SP-227—1

Managing Deflection, Shortening and Cracking Arising from Restrained Contraction

by S. J. Alexander

Synopsis: Concrete shrinks. Steel doesn't, and the resistance of reinforcement to shrinkage causes deflection of slabs and beams and shortening of columns and walls. A simple visualization is given, and used to derive formulae for analysis. Current methods of calculating shrinkage curvature and deflection in reinforced sections are examined and compared, concluding that the ACI method appears realistic while the UK and European methods significantly over-estimate the deflection.

Restraint by differential contraction between an insitu concrete overlay and an older substrate produces tension in the overlay and curvature and deflection of the composite unit. A method for calculating this is given, and the resulting effects are found to be significant in certain circumstances. The method is extended to consider shrinkage in insitu slabs in steel-concrete composite construction. The deflection from shrinkage is found to be approaching span/750, and cracking in the slabs is predicted in some cases.

External restraint to contraction induces tensile stresses, and a rational approach to providing sufficient reinforcement to control cracking in direct tension is given. It is particularly relevant to elements which need to be water-resisting, and a case study of a basement is presented. The reinforcement needed to control cracking reliably is found to exceed most current US, UK and European recommendations.

Keywords: composite beams; cracking; creep; deflection; design; early age contractions; overlays; reinforced concrete; restraint; serviceability; shortening; shrinkage; structural analysis

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INTRODUCTION

Concrete contracts and shrinks. The main effects to be considered in design are early age contraction, temperature drop and long-term drying shrinkage. Restraint arises in a number of ways. Embedded reinforcement causes deflection of slabs and beams and contributes to shortening of columns and walls. Casting concrete against a previous pour or as an overlay on top of an older substrate induces tensile stresses that can cause cracking. So do metal decking and supporting beams in steel-concrete composite construction. Similarly, external restraint from rigid elements such as in-plane walls, piles and pile caps, and even friction from the underlying soil, can cause cracking. Shrinkage is also significant in concrete roads and industrial ground floors, but the paper is limited to typical structural elements in buildings.

The paper considers how to quantify these effects, and makes recommendations for managing them.

LIFE CYCLE OF CONCRETE

Concrete typically goes through the following life cycle:

Early age contractions. In the first three to six days, the hydration process first causes the concrete to heat up – from a casting temperature that is usually already a few degrees above ambient. During this period, it behaves in a plastic manner so the only result is an increase in volume. However, when it cools to ambient temperature (Figure 1) it has hardened enough to go into tension and may crack if it is restrained. At the same time, shrinkage (a complex combination of actions, see Altoubat and Lange (1)) also occurs, adding to the contractions.

Strength gain. In the following three to four weeks (longer in mixtures with flyash or slag), both compressive and tensile strength increase.

Temperature. In the first year, it will be subjected to the seasonal temperature and humidity cycle if it is exposed, or to indoor conditions if it is enclosed. Daily variations will also occur.

Shrinkage. Over a longer period, perhaps two to ten years, long-term drying shrinkage will gradually occur.

It is important to recognise the cumulative effect of all the contractions, that is early age contractions (partially relieved by creep, see below) plus temperature drop plus shrinkage, if they are restrained. Thus only a small temperature drop or amount of shrinkage can be enough to initiate a crack in concrete with locked-in tensile stress from early age contraction. However, note that concrete and steel can usually be assumed to have the same coefficient of thermal expansion, so that embedded reinforcement does not form a restraint to thermal movements. Also, if the only restraint present is that to early age contraction from a recent adjacent pour, shrinkage and temperature drop can usually be assumed to apply equally to the whole structure and any long-term differential therefore ignored.

Alleviation by creep

Some alleviation of sustained tensile stresses, that is those arising from early age contractions and long-term shrinkage but only seasonal temperature, is provided by creep. Altoubat and Lange found that early age contractions were reduced very rapidly by as much as 50 %. Interestingly, the same reduction is included in BS 8007 (2) where it is explained as a ‘restraint factor’.

European guidance on creep and shrinkage is published in BS 5400-4 (3), but appears to be largely additional to early age effects. Figure 2 is derived from BS 5400-4, and shows the reduction over time of both early thermal contraction and shrinkage. Early thermal contraction stresses are reduced quite rapidly. The figure is based on fairly conservative assumptions (loading at 3.5 days, section 300 mm [12 in] thick exposed on one side only, *rh* 80 %), yet the reduction is estimated to be 30 % at four months and 60 % at two years. Long-term shrinkage is different. Although each increment is lessened by creep, the shrinkage builds up quite slowly so the effect of creep is even more delayed. The figure shows that the effect is hardly significant up to one year, although ultimately the reduction is about one-third.

