Requirements For Seismic-Resistant Flat Plates: Strength and Ductility

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SYNOPSIS: Lateral displacement of multi-story flat plate concrete buildings in an earthquake induces moment reversals between columns and slabs. The amplitude of the transferred moment depends upon the story drift, defined as the displacement of one floor relative to the floor above or below. Flat plate buildings must have a lateral force-resisting system that limits the design story drift ratio to 0.025; where the design story drift includes plastic deformation and is defined as the design story drift divided by the distance between the mid-surfaces of the flat plates of two consecutive floors. The moments transferred from the columns to the slabs have to be resisted by flexural and shear reinforcements, whose magnitudes and detailing provide the slabs with the strength and the ductility to undergo the design story drift without failure.

The design of shear reinforcement for the moment transfer in an earthquake, as required by ACI 318, considers either the strength or the ductility, not both. ACI 421.2R-10 recommends and justifies a design procedure for the shear reinforcement providing the strength as specified by ACI 318; in addition, it recommends a minimum amount and extension of shear reinforcement that provides a level of ductility adequate for a design-story drift ratio = 0.025 (the upper permissible level in several codes). The design procedure is presented with examples.

KEYWORDS: column; drift; ductility; earthquake; flat plate; flexure; punching shear; seismic; strength.

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INTRODUCTION

For seismic design, the International Building Code 2006[1] (IBC-06) requires that: $DR_u \leq 0.007$ to 0.025, depending upon the type of building and its usage; where DR_u is design story drift ratio defined as the difference of displacements, including plastic deformations, between the top and bottom of a story divided by the story height. The present paper concentrates on the punching shear design, assuming that DR_u has been determined.

For seismic design, Section 21.13.6 of ACI 318-11[2] permits option (a) or (b) for punching shear in slab-column connections. Option (a) is a strength design to resist V_u combined with the unbalanced moment, M_u transferred between the slab and the column under the design displacements to satisfy the requirement of Sections 11.11.7 and 13.5.3.2[2]. Option (b) requires shear reinforcement when the design story drift ratio exceeds the larger of 0.005 and $\left[0.035 - 0.05\left(V_{ug}/\phi V_c\right)\right]$; these limits are presented by line *abc* in Fig. 1. It also specifies the amount of shear reinforcement and the extent of the shear-reinforced zone. V_{ug} is the factored shear force due to gravity loads; ϕ (= 0.75) is the strength reduction factor; V_c is the shear strength provided by concrete in absence of shear reinforcement. The two alternatives are here studied in view of the strength required in Sections 11.11 and 13.5.3.2 of the code[2].

Option (a)[2] requires calculation of the shear forces and the moments caused by the drift. ACI 318-11[2], Chapter 8, permits elastic analysis to calculate the internal forces. (No other method is required for seismic-resistant structures). An elastic analysis of the plane frame idealization in Fig. 2a, subjected to imposed drift ratio DR_e, can be used; where

$$DR_e = DR_u / \left(C_d / I_E\right) \qquad (1)$$

C_d is a dimensionless coefficient, representing the inherent inelastic deformability of the lateral force-resisting system (LFRS); I_E is the occupancy importance factor; (C_d/I_E) = 2.67 to 6.5 (IBC-06[1] or ASCE 7-08[3] (Section 9.5.5.7.1)). A similar relationship to Eq. 1 between DR_e and DR_u is given in the National Building Code of Canada 2005, Section 4.1.8.13[4]. The flexural reinforcement at critical sections of the slab must be designed (according to 13.3 of ACI 318-11[2]) for the moments determined by the analysis of the effect of the drift, combined with the moments due to factored gravity loads. Insufficient flexural reinforcement can invoke premature punching failure[5,6].

Section 13.7 of ACI 318-11[2] specifies the *I*-values in Figs. 2a and 2b to be used in the elastic analysis of the internal forces due to earthquake-induced drift and due to gravity loads, respectively. In Fig. 2c, the equivalent moment of inertia of the column, I_{ec} accounts for an assumed torsional strip in accordance with Section 13.7.5[2].

While option (a) gives attention to the strength, option (b) focuses on the ductility. With option (b), the design criterion that permits the absence or requires shear reinforcement – and specifies its amount – is based on experiments that give an empirical relationship between the parameter $V_u/(\phi V_c)$ and the drift ratio capacity. The present paper claims that this relationship is not universal, because it does not include other parameters that affect the drift ratio capacity, including the flexural reinforcement ratio and the column's aspect ratio / flexural rigidity. The paper justifies and recommends a design with a combination of the two options to ensure the strength and the ductility.

