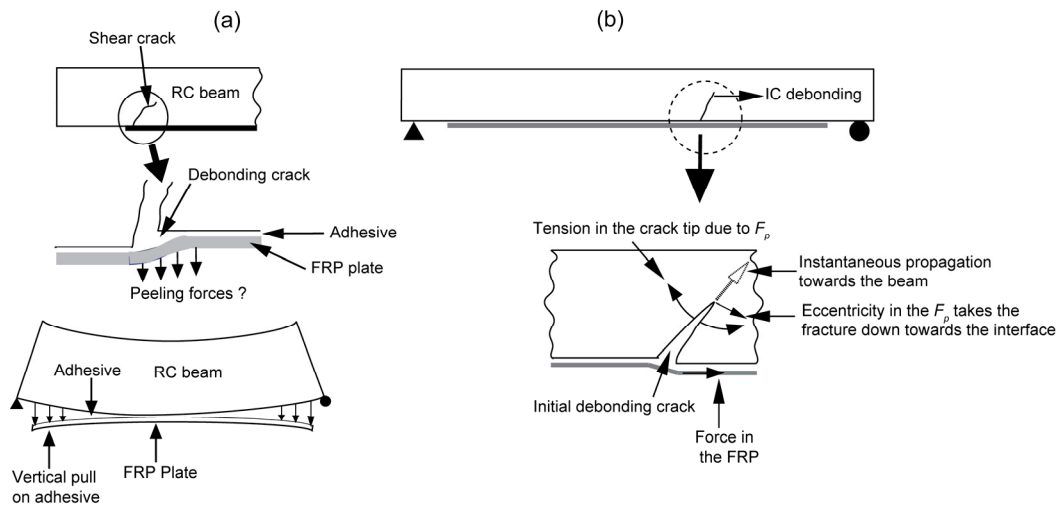


Figure 2. – Tensile stresses of significant magnitudes are developed in the crack tip
(a) PE debonding (b) IC debonding

MECHANISM OF FRP DEBONDING

PE debonding forms when an interface crack forms due to the widening of a shear crack in the vicinity of the plate end, whereas IC debonding is triggered by an interface crack formed due to the widening of a critical flexural crack in the high moment zone (Fig. 1b). Shear and peeling stress concentrations develop due to geometric restraints and also due to the relative vertical movements of the faces of the critical shear/flexural crack, which triggers further propagation of the already-formed interface crack, causing separation of FRP from the concrete beam⁴.

The whole concrete cover of the beam usually separates during PE debonding, whereas a concrete layer of only a few millimeters thick separates during IC debonding⁹. This observation has led some researchers to analyze PE debonding as a shear failure of the RC beam and IC debonding as an interface failure⁹. However, both modes are essentially fractures in the concrete substrate and the difference in the fracture path is due to the effect of the difference in the magnitudes of force in the FRP (F_p) in the corresponding locations. The principal interfacial stress

in the vicinity of the debonding crack will be at about 45° to the interface, and thus it is expected that the crack will move into the beam. PE-debonding cracks usually move up to the level of tension steel bars and the final failure occurs at this level. However, during IC debonding, the large force F_p that acts eccentrically to the crack tip takes the fracture back down towards the interface, and debonding propagates in the concrete just a few millimeters above the interface (Fig. 2). The present GEBA model has been used to analyse the both modes of FRP debonding³.

Experimental studies reported in the literature suggest that PE debonding is the likely failure mode of most strengthened beams⁹, and hence, a number of studies investigated the effectiveness of the use of plate-end anchoring systems (e.g. FRP jackets¹⁰) and the use of long FRP plates right up to the beam end¹¹ as methods to resist PE debonding. Although the methods improved the strength and the ductility of beams, the fact that PE debonding still takes place may be due to the stress concentrations developed due to the anchoring devices. The use of long FRPs usually eliminated PE debonding, but this can lead to premature IC debonding¹¹.