Note that these reductions apply to the tensile stress in the concrete. Thus if it is cracked the initial stress will be the stress at which it cracked and the reduction should be applied to that stress.

RESTRAINT FROM EMBEDDED REINFORCEMENT

The easiest way to understand the restraining effect of reinforcement is to envisage a unit length taken from a member. Imagine firstly that the reinforcement is disconnected from the concrete, for example by being greased. When shrinkage takes place, the ends of the reinforcement will be left projecting from the face. The reinforcement is then pushed back into the concrete and at the same time the concrete is pulled out so that the surface is flush once again (Figure 3).

Symmetrically reinforced sections

If the reinforcement is symmetrical and the compressive strain in it is ε_s, the total force in the reinforcement is $\varepsilon_s E_s A_s$. The tensile strain in the concrete is $\varepsilon_{cs} - \varepsilon_s$ where ε_{cs} is the free (ie unrestrained) shrinkage. A straightforward method of estimating ε_{cs} taking relevant parameters into account is given in ACI 209R-92 (4) and is similar in principle to the European method in BS 5400-4. The tensile force in the concrete is $(\varepsilon_{cs} - \varepsilon_s) E_c A_c$. Equating these two forces gives the restrained shrinkage as

$$\varepsilon_{cr} = \varepsilon_s = \varepsilon_{cs} / (1 + m\,\rho_n) \tag{1}$$

where m is the modular ratio ($= E_s / E_c$) and ρ_n is the ratio of reinforcement area to concrete area.

Each increment of shrinkage occurs at a different age before being subsequently relieved by creep, so it is not immediately obvious how to evaluate m. A simple spreadsheet analysis by the author has shown that the 30-years effect is given quite accurately by imposing the total shrinkage in one step at an age of 150 days. For a typical structure indoors (rh = 45%, f_{cu} = 28/35 MPa [4000 psi], h_e = 250 mm [10 in]), this gives the creep coefficient ν = 1.6, whence E_{eff} = 11.5 GPa [1670 ksi] and m = 17.5.

Shrinkage deflection in reinforced members

If the section is reinforced asymmetrically – as is usual in a beam or slab – the restraint provided by the reinforcement is lopsided. The shortening described above for symmetrical sections takes place, but in addition the asymmetry produces curvature, which leads to deflection.

Once again, the strain ε_s in the reinforcement is used to define the force, and the stress in the concrete is calculated by the conventional formula $F/A + My/I$. The resulting strain at the level of the reinforcement is equated to $\varepsilon_{cs} - \varepsilon_s$, giving the uncracked curvature as:

$$\kappa_{un} = \varepsilon_{cs}\, m\, S_s / (I_g + m\,\rho_n\, I_c) \tag{2}$$

where S_s is the net first moment of area of the reinforcement about the neutral axis of the concrete section (ie deducting S of any compression reinforcement), I_c is the second moment of area of the concrete section, and I_g is the second moment of area of the gross section, ie including the reinforcement multiplied by (m-1).

This differs from the British code of practice BS 8110-2 (5) in that the second term in parentheses is excluded and the points about calculating ν at 150 days and deducting the S of compression reinforcement are not made. More importantly, both BS 8110-2 and the forthcoming Eurocode EN 1992-1-1 (6) state that the cracked section properties should be used if the section is cracked under load. This needs further consideration.

At a crack, the concrete below the neutral axis cannot transmit shrinkage strain, and it can be shown that the fully cracked curvature is

$$\kappa_{cr} = \varepsilon_{cs} / (d - c/2) \tag{3}$$

Compression reinforcement can be allowed for by reducing ε_{cs} by $1 + m\,\rho'$ where ρ' is $A_s' / b\,c$ and m is determined as above. The actual behaviour is presumably an alternation between cracked and uncracked zones as shown in Figure 4.

The method recommended in the US is equation 2.26 in ACI 435R-95 (7):

$$\delta_{sh} = k_{sh}\,(A_{sh}\,\varepsilon_{sh} / h)\,L^2 \tag{4}$$

where $A_{sh} = 0.7\,\rho^{1/3}$ for $\rho' = 0$ (modified for $\rho' > 0$). The curvature is the term in parentheses.

In order to compare the shrinkage curvatures obtained by different methods, they have all been reduced to the form

$$\kappa = k_\rho\,\varepsilon_{cs} / d \tag{5}$$

and plotted in Figure 5 against values of $\rho_d = 100\,A_s / b\,d$. Where a method did not lend itself to this formula directly, values were derived from the model of a 300 mm (12 in) thick solid slab, taking $d = 0.9\,h$ and $m = 17.5$. The tension reinforcement is assumed to be the amount required for ultimate load, ie stressed to 300 MPa [105 ksi] under service load.

