5. The actual support conditions apparently do not need to conform with those assumed in order to obtain a response (at least in the middle half of the structure) to load that agrees well with theoretical analysis.

To gain a better understanding of the behavior of reinforced concrete plates at the initiation of buckling, a fundamental experimental study on plates was conducted at Kansas State University (Ref. 27). It involved the testing of twenty-four rectangular, reinforced concrete plates, simply-supported along all edges and subjected to uniaxial compressive loads.

The plates were constructed of normal weight concrete with a nominal 28 day strength of 3000 psi. They were all 8 ft. high and 4 ft. wide. Thickness varied from 0.75 in. to 1.25 in. The steel ratio varied from 0.2 to 1.0 percent. The plates were simply supported with the load applied along the shorter edges.

Results of this study apply to those folded plate structures in which the buckled regions of individual plates can be considered as simply supported and where either N_x or N_y dominates the behavior of the plate and whose distribution is sufficiently uniform.

Four equations (Ref. 28) were subsequently developed by the authors for the purpose of predicting the buckling of reinforced concrete plates of the type tested.

1. Orthotropic Case - Tangent Modulus Theory

2. Orthotropic Case - Double Modulus Theory

3. Isotropic Case - Tangent Modulus Theory

4. Isotropic Case - Double Modulus Theory

Assumptions used in the development of these equations were:

1. Small deflection theory prevails and initial imperfections are neglected.

2. The load is perfectly uniaxial.

3. Only the initial, short-term buckling is considered.

4. The plate is simply supported.

5. The concrete is assumed to be uncracked at the onset of buckling.

6. Interaction between vertical and horizontal reinforcement is neglected. The steel remains elastic. The steel is assumed to be a two-way isotropic mesh.

Predictions based on the four equations were compared with experimental data in Figure 5. There is not a large difference in the predictions from the four formulas. The greatest difference (between the isotropic-tangent modulus case and the orthotropic-double modulus case) is about 25%. The isotropic-tangent modulus formula gave the lowest buckling loads and is recommended by the authors. The equations for this case are given next.

For $\varepsilon_{cr} \leq \varepsilon_y$,

$$P_{crd} = C_s bh\,[f_{cr}(1-\rho) + E_s \varepsilon_{cr} \rho]. \tag{1a}$$

For $\varepsilon_{cr} \geq \varepsilon_y$,

$$P_{crd} = C_s bh\,[f_{cr}(1-\rho) + f_y \rho]; \tag{1b}$$

where C_s = factor of safety coefficient,

$$f_{cr} = 0.425\, f'_c\, B(-B+\sqrt{4+B^2}), \tag{2}$$

$$B = \frac{\pi^2}{6\varepsilon_o(1-\rho)} \left(\frac{1}{m} + m\right)^2 \left(\frac{b}{h}\right)^2, \tag{3}$$

$$m = \frac{a}{b} \text{ if } \frac{a}{b} < 1,$$

$$m = 1 \text{ if } \frac{a}{b} > 1,$$

$$\varepsilon_{cr} = 1 + 1/2(B-\sqrt{4+B^2}), \tag{4}$$

$$\varepsilon_{cr} = e_{cr}\varepsilon_o. \tag{5}$$

These parameters are

- P_{crd} - buckling load;
- ε_y - yield strain in steel;
- ε_{cr} - strain in concrete at buckling;
- f_{cr} - critical buckling stress;
- f'_c, ε_o - strength of concrete cylinder and strain at which it is attained;
- E_s - modulus of elasticity of steel;
- a,b,h - length, width and thickness respectively, of plate;
- e_{cr} - nondimensional strain in concrete at buckling;
- ρ - total steel ratio.

Although this formula is easy to apply, its applicability is limited to a fairly narrow class of folded plate structures.

Martin (Ref. 29,34) reported a full-scale load test of a single, precast post-tensioned lightweight concrete folded plate unit. The cross section of the unit is shown in Figure 6 and the span was 101 ft. The unit was tested by filling the trough with water and placing concrete blocks on the top of the inclined plates. The unit failed at a load which was 62 percent of the design load. The buckling load was estimated by simplified methods and conservative assumptions as customary in design.

The assumptions were:

1. Reduced modulus of elasticity due to creep of concrete.

2. Each outside plate was assumed as supported on three sides and under uniform longitudinal compression.

The buckling stress was estimated as 1022 psi while the test buckling occurred at 913 psi stress at the extreme top fiber. At failure one cantilever rotated out.

In the discussion (30, 31, 32, 33, 34) two main points were raised.

1. The method of calculating the buckling was too conservative. More exact methods yield higher buckling loads. Also the duration of the test was too short to justify a substantial reduction of E due to creep.

2. The detail of reinforcement for negative moment at the lower corner could not develop the full stresses in the reinforcement as proved by numerous tests on beams with reentrent corners.