ANALYSIS OF FRP DEBONDING USING GEBA METHOD

Fracture mechanics investigates the possible propagations of existing cracks and it better simulates the mechanism of interface debonding than any other method. Even though numerous cracks are inevitably present in the interface between the FRP and the concrete, most are either not long enough, or not weak enough to trigger failure. Only the propagation of a dominant crack triggers failure and it is this that is analyzed by the GEBA method. Analytical methods that compare interfacial stress concentrations with the interface strength might not be able to distinguish the critical crack that triggers debonding from other minor flaws. Conventional fracture mechanics analyses that determine G_R at the crack tip, such as the J integral method, cannot be performed because the microstructure in the vicinity of the interface is unknown. Nonlinear finite element packages use special types of element to model crack tips and the FPZ, such as collapsing elements to model stress singularity at a crack tip and special “spring” type elements that can incorporate the effects of the cohesive forces in the FPZ. However, because of the heterogeneous nature of concrete the details can never be known in sufficient detail, even for laboratory specimens, let alone when designing a new structure.

As an alternative, the present GEBA model predicts FRP debonding by comparing two governing parameters (G_R and G_{CI}), both of which can be determined to an appropriate accuracy. An essential first stage of the calculation of G_R associated with an existing crack, is the determination of the energy state of the beam at a given applied load, derived using an appropriate $M-\kappa$ model that is discussed below.

Moment–curvature analysis of strengthened beams

Although the analysis of a RC beam section with the assumptions that the section was uncracked or fully-cracked is straightforward, an accurate $M-\kappa$ analysis of a partially-cracked section whilst incorporating the effects of tension-stiffening of cracked concrete is not trivial. Branson’s model (Eq. 1) incorporates the effects into the section’s stiffness indirectly by defining an effective stiffness (I_{eff}) as an interpolation between those of the uncracked (I_{un}) and fully-cracked (I_{fc}) sections, primarily with a view to being able to predict the deflections of beams (I_{un} and I_{fc} can be determined to an acceptable accuracy from an elastic and a cracked-elastic analysis respectively). The interpolation coefficient (K in Eq. 1) takes account of the current cracking level of the section and is defined as the ratio between the moment that causes the first flexural crack in the section (M_{cr}) and the current applied moment (M_{app}). The model has been widely validated against experimental results of deflections of RC beams, and also, with appropriate modifications, the method has been used in the analyses of prestressed concrete beams¹¹¹².

$$I_{eff} = K I_{un} + (1 - K) I_{fc} \quad \text{where} \quad K = \left(M_{cr} / M_{app} \right)^4 \quad (1)$$

I_{eff} in Eq. (1) is the effective second moment of area of the equivalent transformed concrete section of modulus E_c , so curvature of the section (κ) can be determined as:

$$\kappa = M_{app} / (E_c I) \quad (2)$$

A modified Branson’s model for strengthened beams

In a strengthened beam, the $M-\kappa$ relationships of uncracked/fully-cracked sections can be determined as those of a conventional beam whilst taking account of the effect of the force in the FRP (F_p). The objective is to use the Branson's concept to determine stiffness and hence the curvature in a partially-cracked section and is discussed below. However, the original Branson's model will need modifying here because it is not correct to consider FRP as a second layer of internal steel reinforcement and it is also necessary to consider the strain state when the FRP is partially debonded. The earlier work of the present authors² has shown that, if the effect of F_p was incorporated as an external prestressing force on the RC beam alone, the analyses can be simplified conceptually since it allows analyzing the RC beam portion as a conventional beam (Fig. 3). However, this complicates the analysis since now it requires analyzing the RC section for a combined action of a compressive force and moment, both acting at the section's centroid. At any given location along the beam, the moment due to the applied load is generally known (M_{app}). This acts on the combined beam section (i.e. RC beam section + FRP plate). The portion of this applied moment which is resisted by the RC section alone (M_{eff}) can be determined if the location of the centroid is known. The energy in the RC beam can be then determined as $\int_L M_{eff} \kappa dx + \int_L F_p \varepsilon_0 dx$ (κ – curvature, ε_0 – strain in the

beam section at the location of centroid and L – beam span). The determination of the energy in the FRP is trivial since FRP is assumed to be linear elastic. The separation of the energies due to moment and force in this way is only valid if the actions are calculated relative to the section's centroid, the determination of which is discussed below.