RESEARCH SIGNIFICANCE

A study of the major parameters affecting the drift capacity of the connections of columns with flat plates is presented to show a need for a change in the ACI 318-11 code[2]. The section that needs the change and the recommended revision are indicated.

PUNCHING SHEAR DESIGN EQUATIONS

The design equations of ACI 318-11[2] that will be used in this paper are:

$$v_u \leq \phi v_n \tag{2}$$

$$v_u = \frac{V_u}{b_o\, d} + \frac{\gamma_v\, M_u\, x}{J_c} \tag{3}$$

where v_u = the maximum shear stress; v_n = the nominal shear strength (MPa or psi); b_o = the perimeter length of the shear critical section at $d/2$ from the column face (Fig. 3), or from the outermost peripheral line of shear reinforcement; d = effective depth of slab = the average of distances from extreme compression fiber to the centroid of the tension reinforcements running in two orthogonal directions; γ_v = the fraction of unbalanced moment transferred by eccentricity of shear; x = coordinate of the point at which v_u is calculated; J_c = property of the shear critical section. ACI 421.1R-08[7] gives equations for J_c and γ_v for shear critical sections of any polygonal shape. Equation 4 or 5 gives, respectively, the nominal shear strength, v_n in the absence or in the presence of shear reinforcement (stirrups or headed shear studs).

$$\left.\begin{aligned} & v_n \equiv v_c = \text{the least of}: \lambda\sqrt{f'_c}/3; \left[2+(4/\beta)\right]\lambda\sqrt{f'_c}/12; \left[(\alpha_s\, d/b_o)+2\right]\lambda\sqrt{f'_c}/12\ (\text{MPa}) \\ \text{or}\ & v_n \equiv v_c = \text{the least of}: 4\lambda\sqrt{f'_c}; \left[2+(4/\beta)\right]\lambda\sqrt{f'_c}; \left[(\alpha_s\, d/b_o)+2\right]\lambda\sqrt{f'_c}\ (\text{psi}) \end{aligned}\right\} \tag{4}$$

$$\begin{aligned} & v_n = 0.17\sqrt{f'_c} + v_s \leq 0.5\sqrt{f'_c}\ (\text{MPa})\ \text{or}\ v_n = 2\sqrt{f'_c} + v_s \leq 6\sqrt{f'_c}\ (\text{psi}) \quad \text{Stirrups} \\ & v_n = 0.25\sqrt{f'_c} + v_s \leq 0.67\sqrt{f'_c}\ (\text{MPa})\ \text{or}\ v_n = 3\sqrt{f'_c} + v_s \leq 8\sqrt{f'_c}\ (\text{psi}) \quad \text{Headed shear studs} \end{aligned} \tag{5}$$

$$v_s = A_v\, f_{yt}/(b_o\, s) \tag{6}$$

where λ = modification factor reflecting the reduced mechanical properties of lightweight concrete, relative to normal-weight concrete of the same compressive strength; f'_c = the specified compressive strength of concrete; β ($\geq$ 1.0) is the aspect ratio of column; α_s = 40, 30 or 20 for interior, edge or corner columns, respectively; A_v = the area of the vertical legs of the shear reinforcement on one peripheral line parallel to the column face; f_{yt} = the specified yield strength of the shear reinforcement; s = the spacing between peripheral lines of the shear reinforcement. At the critical section at $d/2$ from the outermost peripheral line of shear reinforcement, the nominal shear strength is:

$$v_n = 0.17\lambda\sqrt{f'_c}\ (\text{MPa}) = 2\lambda\sqrt{f'_c}\ (\text{psi}) \tag{7}$$

Section 13.5.3.2 of ACI 318-11[2] requires the provision of flexural reinforcement within a specified width to resist a fraction of the unbalanced moment given by (1-γ_v) M_u. When shear reinforcement is required in option (b) of Section 21.13.6[2], it must satisfy Eqs. 8 and 9:

$$v_s \geq 0.29\sqrt{f'_c}\ (\text{MPa})\ \left[3.5\sqrt{f'_c}\ (\text{psi})\right] \tag{8}$$

$$b_s \geq 4h + d/2 \tag{9}$$

where h = the slab thickness; b_s = the distance from the column face to the shear critical section at $d/2$ from the outermost peripheral line of shear reinforcement.

