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**Table 22.6.5.2— $v_c$  for two-way members without shear reinforcement**

$v_c$		
Least of (a), (b), and (c):	$0.33\lambda_s\lambda\sqrt{f'_c}$	(a)
	$\left(0.17 + \frac{0.33}{\beta}\right)\lambda_s\lambda\sqrt{f'_c}$	(b)
	$\left(0.17 + \frac{0.083\alpha_s d}{b_o}\right)\lambda_s\lambda\sqrt{f'_c}$	(c)

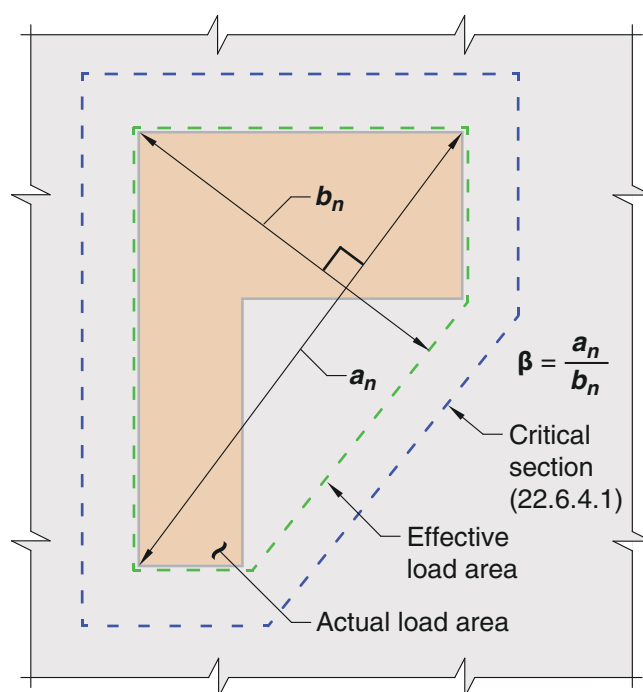
Notes:

(i)  $\lambda_s$  is the size effect factor given in 22.5.5.1.3.(ii)  $\beta$  is the ratio of long to short sides of the column, concentrated load, or reaction area.(iii)  $\alpha_s$  is given in 22.6.5.3.

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slabs subjected to bending in two directions is limited to  $0.33\lambda_s\lambda\sqrt{f'_c}$ . However, tests (Joint ACI-ASCE Committee 426 1974) have indicated that the value of  $0.33\lambda_s\lambda\sqrt{f'_c}$  is unconservative when the ratio  $\beta$  of the lengths of the long and short sides of a rectangular column or loaded area is larger than 2.0. In such cases, the actual shear stress on the critical section at punching shear failure varies from a maximum of approximately  $0.33\lambda_s\lambda\sqrt{f'_c}$  around the corners of the column or loaded area, down to  $0.17\lambda_s\lambda\sqrt{f'_c}$  or less along the long sides between the two end sections. Other tests (Vanderbilt 1972) indicate that  $v_c$  decreases as the ratio  $b_o/d$  increases. Expressions (b) and (c) in Table 22.6.5.2 were developed to account for these two effects.

For shapes other than rectangular,  $\beta$  is taken to be the ratio of the longest overall dimension of the effective loaded area to the largest overall perpendicular dimension of the effective loaded area, as illustrated for an L-shaped reaction area in Fig. R22.6.5.2. The effective loaded area is that area totally enclosing the actual loaded area, for which the perimeter is a minimum.

**Fig. R22.6.5.2—Value of  $\beta$  for a nonrectangular loaded area.**

**22.6.5.3** The value of  $\alpha_s$  is 40 for interior columns, 30 for edge columns, and 20 for corner columns.

**22.6.5.4** For prestressed, two-way members, it shall be permitted to calculate  $v_c$  using 22.6.5.5, provided that (a) through (c) are satisfied:

**R22.6.5.3** The terms “interior columns,” “edge columns,” and “corner columns” in this provision refer to critical sections with a continuous slab on four, three, and two sides, respectively.