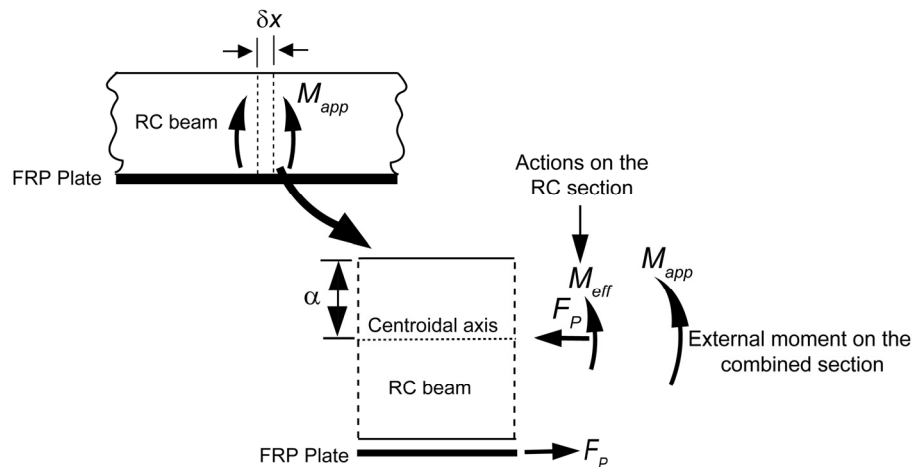


Figure 3. – Taking the effect of FRP as an external prestressing force requires analyzing the RC section for the combined action of M_{eff} and F_p

Complexities over conventional Branson analysis

The original Branson's model only applies to RC beams subject to pure bending, which can be regarded as a simple couple, so there is no need to define a particular reference axis. This will no longer be true in the analysis of strengthened beams because the RC section has to be analyzed under M_{eff} and F_p (Fig. 3). The modification of Branson's model to take account of the effects of the axial force is discussed below.

Force in the FRP

Branson's original model is only concerned with stiffness and is not used to determine the strains in the beam that are assumed to be adequate because separate checks (either permissible stress or section strength) would be performed in association. However, in a strengthened beam, if the FRP is bonded to the beam section then the strain in the FRP is locally compatible with that in the extreme tension fiber of the RC section, and if the FRP is partly debonded over a zone in the beam span, then the extension of the FRP over the unbonded region is compatible with that of the extreme tension fiber of the RC beam over the same zone². The new model, therefore, requires the satisfaction of a compatibility condition between the FRP and the concrete, which means strong assumptions need to be made about the strains, and hence stresses, in the beam section; these have to be determined from the effective stiffness. As a result, F_p at a given location in the beam span cannot be known *a priori*, so it is treated as a variable and determined numerically using a least-squares method². Once an accurate value for F_p is known, all other parameters may be evaluated.

Location of equivalent centroid

It is impossible to find an axis in a cracked RC beam section that satisfies the requirements of centroid in a linear elastic analysis (i.e. $F_p = \int_A \sigma dA$ and $M_{eff} = \int_A \sigma y dA$ where A is the cross sectional area and y is the distance from the centroid; strain energy given by $\int_L M_{eff} \kappa dx + \int_L F_p \varepsilon_0 dx$). The concept was thus developed of an “equivalent centroid” (α) of strengthened RC beam sections that will allow the separation of M_{eff} and F_p approximately, and hence, to determine the energy state in the usual way to an accuracy good enough to be used in the debonding analyses². For uncracked and fully-cracked sections the σ - ε distributions are reliably known from respective section analyses, and hence, the relevant equivalent centroids can be determined by considering the equivalent transformed sections (α_{un} and α_{fc} respectively), whilst taking account of the secant modules of non-linear materials. The centroidal location of a partially-cracked section (α_{eff}) will then be interpolated between respective α_{un} and α_{fc} using Branson’s concept; this analysis is presented elsewhere². It should also be noted that the materials are non-linear, and the secant modulus varies with stress, and hence, the location of α changes with the applied load. Thus, there is no fixed centroid that is a section property.