DUCTILITY DEPENDENCE ON FLEXURAL REINFORCEMENT RATIO

A frequently used test setup is shown in Fig. 4a; it represents a specimen of a square column connection with a simply supported square slab subjected to a shear force *V*. The nominal strength is the smaller of:

$$V_n = v_n \; b_o \, d \quad \text{(Punching shear failure mode)} \tag{10}$$

and $$V_f = 8\,m\,l_s/(l_1 - c) \quad \text{(Flexural yield-line collapse mode; Fig. 4b)} \tag{11}$$

$$m = \rho\, f_y\, d^2 \left[1 - 0.59\left(\rho\, f_y / f_c'\right)\right] \tag{12}$$

where l_s, l_1 and c are dimensions shown in Fig. 4b; m = the ultimate flexural strength of a strip of the slab of unit width in each of two orthogonal directions; ρ and f_y = the ratio of the top flexural reinforcement in a strip of unit width and its specified yield strength.

The load-deflection graph, e.g. Fig. 4c, is often used to give a ductility index, which is defined as the ratio of the deflection at a point on the descending part of the graph (e.g. point B or C) where the load drops to a specified fraction of its maximum value, to the deflection at a point on the ascending part where the graph ceases to be linear; specifying these two points is not needed in the following discussion.

When the nominal shear strength is sufficiently large, $V_{n1} > V_f$, the load-deflection graph will be OAC (Case I, Fig. 4c), exhibiting large deflection, D_C at which the yield-line fracture pattern in Fig. 4b is developed; the maximum load value in the graph will be V_f that can be predicted by Eq. 11. The ductility index in this case is high. At this stage, the cracks in the vicinity of the column extend deep in the slab; concrete crushing occurs at the compression face and the connection may exhibit secondary failure by punching before extensive plastic deflection.

When the nominal shear strength is small, e.g. a slab without shear reinforcement, $V_{n2} < V_f$, the load-deflection graph extends from O to A, where sudden punching failure occurs at a deflection $D_A << D_C$. For such a slab, the lack of ductility can be explained by the limited extent of the zone at which the flexural reinforcement yields. Here, the ductility index is low (Case II).

Yielding of the flexural reinforcement starts well before the development of the full yield-line pattern that can produce collapse (Case I). The ductility index depends upon the extent of yielding of the flexural reinforcement. Consider Case III, where the nominal shear strength, V_{n3} is such that $V_{n2} < V_{n3} < V_f$. This can occur by the provision of adequate and properly detailed shear reinforcement, to prevent failure within the shear-reinforced zone, but the extension of the shear-reinforced zone is limited; the load-deflection curve will be OAB. At point B, punching failure occurs outside the shear-reinforced zone, exhibiting a ductility index somewhere in between the two extremes of Cases I and II. The *V*-values marked in the graph of Fig. 4c are for a connection having: c = 250 mm (9.84 in.); h = 150 mm (5.91 in.); d = 114 mm (4.49 in.); l_s = 1.9 m (74.8 in.); l_1 = 1.8 m (70.9 in.); ρ = 0.0116; f_c' = 30 MPa (4400 psi); f_y = 400 MPa (58 ksi). The shear reinforcement considered has: A_v = 570 mm^2 (0.88 in.^{2}); f_{yt} = 345 MPa (50 ksi); s = 85 mm (3.3 in.); b_s = 929 mm (36.6 in.); at $d/2$ from the outermost peripheral line of shear reinforcement, b_o = 6310 mm (248 in.).

Appropriately designed shear reinforcement can delay failure until the development of the full yield-line pattern, exhibiting extensive ductility. For this to occur, the shear reinforcement must be designed to ensure that punching shear strengths within and outside the shear-reinforced zone are greater than V_f. The increase of the shear reinforcement beyond what is needed to satisfy these requirements does not change the graph OAC in Fig. 4c. Examples of the three graphs of the typical Cases I, II and III in Fig. 4c can be seen in results of experiments reported in the literature[8,9].

From this discussion, it can be seen that in the test setup in Fig. 4a, the ductility would generally depend upon the value of V_f with respect to V_n. Because V_f is mainly dependent upon ρ, it follows that the ductility is also dependent on ρ. Application of two equal and opposite horizontal forces at the tips of the column stubs, combined with V, would transfer unbalanced moment, M between the column and the slab. If V is kept constant at a level below V_n and M is gradually increased, a graph of M versus DR would be similar to one of the graphs of Fig. 4c; however, the yield-line pattern at which flexural failure occurs will be different from that in Fig. 4b. By the same reasoning as above, it can be seen that ρ significantly affects the ductility, represented by the drift ratio capacity, DR_u (the value of DR at maximum M or after its drop by 20%).