**R22.6.5.4** For prestressed two-way members, modified forms of expressions (b) and (c) in Table 22.6.5.2 are specified. Research (ACI 423.3R) indicates that the shear strength of two-way prestressed slabs around interior columns is

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- (a) Bonded reinforcement is provided in accordance with 8.6.2.3 and 8.7.5.3
- (b) No portion of the column cross section is closer to a discontinuous edge than four times the slab thickness  $h$
- (c) Effective prestress  $f_{pc}$  in each direction is not less than 0.9 MPa

**22.6.5.5** For prestressed, two-way members conforming to 22.6.5.4,  $v_c$  shall be permitted to be the lesser of (a) and (b)

$$(a) \ v_c = 0.29\lambda\sqrt{f'_c} + 0.3f_{pc} + \frac{V_p}{b_o d} \quad (22.6.5.5a)$$

$$(b) \ v_c = 0.083 \left( 1.5 + \frac{\alpha_s d}{b_o} \right) \lambda \sqrt{f'_c} + 0.3f_{pc} + \frac{V_p}{b_o d} \quad (22.6.5.5b)$$

where  $\alpha_s$  is given in 22.6.5.3; the value of  $f_{pc}$  is the average of  $f_{pc}$  in the two directions and shall not exceed 3.5 MPa;  $V_p$  is the vertical component of all effective prestress forces crossing the critical section; and the value of  $\sqrt{f'_c}$  shall not exceed 5.8 MPa.

**22.6.6** *Two-way shear strength provided by concrete in members with shear reinforcement*

**22.6.6.1** For two-way members with shear reinforcement,  $v_c$  at critical sections shall be calculated in accordance with Table 22.6.6.1.

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conservatively calculated by the expressions in 22.6.5.5, where  $v_c$  corresponds to a diagonal tension failure of the concrete initiating at the critical section defined in 22.6.4.1. The mode of failure differs from a punching shear failure around the perimeter of the loaded area of a nonprestressed slab calculated using expression (b) in Table 22.6.5.2. Consequently, the expressions in 22.6.5.5 differ from those for nonprestressed slabs. Values for  $\sqrt{f'_c}$  and  $f_{pc}$  are restricted in design due to limited test data available beyond the specified limits. When calculating  $f_{pc}$ , loss of prestress due to restraint of the slab by structural walls and other structural elements should be taken into account.

**R22.6.6** *Two-way shear strength provided by concrete in members with shear reinforcement*

Critical sections for two-way members with shear reinforcement are defined in 22.6.4.1 for the sections adjacent to the column, concentrated load, or reaction area, and 22.6.4.2 for the section located just beyond the outermost peripheral line of stirrup or headed shear stud reinforcement. Values of maximum  $v_c$  for these critical sections are given in Table 22.6.6.1. Limiting values of  $v_u$  for the critical sections defined in 22.6.4.1 are given in Table 22.6.6.3.

The maximum  $v_c$  and limiting value of  $v_u$  at the innermost critical section (defined in 22.6.4.1) are higher where headed shear stud reinforcement is provided than the case where stirrups are provided (refer to R8.7.7). Maximum  $v_c$  values at the critical sections defined in 22.6.4.2 beyond the outermost peripheral line of shear reinforcement are independent of the type of shear reinforcement provided.

**R22.6.6.1** For two-way slabs with stirrups, the maximum value of  $v_c$  is taken as  $0.17\lambda_s\lambda\sqrt{f'_c}$  because the stirrups resist all the shear beyond that at inclined cracking (which occurs at approximately half the capacity of a slab without shear reinforcement (that is,  $0.5 \times 0.33\lambda_s\lambda\sqrt{f'_c} = 0.17\lambda_s\lambda\sqrt{f'_c}$ ) (Hawkins 1974). The higher value of  $v_c$  for two-way slabs with headed shear stud reinforcement is based on research (Elgabry and Ghali 1987).