Modified interpolation coefficient

When the amount of cracking of a RC section increases, the tension-stiffening effects eventually become ineffective. In Branson’s model, however, the stiffness in the section becomes asymptotic to the fully-cracked state but never reaches it. That model was intended to represent sections at working loads and well below yield of the steel bars. However, the fully-cracked state will be reached in strengthened beams because that is why they needed strengthening in the first place. In the present model, it was assumed that a beam section will be fully-cracked at the moment that causes first yielding of tension steel (M_y); and the interpolation coefficient (K_p) is determined by considering the magnitudes of M_{cr} , M_y and M_{app} ². This modification does not cause significant changes to the predictions from the original model for conventional RC beams but avoids a discontinuity in stiffness when the steel yields (an example is shown in Fig. 4).

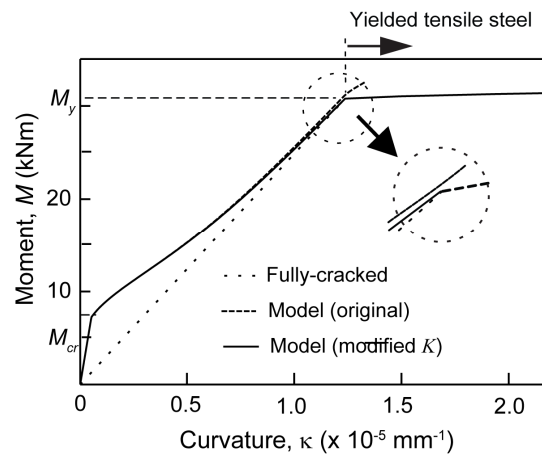


Figure 4. – For a conventional RC beam, the model predictions with the assumption of the fully-cracked state at M_y do not cause significant changes to the results at moments below M_y
(Curvature - 1.10^{-5} mm = $2.5.10^{-4}$ ins; Moment - 10 kNm = 7376 ft-lbs)

As a further complication, because of the presence of the axial load, the effective moment acting on the RC section alone depends on the choice of the axis about which it acts. The obvious choice would be the centroid, but as shown above this is not at a fixed location, either along the beam or as the loading increases. To avoid complications, it was decided to use a fixed axis about which to calculate the effective moment used to determine the interpolation

factor K_p ; the mid-depth axis of the beam was chosen, the corresponding moments are M_{cr-m} , M_{y-m} , and M_{eff-m} respectively (Eq. 3).

$$K_p = (M_{cr-m}/M_{eff-m})^4 \left[1 - \left\{ (M_{app-m} - M_{cr-m}) / (M_{y-m} - M_{eff-m}) \right\}^4 \right] \quad (3)$$

Equivalent elastic stiffness

As discussed previously, M_{eff} and κ of uncracked and fully-cracked sections can be directly determined from respective section analyses. The objective is to use Branson's concept to determine effective stiffness and hence κ of partially-cracked sections. Since the present model is to be applied to sections where the nonlinearity of material behavior needs to be taken into account, the cracked-elastic analysis used in the Branson's model is not applicable. Since the Young's modulus of concrete is no longer fixed, there is no value in defining an equivalent second moment of area. Instead, an equivalent elastic stiffness (B) is defined in place of the product of E_c and I used in the original model. The values of B for uncracked and fully-cracked sections (B_{un} and B_{fc} respectively) can be determined from the direct section analyses². For a partially-cracked section, the corresponding B_{un} and B_{fc} are first calculated, and that of the actual section (B_{eff}) is then interpolated (Eq. 4). The location of α_{eff} and hence M_{eff} of the actual section is known so combining M_{eff} with B_{eff} the κ of the section can be determined (Eq. 5). Flowcharts of the complete process of determining M_{eff} and κ of sections with bonded or partially debonded FRPs are shown in Fig. 5 (a) and (b) respectively.