DUCTILITY DEPENDENCE ON THE RATIO $V_u/(\phi V_c)$

Several researchers[10-17] have investigated the effect of the ratio $V_u/(\phi V_c)$ on the ductility of slab-column connections transferring V_u combined with cycles of lateral displacements (drifts) of increasing magnitude; in these investigations, the variation of the drift ratio capacity, DR_u versus $V_u/(\phi V_c)$ have been plotted. Kang and Wallace[16] plotted the points $\left[V_u/(\phi V_c), DR_u\right]$ of 76 interior column tests; all the columns were square except six. The graph clearly shows the trend that the ductility drops as the ratio $V_u/(\phi V_c)$ increases. However, DR_u does not depend uniquely upon the value of $V_u/(\phi V_c)$; the graph of Kang and Wallace[16] does not indicate how the ductility is affected by other parameters including ρ and the flexural rigidity of the column.

It is noted that the graph of Kang and Wallace[16] is for 76 tests in which the maximum $V_u/(\phi V_c)$ did not exceed 0.9 and this parameter is greater than 0.85 only for 4 tests. However, practical cases where $V_u/(\phi V_c) > 0.9$ are common. Equation 13 shows that for a non-prestressed flat plate, $V_u/(\phi V_c)$ can exceed 0.90 in practice.

$$V_u/(\phi V_c) = 1.0 \text{ when } \begin{cases} q_u \text{ (kPa)} \cong 3.6\left[(c/h)+0.8\right] \\ q_u \text{ (psf)} \cong 75\left[(c/h)+0.8\right] \end{cases} \tag{13}$$

where q_u = the sum of the factored dead and live loads; c = side of square column; h = slab thickness. As example, c = 305 mm (12 in.) and h = 203 mm (8 in.); Eq. 13 gives q_u = 8.3 kPa (173 psf). The weight of 203 mm (8 in.) slab plus super-imposed dead load with a factor of 1.2 can amount to 7.2 kPa (150 psf); this leaves only 1.1 kPa (23 psf) for the factored live load to make $V_u/(\phi V_c) = 1.0$. Equation 13 can be derived assuming: $d = 0.8h$; square panels of span = $35h$; f'_c = 30 MPa (4400 psi); $\phi V_c = 0.75\left(\sqrt{f'_c}/3 \text{ MPa}\right) b_o\ d \left[0.75\left(4\sqrt{f'_c} \text{ psi}\right) b_o\ d\right]$; $V_u = q_u\ (35\ h)^2$; $b_o\ d = 4\left(c + 0.8\ h\right) 0.8\ h$. By a similar derivation, it can be shown that $V_u/(\phi V_c)$ reaches 1.0 more often in practice in prestressed slabs, where the ratio of span to $h > 35$.

Figure R21.13.6 of ACI 318-11[2] is a graph illustrating the criterion for requiring shear reinforcement in option (b) of 21.13.6. The graph is the same as the broken line *abc* in Fig. 1 of the present paper, with the abscissa representing $V_{ug}/(\phi V_c)$ and terminating at 0.75. A possible interpretation is: the graph and option (b) are intended for the case where $V_{ug}/(\phi V_c)$ does not exceed 0.75; this would also limit their validity to the case where shear reinforcement is not needed for pure gravity load and would allow only option (a) in other cases. The interpretation of 21.13.6(b) and Fig. R21.13.6 of the code[2] in this way is not pursued further here; it is more useful to focus on the design in a general case, without the mentioned restrictions. The examples below show that design of shear reinforcement based on 21.13.6(b)[2] does not ensure adequate strength for relatively high values of $V_u/(\phi V_c)$ or DR_u.

DUCTILITY DEPENDENCE ON THE ASPECT RATIO OR THE FLEXURAL RIGIDITY OF COLUMNS

As mentioned earlier, there is not sufficient experimental data to give the variation of DR_u with $V_u/(\phi V_c)$ for varying aspect ratio or flexural rigidity of columns. The unbalanced moment, M_u increases with the flexural rigidity of the column; also, the value of γ_v for a rectangular column, is higher compared to a square column. It will be shown below that for any design drift ratio, the unbalanced moment and the maximum shear stress are higher for rectangular columns compared to square columns. Thus, it is logical to expect that punching would occur at a lower drift ratio as the column aspect ratio, c_x/c_y exceeds 1.0 (Fig. 5); with x being the drift direction.