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**Table 22.6.6.1— $v_c$  for two-way members with shear reinforcement**

Type of shear reinforcement	Critical sections	$v_c$		
Stirrups	All	$0.17\lambda_s\lambda\sqrt{f'_c}$		(a)
Headed shear stud reinforcement	According to 22.6.4.1	Least of (b), (c), and (d):	$0.25\lambda_s\lambda\sqrt{f'_c}$	(b)
			$0.17\left(1 + \frac{2}{\beta}\right)\lambda_s\lambda\sqrt{f'_c}$	(c)
			$0.083\left(2 + \frac{\alpha_s d}{b_o}\right)\lambda_s\lambda\sqrt{f'_c}$	(d)
	According to 22.6.4.2		$0.17\lambda_s\lambda\sqrt{f'_c}$	(e)

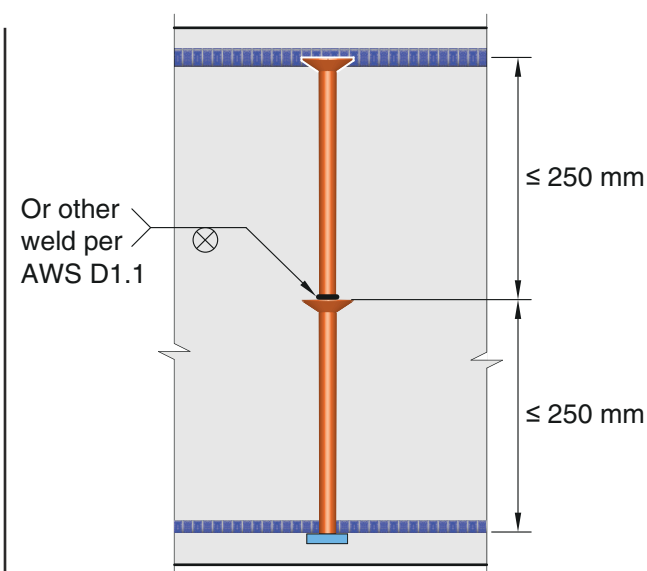
Notes:

(i)  $\lambda_s$  is the size effect factor given in 22.5.5.1.3.(ii)  $\beta$  is the ratio of long to short sides of the column, concentrated load, or reaction area.(iii)  $\alpha_s$  is given in 22.6.5.3.

**22.6.6.2** It shall be permitted to take  $\lambda_s$  as 1.0 if (a) or (b) is satisfied:

- (a) Stirrups are designed and detailed in accordance with 8.7.6 and  $A_v/s \geq 0.17\sqrt{f'_c} b_o/f_{yt}$ .
- (b) Smooth headed shear stud reinforcement with stud shaft length not exceeding 250 mm is designed and detailed in accordance with 8.7.7 and  $A_v/s \geq 0.17\sqrt{f'_c} b_o/f_{yt}$ .

**R22.6.6.2** The size effect in slabs with  $d > 250$  mm can be mitigated if a minimum amount of shear reinforcement is provided. The ability of ordinary (smooth) headed shear stud reinforcement to effectively mitigate the size effect on the two-way shear strength of slabs may be compromised if studs longer than 250 mm are used. Until experimental evidence becomes available, it is not permitted to use  $\lambda_s$  equal to 1.0 for slabs with  $d > 250$  mm without headed shear stud reinforcement with stud shaft length not exceeding 250 mm. Stacking or “piggybacking” of headed shear studs, as shown in Fig. R22.6.6.2, introduces an intermediate head that contributes to further anchor the stacked stud.



**Fig. R22.6.6.2—Stacking (piggybacking) of headed shear stud reinforcement.**

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**22.6.6.3** For two-way members with shear reinforcement, effective depth shall be selected such that  $v_u$  calculated at critical sections does not exceed the values in Table 22.6.6.3.

**Table 22.6.6.3—Maximum  $v_u$  for two-way members with shear reinforcement**

Type of shear reinforcement	Maximum $v_u$ at critical sections defined in 22.6.4.1	
Stirrups	$0.5\phi\sqrt{f'_c}$	(a)
Headed shear stud reinforcement	$0.66\phi\sqrt{f'_c}$	(b)

**22.6.7** Two-way shear strength provided by single- or multiple-leg stirrups

**22.6.7.1** Single- or multiple-leg stirrups fabricated from bars or wires shall be permitted to be used as shear reinforcement in slabs and footings satisfying (a) and (b):

(a)  $d$  is at least 150 mm

(b)  $d$  is at least  $16d_b$ , where  $d_b$  is the diameter of the stirrups

**22.6.7.2** For two-way members with stirrups,  $v_s$  shall be calculated by:

$$v_s = \frac{A_v f_{yt}}{b_o s} \quad (22.6.7.2)$$

where  $A_v$  is the sum of the area of all legs of reinforcement on one peripheral line that is geometrically similar to the perimeter of the column section, and  $s$  is the spacing of the peripheral lines of shear reinforcement in the direction perpendicular to the column face.