$$B_{eff} = K_p B_{uc} + (1 - K_p) B_{fc} \quad \text{where } K_p \text{ is from Eq. (3)} \quad (4)$$

$$\kappa = M_{eff} / B_{eff} \quad (5)$$

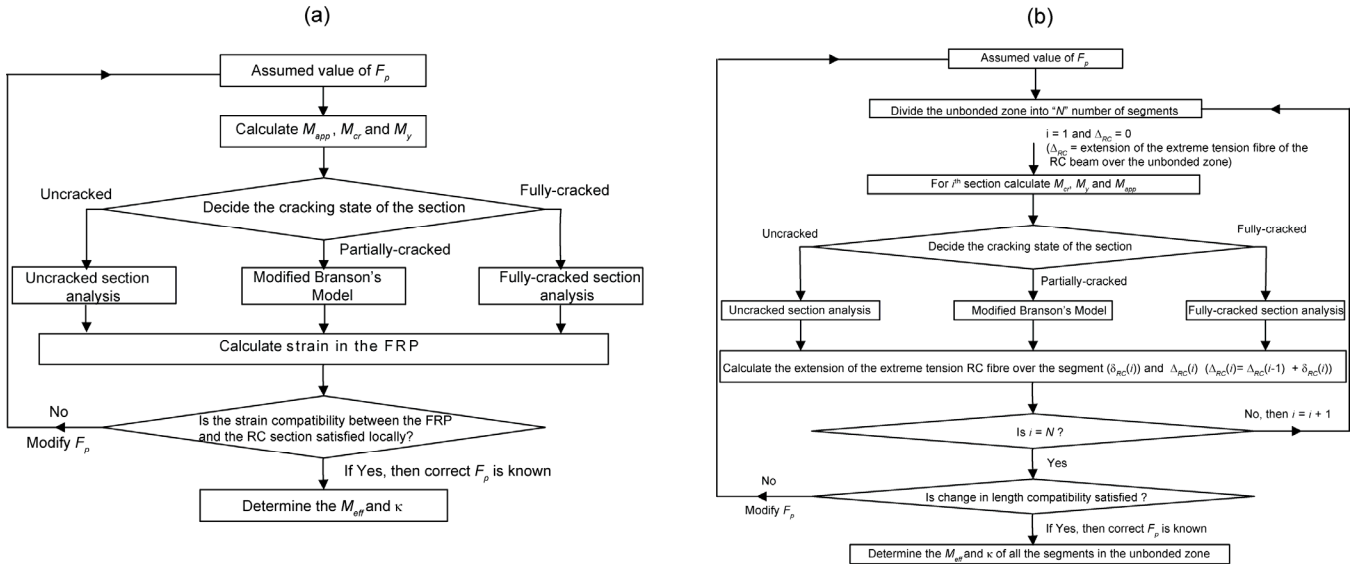


Figure 5. – Step-by-step procedure to calculate M_{eff} and κ in sections with (a) fully-bonded (b) unbonded FRP

VALIDATION OF THE PRESENT M- κ MODEL

The M- κ model was applied to several sets of beam tests reported in the literature and the model was found to be accurate enough for a model of RC beams². All the beams analyzed in the study were tested as simply-supported beams and a large database of specimens, including a variety of material/geometric properties, was investigated. The axial force in the beam can be either externally applied or exists due to unbalanced stress resultants acting on the RC section as in the case of strengthened beams. Comparisons with the test data and the present model were made under both of these categories. The model was also used to determine strain and deflection profiles of strengthened beams. A single example for each of M- κ and strain profile comparison in strengthened beams are shown in Fig. 6.