SHEAR STRESS ASSOCIATED WITH DRIFT

The relationship between $V_u/(\phi V_c)$ and DR_u that satisfies Section 11.11[2] is here derived. Section 11.11.2[2] requires shear reinforcement when $v_u > \phi v_n$ (with $v_n = v_c$, Eq. 4). Express v_u at the critical section in Fig. 3 as the sum of the factored maximum shear stress v_{ug} due to gravity loads and v_{uE} due to earthquake; thus, the criterion that permits absence of shear reinforcement is:

$$v_{uE} \le \phi v_c - v_{ug} \tag{14}$$

v_{uE} can be expressed by Eq. 3:

$$v_{uE} = [V_{uE}/(b_o\, d)] + (\gamma_v\, M_{uE}\, x/J_c) \tag{15}$$

Here the subscript E refers to the effect of the drift induced by earthquake; the effect of the vertical component of ground motion is not included. IBC-06[1] and ACI 318-11[2] (Section 8.3.1) permit the calculation of V_{uE} and M_{uE} by linear analysis. The frame in Fig. 2a is subjected to an imposed displacement = $DR_e\ l_c$; DR_e is a fraction of DR_u specified by Eq. 1; l_c is the story height = distance between mid-surfaces of slabs of two consecutive floors. For the column on the axis of symmetry in Fig. 2a, the drift produces no shear force; thus, V_{uE} = 0, V_u= V_{ug} and v_{uE} is proportionate to DR_e. The maximum shear stress due to earthquake can thus be expressed as linear function of DR_e or DR_u as:

$$v_{uE} = v_e\ DR_e \text{ with } DR_e \leq 0.0094 \tag{16}$$

$$v_{uE} = \left(\frac{v_e}{C_d/I_E}\right) DR_u \text{ with } DR_u \leq 0.025 \tag{17}$$

v_e = constant (MPa or psi), whose value increases with the increase of the flexural rigidity / aspect ratio (c_x/c_y, Fig. 3). The constant v_e represents the maximum shear stress corresponding to DR_e = 1. IBC-06[1] specifies a maximum value of DR_u = 0.007 to 0.025 and gives values of (C_d/I_E) varying between 2.67 and 6.5; this sets an upper limit of DR_e below 0.94 percent. Thus, IBC-06[1] permits the elastic analysis to calculate the unbalanced moment for $DR_e \leq$ 0.94 percent. In option (b) of 21.13.6[2], $V_{ug} = v_{ug}\ b_o\ d$ = the factored shearing force due to factored gravity loads. Substitution of Eq. 17 in Eq. 14 and noting that $v_{ug} = V_u/(b_o\, d)$ and $V_c = v_c\, b_o\, d$, gives the limit of DR_u to permit absence of shear reinforcement:

$$DR_u \leq \phi\left(\frac{v_c}{v_e}\right)\left(\frac{C_d}{I_E}\right)\left[1 - \frac{V_u}{\phi V_c}\right] \text{ ; with } DR_u \leq 0.025 \tag{18}$$

Equation 18 is represented in Fig. 5 by a series of straight lines (e.g. *ef* and *eg*), whose slope depends on v_e (Appendix A). Line *ef* in Fig. 5 represents Eq. 18 for the connection of a slab with square columns, $c_x \times c_y$ = 300 mm×300 mm (11.8 in.×11.8 in.) (Example 1). For any point above the line, option (a) of 21.13.6[2] requires shear reinforcement to satisfy Section 11.11[2]. Line *eg* is for the slab-column connection of Example 1, with the square columns replaced by rectangular columns, 500 mm×250 mm (19.7 in.×9.8 in.); the slope of the line is changed resulting in a wider zone for which shear reinforcement is required. The slopes of lines *ef* and *eg* are calculated in Appendix A. This shows that the shear reinforcement requirement depends on the aspect ratio / the flexural rigidity of the column.

EXAMPLE 1: DESIGN ACCORDING TO OPTION (a) OF 21.13.6, ACI 318-11[2]

Consider a solid flat plate for an office building with equal span lengths in the x- and y-directions, $L_x = L_y$ = 6.0 m (19.7 ft), floor height, l_c = 3.50 m (11.5 ft), column size, $c_x \times c_y$ = 300 mm×300 mm (11.8 in.×11.8 in.), and slab thickness, h = 200 mm (7.87 in.). The floor is designed for a service live load of 2.4 kPa (50 psf), a super-imposed dead load of 1.3 kPa (27 psf), and a self-weight of 4.7 kPa (98 psf). The earthquake-excited motion is in the x-direction, where the structure has six bays (Fig. 2), with many bays in the y-direction. The structure has a LFRS that limits the ultimate inter-story drift ratio, including inelastic deformations, to DR_u = 0.02; assume that the ratio C_d/I_E = DR_u/DR_e = 3.5. The elastic drift (Eq. 1):

$$DR_e\, l_c = 0.02(3.5)/3.5 = 0.02 \text{ m (0.79 in.)}$$

It is required to calculate V_u, M_u and v_u at the central column and design the shear reinforcement, if required. Other data are: concrete cover = 20 mm (0.79 in.); f'_c = 30.0 MPa (4350 psi); E_c = 24.6 GPa (3580 ksi); the top flexural reinforcement is composed of 16 mm (0.63 in.) bars, spaced at 150 mm (5.91 in.). The effective slab depth, d = 200–20–16 = 164 mm (6.46 in.). If required, use headed shear stud reinforcement with f_{yt} = 345 MPa (50 ksi).