**22.6.8** Two-way shear strength provided by headed shear stud reinforcement

**22.6.8.1** Headed shear stud reinforcement shall be permitted to be used as shear reinforcement in slabs and footings if the placement and geometry of the headed shear stud reinforcement satisfies 8.7.7.

**22.6.8.2** For two-way members with headed shear stud reinforcement,  $v_s$  shall be calculated by:

$$v_s = \frac{A_v f_{yt}}{b_o s} \quad (22.6.8.2)$$

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**R22.6.7** Two-way shear strength provided by single- or multiple-leg stirrups

**R22.6.7.2** Because shear stresses are used for two-way shear in this chapter, shear strength provided by transverse reinforcement is averaged over the cross-sectional area of the critical section.

**R22.6.8** Two-way shear strength provided by headed shear stud reinforcement

Tests (ACI 421.1R) show that headed shear stud reinforcement mechanically anchored as close as practicable to the top and bottom of slabs is effective in resisting punching shear. The critical section beyond the shear reinforcement is generally assumed to have a polygonal shape (refer to Fig. R22.6.4.2a, R22.6.4.2b, and R22.6.4.2c). Equations for calculating shear stresses on such sections are given in ACI 421.1R.

**R22.6.8.2** Because shear stresses are used for two-way shear in this chapter, shear strength provided by transverse reinforcement is averaged over the cross-sectional area of the critical section.

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where  $A_v$  is the sum of the area of all shear studs on one peripheral line that is geometrically similar to the perimeter of the column section, and  $s$  is the spacing of the peripheral lines of headed shear stud reinforcement in the direction perpendicular to the column face.

**22.6.8.3** If headed shear stud reinforcement is provided,  $A_v/s$  shall satisfy:

$$\frac{A_v}{s} \geq 0.17 \sqrt{f'_c} \frac{b_o}{f_{yt}} \quad (22.6.8.3)$$

## 22.7—Torsional strength

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## R22.7—Torsional strength

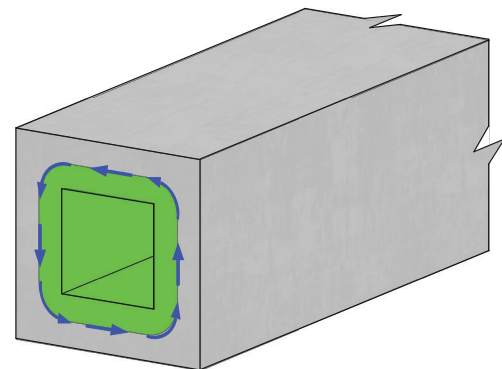
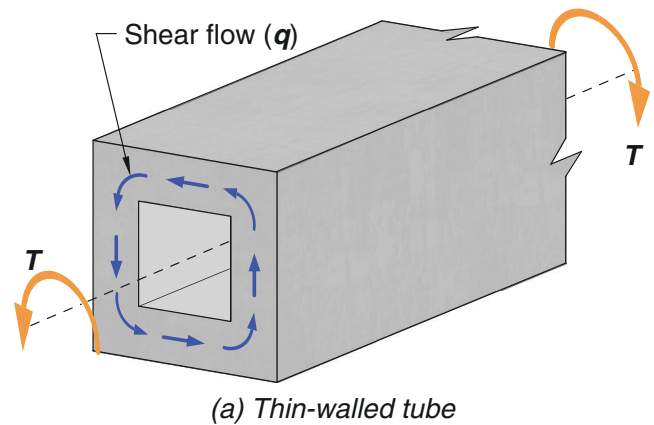
The design for torsion in this section is based on a thin-walled tube space truss analogy. A beam subjected to torsion is idealized as a thin-walled tube with the core concrete cross section in a solid beam neglected as shown in Fig. R22.7(a). Once a reinforced concrete beam has cracked in torsion, its torsional strength is provided primarily by closed stirrups and longitudinal bars located near the surface of the member. In the thin-walled tube analogy, the strength is assumed to be provided by the outer skin of the cross section roughly centered on the closed stirrups. Both hollow and solid sections are idealized as thin-walled tubes both before and after cracking.