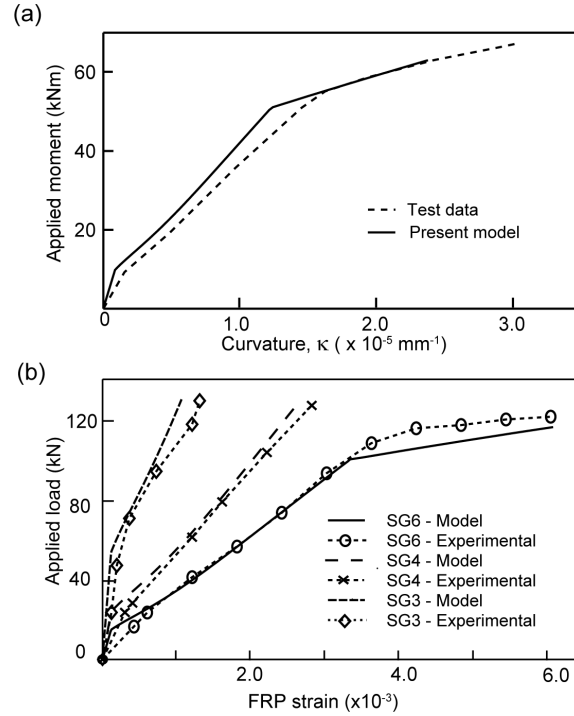


Figure 6. – (a) M – κ comparison for Beam A3.1¹³ (b) FRP strain comparisons along the span of Beam CB4-2S¹⁴
(Curvature - $1.10^{-5} \text{ mm} = 2.5.10^{-4} \text{ ins}$; Force – 100 kN = 22500 lbs; Moment - 10 kNm = 7376 ft-lbs)

Fig. 6a shows M – κ comparisons for a strengthened beam (Beam A3.1) tested by Spadea et al. (1998)¹³. It shows that the model can successfully predict behavior for all uncracked, partially-cracked, and fully-cracked regimes. The predicted M_{cr} and M_y are slightly higher than those actually observed, which may be attributed to the overestimation of the concrete tensile strength or the yield strength of steel. The small variations in the stiffness predictions may be attributed to a slight overestimation of the material stiffness. Comparisons with the measured strains in the FRP at three different span locations in Beam CB4-2S tested in four-point bending by Alagusundaramoorthy et al.¹⁴ are shown in Fig. 6b. Locations SG6, SG4 and SG3 quoted in the figure correspond to positions in the constant moment zone, center region of one of the shear spans and at distance one quarter of the shear span from the beam support, respectively. Good correlations can be observed in all cases. This shows that, not only does the present model correctly predict the curvatures, but it also correctly predicts the neutral axes, from which it can be assumed that the strain profiles will be correct.

DETERMINATION OF G_R ASSOCIATED WITH EXTENSION OF AN INTERFACE CRACK

In the GEBA model the G_R associated with a small potential extension of a given interface crack is required to compare with G_{CI} to decide whether the crack will propagate. The objective is to use the present M – κ model to determine G_R at a given applied load. When the crack extends, the beam loses some of its stiffness, so work is done by the external loads. The curvature increases in the beam, storing some of this extra work as strain energy, but some is available to cause the crack to propagate. Thus, according to the global energy balance of the system, the G_R is the rate of change of the system's total potential energy, Π_{sys} (i.e. the sum of the potential energies in the applied loads (W_{load}) and the work done on the beam (W_{beam})) with respect to the crack length (a) (Eq. 6).

$$G_R = -1/b_p (\partial \Pi / \partial a) = -1/b_p (\partial W_{beam} / \partial a + \partial W_{ext} / \partial a) \quad (b_p = \text{width of the FRP}) \quad (6)$$