ACI 318-11[2] requires seismic design for: ($1.2D + E + 0.5L$). The symbol E in the load combinations represents the effect of drift, excluding the effect of the vertical component of ground motion. The factored gravity load = 8.4 kPa (175 psf); corresponding V_{ug} = 314 kN (70.6 kips) and M_{ug} = 0. From an elastic analysis of the equivalent frame in Fig. 2a, subjected to lateral elastic displacement, $DR_e\ l_c$ = 20 mm (0.79 in.), the shearing force and unbalanced moment at the central interior column are: V_{uE} = 0 and M_{uE} = 117 kN-m (1035 kip-in.). Appendix A determines the moments of inertia of the members of the equivalent frames used in the analysis.

Punching shear strength design is required for V_u = 314 kN (70.6 kips) combined with M_u = 117 kN-m (1035 kip-in.). Properties of the shear critical section at $d/2$ from the column face (Appendix B of ACI 421.1R-08[7]) are: b_o = 1.856 m (73.07 in.); x = 232 mm (9.13 in.); $\gamma_v = 0.4$; $J_c = 10.92\times10^{-3}$ m^4 (26.24×10^3 in.^{4}). The maximum shear stress is (Eq. 3): v_u = 2.03 MPa (294 psi). This value is greater than ϕv_c (= 1.37 MPa (199 psi); Eq. 4); thus, shear reinforcement is required.

Because v_u is less than $0.5\phi\sqrt{f_c'}$ = 2.05 MPa (298 psi), $s \le 0.75d$ = 123 mm (4.84 in.). Use eight rails of headed shear stud reinforcement, arranged as shown in Fig. 6a, having nominal diameter = 12.7 mm (1/2 in.), s = 120 mm (4.72 in.), s_o = 65 mm (2.56 in.) and A_v = 1013 mm^2 (1.57 in.^{2}); Eqs. 5 and 6 give: v_s = 1.58 MPa (216 psi); ϕv_n = 2.21 MPa (320 psi) > v_u = 2.03 MPa (294 psi). Thus, the chosen A_v and s are satisfactory. At $d/2$ from the outermost peripheral line of studs, b_o = 4148 mm (163.3 in.); x = 657 mm (25.9 in.); $\gamma_v = 0.4$; $J_c = 139.2\times10^{-3}$ m^4 (334.4×10^3 in.^{4}) and v_u = 0.68 MPa (98 psi). This stress value is less than the factored concrete strength, $\phi v_n = 0.17\,\phi\sqrt{f_c'} = 0.69$ MPa (99 psi) (Eq. 7) indicating that the extension of the shear-reinforced zone is adequate (Fig. 6a).

Flexural reinforcement to resist $(1-\gamma_v)\,M_{uE}$ has to be provided in the slab and detailed according to 13.5.3.2[2].

EXAMPLE 2: DESIGN ACCORDING TO OPTION (b) OF 21.13.6, ACI 318-11[2]

For the column-slab connection in Example 1, V_u = 314 kN (70.6 kips), V_c = 556 kN (125 kips), $V_u/(\phi V_c)$ = 0.75 and DR_u = 0.02. The point $\left[V_u/(\phi V_c),\ DR_u\right]$ lies above the bilinear limit abc in Fig. 1; thus, shear reinforcement is required. The shear reinforcement has to satisfy Eqs. 6, 8 and 9. For eight rails of 12.7 mm (1/2 in.) studs,

$$s \le \frac{A_v\, f_{yt}}{b_o\left(0.29\sqrt{f_c'}\right)} = \frac{1013(345)}{1856\left(0.29\sqrt{30}\right)} = 119 \text{ mm (4.7 in.)}$$

Use s_o = 65 mm (2.6 in.) and s = 115 mm (4.53 in.). The shear-reinforced zone should extend such that $b_s \ge 4h + (d/2)$ = 4 (200) + (164/2) = 882 mm (34.7 in.); this extension is achieved by eight studs on each rail as shown in Fig. 6(b). Because, the shearing force and moment transferred between the column and the slab due to earthquake are not calculated, the design with option (b) does not verify that the flexural reinforcement or the shear strength satisfies 13.5.3.2 or 11.11.7[2].