In a closed thin-walled tube, the product of the shear stress  $\tau$  and the wall thickness  $t$  at any point in the perimeter is known as the shear flow,  $q = \tau t$ . The shear flow  $q$  due to torsion acts as shown in Fig. R22.7(a) and is constant at all points around the perimeter of the tube. The path along which it acts extends around the tube at midthickness of the walls of the tube. At any point along the perimeter of the tube, the shear stress due to torsion is  $\tau = T/(2A_o t)$ , where  $A_o$  is the gross area enclosed by the shear flow path, shown shaded in Fig. R22.7(b), and  $t$  is the thickness of the wall at the point where  $\tau$  is being calculated. For a hollow member with continuous walls,  $A_o$  includes the area of the hole.

The concrete contribution to torsional strength is ignored, and in cases of combined shear and torsion, the concrete contribution to shear strength does not need to be reduced. The design procedure is derived and compared with test results in MacGregor and Ghoneim (1995) and Hsu (1997).

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**Fig. R22.7**—(a) Thin-walled tube; and (b) area enclosed by shear flow path.

### 22.7.1 General

**22.7.1.1** This section shall apply to members if  $T_u \geq \phi T_{th}$ , where  $\phi$  is given in Chapter 21 and threshold torsion  $T_{th}$  is given in 22.7.4. If  $T_u < \phi T_{th}$ , it shall be permitted to neglect torsional effects.

**22.7.1.2** Nominal torsional strength shall be calculated in accordance with 22.7.6.

**22.7.1.3** For calculation of  $T_{th}$  and  $T_{cr}$ ,  $\lambda$  shall be in accordance with 19.2.4.

### 22.7.2 Limiting material strengths

**22.7.2.1** The value of  $\sqrt{f'_c}$  used to calculate  $T_{th}$  and  $T_{cr}$  shall not exceed 8.3 MPa.

**22.7.2.2** The values of  $f_y$  and  $f_{yt}$  for longitudinal and transverse torsional reinforcement shall not exceed the limits in 20.2.2.4.

### R22.7.1 General

**R22.7.1.1** Torsional moments that do not exceed the threshold torsion  $T_{th}$  will not cause a structurally significant reduction in either flexural or shear strength and can be ignored.

### R22.7.2 Limiting material strengths

**R22.7.2.1** Because of a lack of test data and practical experience with concretes having compressive strengths greater than 70 MPa, the Code imposes a maximum value of 8.3 MPa on  $\sqrt{f'_c}$  for use in the calculation of torsional strength.

**R22.7.2.2** The upper limit of 420 MPa on the value of  $f_y$  and  $f_{yt}$  used in design is intended to control diagonal crack width.

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**22.7.3 Factored design torsion**

**22.7.3.1** If  $T_u \geq \phi T_{cr}$  and  $T_u$  is required to maintain equilibrium, the member shall be designed to resist  $T_u$ .

**22.7.3.2** In a statically indeterminate structure where  $T_u \geq \phi T_{cr}$  and a reduction of  $T_u$  can occur due to redistribution of internal forces after torsional cracking, it shall be permitted to reduce  $T_u$  to  $\phi T_{cr}$ , where the cracking torsion  $T_{cr}$  is calculated in accordance with 22.7.5.

**22.7.3.3** If  $T_u$  is redistributed in accordance with 22.7.3.2, the factored moments and shears used for design of the adjoining members shall be in equilibrium with the reduced torsion.

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**R22.7.3 Factored design torsion**

In designing for torsion in reinforced concrete structures, two conditions may be identified (Collins and Lampert 1973; Hsu and Burton 1974):

(a) The torsional moment cannot be reduced by redistribution of internal forces (22.7.3.1). This type of torsion is referred to as equilibrium torsion because the torsional moment is required for the structure to be in equilibrium. For this condition, illustrated in Fig. R22.7.3(a), torsional reinforcement must be provided to resist the total design torsional moments.