It was therefore suggested in the discussion that the case was not of buckling failure but rather transverse bending failure at the corner. Martin, in recent correspondence, replied that deformations in the lateral wings indicated buckling prior to final collapse.

In the final condition the V elements will become part of a continuous structure. The plates will not be cantilevered anymore and the capacity of the structure will increase many folds both in transverse bending and buckling.

A relatively simple method for predicting local buckling is presented in Ref. 35. This approximate method, based on the energy principle, produces a closed form solution for a plate buckling zone of known dimensions. It is directly applicable to end supported folded plates consisting of an isotropic linearly elastic material with or without transverse interior stiffeners when subjected to any reasonable loading condition.

The buckled panel is assumed to be simply supported along nodal lines and elastically supported with regard to rotation only along longitudinal edges. It is further assumed that the normal deflections along all edges of the panel are zero, force distributions acting on the panel as unchanged as buckling begins and the effect of normal pressure may be disregarded. Other assumptions include (1) the usual geometry and linearly elastic material assumptions associated with the general stress analysis of folded plates; (2) the bending stiffness provided by the elastic restraining medium along the longitudinal panel is unchanged as the result of buckling; and (3) the structure is long (i.e., the length to overall depth ratio is greater than about 15 to 1).

The method requires a knowledge of the distribution of N_x, N_y and N_{xy} due to dead load and unit live load. The size of the panel needs to be specified. For each panel size and distribution of in-plane stresses the method produces a prediction of the critical load which will cause local buckling to occur.

The procedure is relatively straightforward. However, since it does not permit a prediction of the minimum buckling load for a given structure automatically but requires the analyst to select various panel sizes in critical regions of the structure until the minimum load has been obtained; the method is best handled by means of electronic computation. Predictions based on this technique and available experimental data are compared in Table 2.

A simplified method for obtaining the load associated with the onset of local buckling in certain types of long-span end supported concrete plates is presented in Reference 36. The proposed expression for computing a load, q_{cr}, associated with buckling in a panel of a concrete folded plate is given as

$$q_{cr} = - \frac{f_{cr} + f_c^D}{f_c^{q=1}} \tag{6}$$

in which f_c^D and $f_c^{q=1}$ represent the longitudinal stresses in the buckled panel due to dead load and unit live load, q = 1, respectively (compression is minus). The beneficial influence of steel reinforcing is neglected and the plate is assumed to be uncracked prior to buckling.

The buckling stress f_{cr} is given by Eq. (2) where now with m = 1 and ρ neglected,

$$B = \frac{2\pi^2}{3\varepsilon_o} \left(\frac{h}{b}\right)^2; \text{ and} \tag{7}$$

$$\varepsilon_o = \frac{0.85\sqrt{f'_c}}{33w^{1.5}} \left(1+\sqrt{\frac{0.40}{0.85}}\right). \tag{8}$$

The term ε_o represents the strain in a plain concrete cylinder at the 28-day ultimate stress, f'_c (see Ref. 37); h is the plate thickness, and b is the dimension of the square panel. This expression applies to the analysis of long-span (span/depth greater than 15) end supported concrete folded plates, with single or multiple cell cross sections, of the type shown in Figure 7. The buckled zone, located in the most highly compressed region of the plate, is assumed to be square.

The authors (Ref. 36) conclude that the comparison of results obtained from the tangent modulus theory with those predicted by elastic theory indicates that the elastic theory does not yield acceptable results when applied to concrete folded plates. It is also stressed by the authors that a purely elastic method for predicting buckling loads is not directly applicable to any actual concrete shell.

The writers (Ref. 36) tentatively recommend a factor of safety against buckling stress of 2.2 for folded plates with width to thickness ratio less than 64 (the limit of their plate buckling tests) when using the inelastic method described. They also indicate that the effect of surface imperfections (thickness variations, out-of-flatness) may be neglected when the width-thickness ratio is less than 64. Based on purely elastic buckling theory, the factor of safety against buckling as proposed by ACI Committee 334 (Ref. 38) is 5.

Based on the foregoing review of the literature, it is concluded that the field of "stability analysis of folded plate reinforced concrete structures" is still in its infancy. Most of the experimental work done to date has dealt with "small" structures constructed of aluminum (Ref. 21,23,26). The one study (Ref. 28) which dealt with reinforced concrete was restricted to simply-supported (not folded) plates under a very specific loading condition (i.e., planar, uniform compression). The one full-scale load test (Ref. 29) reported of a folded plate structure exhibited a failure mode whose correct classification appears to be subject to some debate (i.e., it is either a buckling failure or a flexural failure). In short, reliable experimental data for reinforced concrete folded plates appear to be essentially nonexistent.

Analysis methods that can be readily used by the practicing engineer appear to be also limited to relatively crude approximate procedures (Ref. 35,36) insofar as local buckling is concerned. Overall instability is not discussed to any appreciable extent by any of the authors cited herein.