COMPARISON OF DESIGNS ACCORDING TO THE OPTIONS OF SECTION 21.13.6[2]

Equation 18 is represented in Fig. 1 by a typical line eg, whose slope depends on the column's aspect ratio / flexural rigidity; a connection, without shear reinforcement, represented by a point of $\left[V_u/(\phi V_c),\ DR_u\right]$ that falls within the shaded zones satisfies option (b) of 21.13.6[2], but lacks the strength required by Section 11.11[2].

From Figs. 6(a) and 6(b), it can be seen that Eq. 9 requires unnecessarily large extension of the shear-reinforced zone. It is impossible to find a case that requires this extension by a strength design based on Section 11.11[2].

To satisfy the requirement: $v_u \le \phi v_n$ (Eq. 2), the designer can limit v_u by increasing the slab thickness or augmenting v_n by increasing f_c' or providing shear reinforcement. In the design with option (b) of 21.13.6[2], the amount of shear reinforcement is prescribed (by Eq. 8) and the calculation of v_u is not done. The prescribed shear reinforcement of Eq. 8 may not be sufficient to bring v_n to the required level. Section 11.11.7[2] sets an upper limit for v_n (Eq. 5 or 6) that cannot be exceeded regardless of the amount of shear reinforcement. Again, by not using Eq. 2 in the design, there is no guarantee that the upper limit for v_n is respected. In summary, the shortcomings of 21.13.6(b)[2] are:

- It may miss to indicate the need for shear reinforcement, or require shear reinforcement having inadequate v_s and extend the shear-reinforced zone farther than needed.
- It may miss to indicate the need to increase the slab thickness or f'_c.
- It does not enable the design of the flexural reinforcement required to resist (1-γ_v) M_u (Section 13.5.3.2[2]).
  Thus, the design with option (b) does not insure that all the requirements of Sections 11.11.7 are satisfied.

EXAMPLE 3: APPLICATION OF 21.13.6[2] TO A RECTANGULAR EDGE COLUMN

Consider the edge column-slab connection of the same structure of Example 1, with all columns having rectangular cross-section, $c_x \times c_y$ = 500 mm×250 mm (19.7 in.×9.8 in.) subjected to the same gravity load. Find the upper limits of DR_u permitted in absence of shear reinforcement by options (a) and (b) of Section 21.13.6[2]. The solution of this example, given below, will show that the upper limits are 0.61 percent with option (a) and 1.13 percent with option (b). This large difference indicates that option (b) cannot safely be a substitute of option (a).

For the edge column-slab connection, the internal forces due to factored gravity loads are: V_{ug} = 168 kN (37.8 kips) and $(M_{ug})_O$ = 44 kN-m (389 kip-in.); where M_{uO} is the unbalanced moment about an axis through the column's centroid, O (Fig. 3b). The critical shear stress occurs for a drift in the negative x-direction; elastic analysis of the equivalent frame in Fig. 2a, subjected to lateral elastic displacement in this direction, $DR_e\, l_c$ (Eq. 1), gives at the edge column the shearing force, $V_{uE} = 2400DR_u$ kN ($540DR_u$ kips) and the unbalanced moment, $(M_{uE})_O = 7800DR_u$ kN-m ($69000DR_u$ kip-in.). Calculate the maximum shear stress, v_u due to the factored internal forces of the combination ($1.2D + E + 0.5L$): V_u = ($168+2400DR_u$) kN (($37.8+540DR_u$) kips), combined with M_{uO} = ($44+7800DR_u$) kN-m (($389+69000DR_u$) kip-in.). The corresponding unbalanced moment at the centroid of the shear critical section at d/2 from the column (Fig. 3b) is: $M_u = M_{uO} + V_u\, x_O$.

Properties of the shear critical section at d/2 from the column face (Fig. 3b) are: b_o = 1.578 m (62.13 in.); x_O = –117 mm (–4.61 in.); x_A = –367 mm (–14.4 in.); x_B = 215 mm (8.46 in.); γ_v = 0.423; $J_c = 9.629\times10^{-3}$ m^4 (23.13×10^3 in.^{4}) (ACI 421.1R-08[7]). The value of $DR_u = 6.1\times10^{-3}$ would bring v_u to its permissible limit without shear reinforcement, ϕv_c (= 1.37 MPa (199 psi); Eq. 4). Thus, option (a) of 21.13.6[2] requires shear reinforcement for $DR_u > 6.1\times10^{-3}$. On the other hand, with $V_{ug}/(\phi V_c)$ = 168/(472 ϕ) = 0.475, the design with option (b) of 21.13.6[2] would permit absence of shear reinforcement with $DR_u \le 0.035 - 0.05\,[V_{ug}/(\phi V_c)] = 11.3\times10^{-3}$.