(b) The torsional moment can be reduced by redistribution of internal forces after cracking (22.7.3.2) if the torsion results from the member twisting to maintain compatibility of deformations. This type of torsion is referred to as compatibility torsion.

For this condition, illustrated in Fig. R22.7.3(b), the torsional stiffness before cracking corresponds to that of the uncracked section according to St. Venant's theory. At torsional cracking, however, a large twist occurs under an essentially constant torsional moment, resulting in a large redistribution of forces in the structure (Collins and Lampert 1973; Hsu and Burton 1974). The cracking torsional moment under combined shear, moment, and torsion corresponds to a principal tensile stress somewhat less than the  $0.33\lambda\sqrt{f'_c}$  used in R22.7.5.

If the torsional moment exceeds the cracking torsional moment (22.7.3.2), a maximum factored torsional moment equal to the cracking torsional moment may be assumed to occur at the critical sections near the faces of the supports. The maximum factored torsional moment has been established to limit the width of torsional cracks.

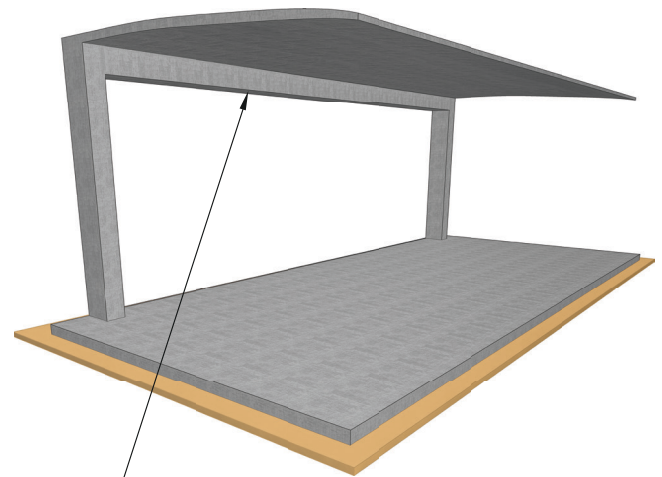
Provision 22.7.3.2 applies to typical and regular framing conditions. With layouts that impose significant torsional rotations within a limited length of the member, such as a large torsional moment located close to a stiff column, or a column that rotates in the reverse directions because of other loading, a more detailed analysis is advisable.

If the factored torsional moment from an elastic analysis based on uncracked section properties is between  $\phi T_{th}$  and  $\phi T_{cr}$ , torsional reinforcement should be designed to resist the calculated torsional moments.



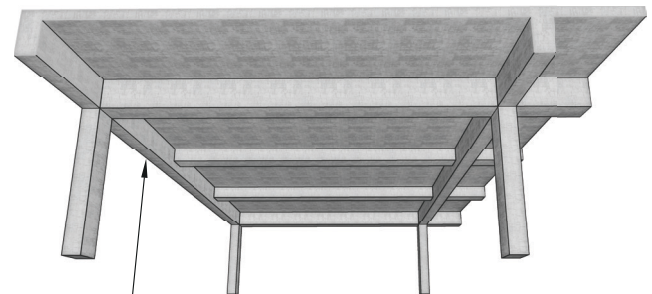
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Design torsional moment may **not** be reduced because moment redistribution is **not** possible

**Fig. R22.7.3a**—Equilibrium torsion, the design torsional moment may not be reduced (22.7.3.1).



Design torsional moment for this spandrel beam may be reduced because moment redistribution is possible

**Fig. R22.7.3b**—Compatibility torsion, the design torsional moment may be reduced (22.7.3.2).

#### 22.7.4 Threshold torsion

**22.7.4.1** Threshold torsion  $T_{th}$  shall be calculated in accordance with Table 22.7.4.1(a) for solid cross sections and Table 22.7.4.1(b) for hollow cross sections, where  $N_u$  is positive for compression and negative for tension.

#### R22.7.4 Threshold torsion

The threshold torsion is defined as one-fourth the cracking torsional moment  $T_{cr}$ . For sections of solid members, the interaction between the cracking torsional moment and the inclined cracking shear is approximately circular or elliptical. For such a relationship, a threshold torsional moment of  $T_{th}$ , as used in 22.7.4.1, corresponds to a reduction of less than 5 percent in the inclined cracking shear, which is considered negligible.