Folded plates are related to cylindrical shells. As the number of plates in each bay is increased the cross section of the folded plate approaches that of a curve. The angle between adjacent planes is shallow and the effectiveness of each fold line as support is reduced. With more such folds, the buckling phenomena will approach that of cylindrical shells, which means the buckled surface can extend over a number of plates.

PRECAST SHELLS

The economy of reinforced concrete shells is offset by the high cost of forms and pouring concrete in slope--hence the temptation to construct shells from precast elements. Cylindrical shells and folded plates lend themselves especially to precasting when used as multiple barrels laid side by side. In spite of the availability of new methods and materials for waterproofing of joints, it is still preferable to have the joints at the crest and not the valleys. Hence the popularity of the V shaped precast element laid side by side. The joint at the top may be poured with reinforcement of the two adjacent elements overlapping at the joint. It is also possible to just connect the elements by bolting. Waterproofing is achieved by caulking the joint.

The idea of constructing folded plates from flat plates seemed very promising (Ref. 39, 40). The concept was to stack flat plates side by side and pull them like a folding door and then pour the joints. Of course the stability of the plates during construction, until the joints have the capacity of converting the single plates into a combined folded plates structure, is a problem. This is the reason why this system which was tried almost twenty years ago did not become popular.

The problem of stability of precast shells is the same as for reinforced concrete shells, once the shell is complete. Of course, the joint between the elements should be considered hinged or continuous depending on the detail.

However, instability conditions of the separate elements may exist. As mentioned before, V elements are susceptible to buckling during transportation and erection and care should be taken to avoid this.

GENERAL PURPOSE COMPUTER PROGRAMS

There now exists a number of finite element, general purpose computer programs in the public domain which may be modified to yield predictions of local or overall buckling loads for folded plate structures. These capabilities are briefly described for ANSYS (Ref. 41).

In the formulation of finite element procedures for ANSYS, the usual stiffness matrix [K] is modified to include the interaction between in-plane forces and lateral bending. This effect is taken into account by introducing a supplementary matrix [K'] in addition to the original elastic stiffness matrix [K]. This supplementary matrix [K'] is called the "stress stiffness" or "geometric stiffness" matrix, whose elements depend not only on the geometry but also on the initial internal stresses. Two basic options are open to the user (i.e., (1) eigenvalue buckling (bifurcation) and (2) large displacement with coordinate updating).

In the first option, the equation that is solved is

$$([K] + \lambda_i\ [K'])\ \phi_i = \{0\}, \qquad (9)$$

where λ_i = ith eigenvalue (used to multiply the loads which generated [K']);

ϕ_i = ith eigenvector of displacements, the other matrices were defined previously.

In the second option, the equation which is solved is

$$[K_{up}]\ \{u\} = \{F^n\} + \{F^{ld}\}, \qquad (10)$$

where $[K_{up}]$ = stiffness matrix based on updated geometry;

$\{u\}$ = displacement vector;

$\{F^n\}$ = applied nodal load vector;

$\{F^{ld}\}$ = large displacement force vector;

The procedure is iterative and may be summarized, leaving out the details, as follows.

First the conventional stiffness matrix [K] and the element load vector are calculated. The loads are applied to the structure and displacements are computed. The load vector $\{F^{ld}\}$ is calculated, based on displacements. The element stiffness matrix is recomputed (resulting in $[K_{up}]$), followed by the determination of a new set of displacements. The process is repeated as often as necessary. If a nonlinear material (i.e., plasticity) is used the loads must be "stepped" up slowly because the results of the problem are path dependent.

At least two general purpose computer programs in the public domain have the capability of performing incremental load type buckling analysis of folded plate structures. This includes ANSYS, (Ref. 41) and NASTRAN (Ref. 42). While NASTRAN does not possess inelastic material options at the present time, large deformation effects may be included for the in-plane forces.

None of the computer programs reviewed in the course of writing this article possesses a reinforced concrete finite element. However, an orthotropic stiffness matrix or a specific stiffness matrix, based on effective properties of concrete and reinforcement, may be derived to provide an input to NASTRAN to perform the analysis.

Analytical procedures using ANSYS and NASTRAN can be used to make a preliminary assessment of the buckling strength of reinforced concrete folded plate structures for design purposes. No analytical technique appears to be available as of this writing which satisfactorily predicts buckling loads without the necessity of numerous trials of possible buckling modes.

CONCLUSIONS AND RECOMMENDATIONS

All the published methods known to the authors for review of folded plate structures to estimate buckling which are presented herein are limited to prismatic shells in which only the local effects in a single plate are considered. The tools for developing much more detailed analyses with greater scope are available but to date these analyses have not appeared in the literature.

In any event, due to a dearth of experimental evidence on concrete folded plate structures (prototypes or models), research efforts in this area are at least as warranted as those efforts necessary to develop more accurate analytical models.

The equations and methods of analysis presented here are based on observations of the behavior of certain types of elastic models and reinforced concrete plates and may be considered to be a reasonable description of that behavior.

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