SEISMIC DESIGN RECOMMENDED BY ACI 421.2R-10[18]

ACI 421.2R-10[18] presents design examples of interior, edge and corner column-slab connections according to option (a) of 21.13.6, ACI 318-11[2]. In addition, based on referenced experimental data, ACI 421.2R-10[18] recommends for ductility a minimum amount of shear reinforcement specified by Eq. 20 and an extent of the shear-reinforced zone according to Eq. 21; the minimum shear reinforcement is recommended only when V_u or DR_u is relatively high (Eq. 19).

$$V_u/(\phi\, V_c) \ge 0.4 \text{ or } DR_u \ge 0.035 - 0.05\left[V_u/(\phi\, V_c)\right] \text{ with } DR_u \ge 0.015 \tag{19}$$

$$v_s \ge 0.25\sqrt{f'_c}\ \text{(MPa)}\ \left[3\sqrt{f'_c}\ \text{(psi)}\right] \tag{20}$$

$$b_s \ge 3.5d + d/2 \tag{21}$$

The design in Example 1 (Fig. 5a) satisfies Eq. 20; with two additional peripheral lines of shear reinforcement, Eq. 21 would be satisfied.

To account for the degradation of concrete shear resistance caused by the reversals of the unbalanced moment, ACI 421.2R-10[18] recommends calculating v_n by Eq. 22, in which the first term is half the corresponding term in Eq. 5:

$$\begin{aligned} v_n &= 0.083\sqrt{f'_c} + v_s\ (\text{MPa}) \text{ or } v_n = \sqrt{f'_c} + v_s\ (\text{psi}) \quad \text{Stirrups} \\ v_n &= 0.125\sqrt{f'_c} + v_s\ (\text{MPa}) \text{ or } v_n = 1.5\sqrt{f'_c} + v_s\ (\text{psi}) \quad \text{Headed shear studs} \end{aligned} \tag{22}$$

In lieu of Eq. 22, it may be possible to reduce the first term in Eq. 5 by a factor dependent upon DR_u (a proposal that needs further research).

CAUTIONARY NOTE

The design for punching by option (b) of 21.13.6[2] is simple to apply, because the shear force and the unbalanced moment associated with the drift of the structure is not considered; at the same time, the option does not ensure satisfying the punching shear or the flexural strength of Section 11.11 or 13.5.3.2[2]. Note that the calculation of M_u is still required to verify that $M_u \leq M_f$; where M_f is the unbalanced moment that transforms the slab into a flexural collapse mechanism. Gesund and Goli[19] give the yield-line patterns and the equations for M_f.

NEED FOR CODE CHANGE

Section 21.13.6 of ACI 318-11[2] permits option (a) or (b) for the seismic punching shear design of the connections of flat plates and columns. Option (a) ensures the strength required in Section 11.11 of the same code; while option (b) is based on the effect of the parameter $V_{ug}/(\phi V_c)$ on the drift capacity. To ensure strength and ductility, it is here recommended to combine the two options: For strength, satisfy 11.11.7.1 and 13.5.3.2; for ductility, require shear reinforcement having minimum v_s and b_s when $V_u/(\phi V_c)$ or DR_u exceeds a specified limit (e.g. see values recommended in ACI 421.2R-10[18]). Also, to account for the degradation of concrete shear resistance caused by the reversals of the unbalanced moment, the contribution of concrete to v_n, in the presence of shear reinforcement, may need to be revisited (e.g. Eq. 22).

CONCLUSIONS

Calculation of the shearing force and the unbalanced moment transferred at a column-slab connection in multi-story buildings due to gravity loads and drift (due to any lateral load) is demonstrated by examples. In an earthquake, the connection must be ductile to withstand drift reversals in the plastic range without loss of its punching shear strength. The ductility, expressed in terms of the design inter-story drift ratio, DR_u, depends upon $V_u/(\phi V_c)$, the flexural reinforcement ratio and the column's aspect ratio / flexural rigidity. An empirical graph of the parameter $V_u/(\phi V_c)$ versus DR_u obtained by tests investigating the effect of this parameter does not ensure the strength for all cases.

The seismic design options (a) and (b) of 21.13.6, ACI 318-11[2] are studied to show that option (b) may not satisfy the strengths required in 11.11.7 and 13.5.3.2[2]. An example is presented to show that the limit of DR_u, at which shear reinforcement is required, is substantially different with the two design options. Revisions of Section 21.13.6[2] and its commentary are proposed to ensure that flat plate-column connections have the strengths required in the code[2]. In addition, for ductility, a specified minimum amount of shear reinforcement is proposed when $V_u/(\phi V_c)$ or DR_u is relatively high.

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