For torsion, a hollow section is defined as having one or more longitudinal voids, such as a single-cell or multiple-cell box girder. Small longitudinal voids, such as ungrouted post-tensioning ducts that result in  $A_g/A_{cp} \geq 0.95$ , can be ignored when calculating  $T_{th}$ . The interaction between torsional cracking and shear cracking for hollow sections is assumed to vary from the elliptical relationship for members with small voids, to a straight-line relationship for thin-walled



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**Table 22.7.4.1(a)—Threshold torsion for solid cross sections**

Type of member	$T_{th}$	
Nonprestressed member	$0.083 \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right)$	(a)
Prestressed member	$0.083 \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{f_{pc}}{0.33 \lambda \sqrt{f'_c}}}$	(b)
Nonprestressed member subjected to axial force	$0.083 \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{N_u}{0.33 A_g \lambda \sqrt{f'_c}}}$	(c)

**Table 22.7.4.1(b)—Threshold torsion for hollow cross sections**

Type of member	$T_{th}$	
Nonprestressed member	$0.083 \lambda \sqrt{f'_c} \left( \frac{A_g^2}{p_{cp}} \right)$	(a)
Prestressed member	$0.083 \lambda \sqrt{f'_c} \left( \frac{A_g^2}{p_{cp}} \right) \sqrt{1 + \frac{f_{pc}}{0.33 \lambda \sqrt{f'_c}}}$	(b)
Nonprestressed member subjected to axial force	$0.083 \lambda \sqrt{f'_c} \left( \frac{A_g^2}{p_{cp}} \right) \sqrt{1 + \frac{N_u}{0.33 A_g \lambda \sqrt{f'_c}}}$	(c)

**22.7.5 Cracking torsion**

**22.7.5.1** Cracking torsion  $T_{cr}$  shall be calculated in accordance with Table 22.7.5.1 for solid and hollow cross sections, where  $N_u$  is positive for compression and negative for tension.

**Table 22.7.5.1—Cracking torsion**

Type of member	$T_{cr}$	
Nonprestressed member	$0.33 \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right)$	(a)
Prestressed member	$0.33 \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{f_{pc}}{0.33 \lambda \sqrt{f'_c}}}$	(b)
Nonprestressed member subjected to axial force	$0.33 \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{N_u}{0.33 A_g \lambda \sqrt{f'_c}}}$	(c)

sections with large voids. For a straight-line interaction, a torsional moment of  $T_{th}$  would cause a reduction in the inclined cracking shear of approximately 25 percent, which was considered to be significant. Therefore, the expressions for solid sections are modified by the factor  $(A_g/A_{cp})^2$  to develop the expressions for hollow sections. Tests of solid and hollow beams (Hsu 1968) indicate that the cracking torsional moment of a hollow section is approximately  $(A_g/A_{cp})$  times the cracking torsional moment of a solid section with the same outside dimensions. An additional multiplier of  $(A_g/A_{cp})$  reflects the transition from the circular interaction between the inclined cracking loads in shear and torsion for solid members, to the approximately linear interaction for thin-walled hollow sections.

**R22.7.5 Cracking torsion**

The cracking torsional moment under pure torsion,  $T_{cr}$ , is derived by replacing the actual section with an equivalent thin-walled tube with a wall thickness  $t$  prior to cracking of  $0.75 A_{cp}/p_{cp}$  and an area enclosed by the wall centerline  $A_o$  equal to  $2 A_{cp}/3$ . Cracking is assumed to occur when the principal tensile stress reaches  $0.33 \lambda \sqrt{f'_c}$ . The stress at cracking,  $0.33 \lambda \sqrt{f'_c}$ , has purposely been taken as a lower bound value. In a nonprestressed beam loaded with torsion alone, the principal tensile stress is equated to the torsional shear stress,  $\tau = T/(2 A_o t)$ . Thus, cracking occurs when  $\tau$  reaches  $0.33 \lambda \sqrt{f'_c}$ , giving the cracking torsional moment  $T_{cr}$  as defined by expression (a) in Table 22.7.5.1.