In order to focus on the basic mechanics, analysis of a simply-supported beam is assumed in the present discussion. The procedure would need modifying if a statically indeterminate beam was to be analyzed when the distribution of moments caused by applied loads would change as the beam's stiffness changes during crack extension. When a RC beam bends, a part of the energy put into the beam by the loads is dissipated in cracking and

steel yielding. The rest is stored as the beam's strain energy, shown schematically in Fig. 7. It is not correct to determine G_R simply as the rate of change of strain energy, as is usually done in a LEFM analysis where it is assumed that the total work done on the beam by the loads is stored as strain energy. A number of existing FRP debonding analyses, however, are based on this incorrect theory⁶. It should be pointed out that this analysis applies only to statically determinate beams, so when debonding takes place at one location, the moment distribution, and hence the cracking state, elsewhere do not change. If the beam were statically indeterminate, changing the stiffness in one location could change the moment distribution, and a more complex set of energy changes would need to be considered.

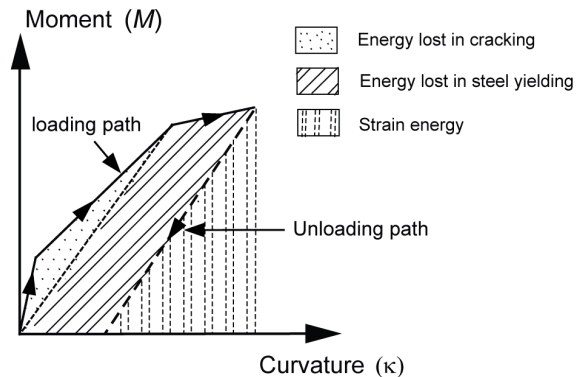


Figure 7. – Only a part of the total work done is stored as beam's strain energy

The method used to calculate G_R

G_R is determined as the change in system's total potential energy ($\Delta\Pi$) per unit area of new interface crack (G_R has the units N/mm). Due to the crack extension, the beam softens, but not uniformly. Over most of the length of the beam the curvature and hence the strain energy remain unchanged; it is only the portion of the beam near the crack tip where significant changes of curvature occur¹.

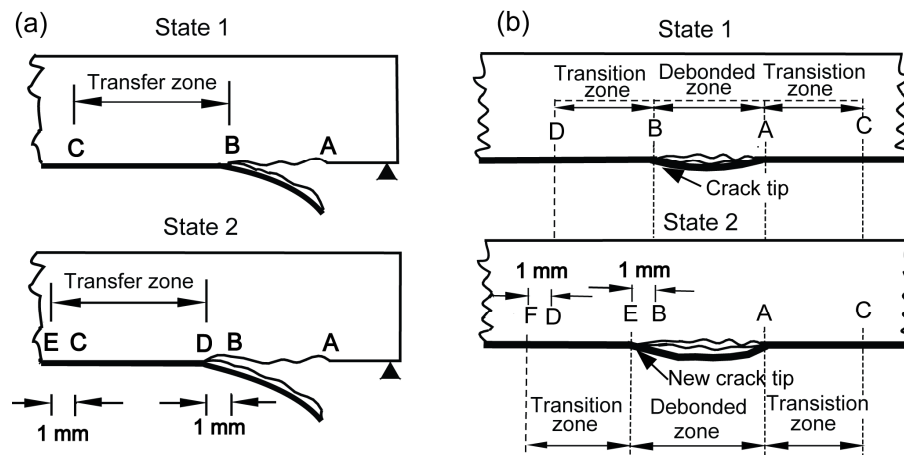


Figure 8. – Energy released zones: before (State 1) and after (State 2) small crack extension
(a) PE debonding (b) IC debonding extension

G_R associated with PE debonding is determined by considering the energy and κ changes take place in beam segments within the plate-end transfer zone (BE in Fig. 8a); for IC debonding κ changes take place in the unbonded zone and the two transition zones (CF in Fig. 8b). Outside these zones, the FRP is fully-bonded to the concrete, so if the load does not change during the debonding increment, the moment and hence the curvatures remain constant.

It is important to know the length of the transition zone. A simplified model, based on the more rigorous interfacial stress analysis of Täljsten¹⁵, was used to determine the distribution of F_p in the plate-end stress transfer