For prestressed members, the torsional cracking load is increased by the prestress given by expression (b) in Table 22.7.5.1. A Mohr's Circle analysis based on average stresses indicates the torsional moment required to cause a principal tensile stress equal to  $0.33 \lambda \sqrt{f'_c}$  is  $\sqrt{1 + f_{pc}/(0.33 \lambda \sqrt{f'_c})}$  times the corresponding torsional cracking moment in a nonprestressed beam. A similar modification is made in expression (c) in Table 22.7.5.1 for members subjected to axial force and torsion.

If the factored torsional moment exceeds  $\phi T_{cr}$  in a statically indeterminate structure, a maximum factored torsional moment equal to  $\phi T_{cr}$  may be assumed to occur at critical sections near the faces of the supports. This limit has been

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**22.7.6 Torsional strength**

**22.7.6.1** For nonprestressed and prestressed members,  $T_n$  shall be the lesser of (a) and (b):

$$(a) \quad T_n = \frac{2A_o A_t f_{yt}}{s} \cot \theta \quad (22.7.6.1a)$$

$$(b) \quad T_n = \frac{2A_o A_t f_y}{p_h} \tan \theta \quad (22.7.6.1b)$$

where  $A_o$  shall be determined by analysis,  $\theta$  shall not be taken less than 30 degrees nor greater than 60 degrees;  $A_t$  is the area of one leg of a closed stirrup resisting torsion;  $A_t$  is the area of longitudinal torsional reinforcement; and  $p_h$  is the perimeter of the centerline of the outermost closed stirrup.

established to control the width of the torsional cracks. The replacement of  $A_{cp}$  with  $A_g$ , as in the calculation of  $T_{th}$  for hollow sections in 22.7.4.1, is not applied here. Thus, the torsional moment after redistribution is larger and, hence, more conservative.

**R22.7.6 Torsional strength**

The torsional design strength  $\phi T_n$  must equal or exceed the torsional moment  $T_u$  due to factored loads. In the calculation of  $T_n$ , all the torsion is assumed to be resisted by stirrups and longitudinal reinforcement, neglecting any concrete contribution to torsional strength. At the same time, the nominal shear strength provided by concrete,  $V_c$ , is assumed to be unchanged by the presence of torsion.

**R22.7.6.1** Equation (22.7.6.1a) is based on the space truss analogy shown in Fig. R22.7.6.1a with compression diagonals at an angle  $\theta$ , assuming the concrete resists no tension and the reinforcement yields. After torsional cracking develops, the torsional strength is provided mainly by closed stirrups, longitudinal reinforcement, and compression diagonals. The concrete outside these stirrups is relatively ineffective. For this reason  $A_o$ , the gross area enclosed by the shear flow path around the perimeter of the tube, is defined after cracking in terms of  $A_{oh}$ , the area enclosed by the centerline of the outermost closed transverse torsional reinforcement.

The shear flow  $q$  in the walls of the tube, discussed in R22.7, can be resolved into the shear forces  $V_1$  to  $V_4$  acting in the individual sides of the tube or space truss, as shown in Fig. R22.7.6.1a.

As shown in Figure R22.7.6.1b, on a given wall of the tube, the shear flow  $V_i$  is resisted by a diagonal compression component,  $D_i = V_i / \sin \theta$ , in the concrete. An axial tension force,  $N_i = V_i (\cot \theta)$ , is required in the longitudinal reinforcement to complete the resolution of  $V_i$ .

Because the shear flow due to torsion is constant at all points around the perimeter of the tube, the resultants of  $D_i$  and  $N_i$  act through the midheight of side  $i$ . As a result, half of  $N_i$  can be assumed to be resisted by each of the top and bottom chords as shown. Longitudinal reinforcement with a strength  $A_t f_y$  is required to resist the sum of the  $N_i$  forces,  $\sum N_i$ , acting in all of the walls of the tube.

In the derivation of Eq. (22.7.6.1b), axial tension forces are summed along the sides of the area  $A_o$ . These sides form a perimeter length  $p_o$  approximately equal to the length of the line joining the centers of the bars in the corners of the tube. For ease in calculation, this has been replaced with the perimeter of the closed stirrups,  $p_h$ .