It can be seen that the ACI method lies 25-30 % of the way between k_{un} and k_{cr} while the BS and EN cracked methods lie 75-80 % of the way between k_{un} and k_{cr}, although this would effectively reduce to 65-70 % when used with the incorrect deflection coefficient 0.104 (see below). Examination of figure 4 suggests that the ACI method is realistic, and that the UK and EN methods significantly over-estimate shrinkage deflection.

Compression reinforcement A_s' has been taken as zero, but similar graphs can be produced incorporating different values of A_s'. Compression reinforcement is very effective in reducing shrinkage curvature. By ACI 435R-95, providing 0.25 % additional compression reinforcement in a section where $A_{s,req} = 0.75$ % reduces the curvature by 29 % while adding it to the tension reinforcement increases it by 10 %.

The 'locked-in' tensile stress at the extreme fibre in the uncracked zone from restrained shrinkage is

$$f_{ct} = \kappa_{un}\,E_{eff}\,(h - c) \tag{6}$$

using the same E_{eff} as used to calculate κ. This stress can be significant after some time and appears to be overlooked in codes of practice when ascertaining whether the section is cracked under load.

Estimating shrinkage curvatures at intermediate ages is not easy. It is possible to set up a spreadsheet that divides the shrinkage into increments which are summed to give the total curvature at the point in time required. However, shrinkage curvature is only a small contribution to the total curvature and it will often be acceptable to make an educated guess.

Deflection is obtained from curvature by the relationship

$$\delta = K \, \kappa L^2 \tag{7}$$

The coefficient K for shrinkage is 0.125 for a span, 0.5 for a cantilever. If the curvature reverses over one or both supports, the coefficient is reduced by the multiplier $1 - \beta/10$ where β is the ratio $(\kappa_A + \kappa_B) / \kappa_C$ in which κ_A, κ_B and κ_C are the curvatures at left support, right support and mid-span respectively.

Confusion arises as both BS 8110-2 and EN 1992-1-1 (but not ACI) state that the shrinkage curvature should be added to the curvature from load. This means that the coefficient K for a simply supported span is taken as 0.104, not 0.125. For a cantilever the difference is even more marked, correctly 0.5 not 0.25. In this paper it is assumed that the various methods are all aiming to give the actual shrinkage curvature; incorporating this curvature into the load calculation will therefore reduce the shrinkage deflection by around 17 % in a span and by 50 % in a cantilever.

Shrinkage deflection calculated by the ACI method is typically $L/1250$ or less, supporting the approach in ACI-318 (8) of including it in the additional deflection from creep.

CONTRACTION OF INSITU CONCRETE OVERLAYS

Casting an insitu concrete overlay onto a precast concrete substrate or base unit is a common way of forming a homogeneous slab. However, the differential between the contraction of the overlay and the lesser or even zero contraction of the substrate produces internal stresses which lead to shortening and deflection of the composite unit, and sometimes to cracking in the overlay. The same theory can be applied when the base unit is prestressed or when it is a steel section (see below).

Consider an overlay cast onto a base section, and firstly allow the contraction of the overlay to take place freely, as if the interface is greased. Then apply a tensile force to the overlay with an equal and opposite compressive force on the base so that the relative movement at the interface is eliminated (Figure 5). The force acts at an eccentricity e to the centroid of the overlay to equalize the curvature in the overlay with that in the base.

By defining the strain in the overlay as ε_r, the force $F = \varepsilon_r A_a E_a$. Applying the equal and opposite force to the base produces a strain at the level of the centroid of the overlay

$$\varepsilon_n - \varepsilon_r = F\,[1/E_b A_b + z(z\text{-}e)/E_b I_b] \tag{8}$$

Eliminating ε_r gives

$$F = \varepsilon_n / [1/E_a A_a + 1/E_b A_b + z(z-e)/E_b I_b] = \varepsilon_n E_b I_b / [E_b I_b/E_a A_a + I_b/A_b + z(z-e)] \qquad (9)$$

In this, suffixes a and b refer to the overlay and the base respectively. A and I are derived from the concrete section including the reinforcement multiplied by the modular ratio m. Reinforced concrete is generally assumed cracked below the neutral axis, prestressed sections uncracked throughout. z is the lever arm between the centroid of the overlay and the centroid of the base. The duration is accounted for by using creep-modified values of E.

The eccentricity e can be found by equating the curvatures (= M/EI) of the overlay and base. This gives

$$F\, e / E_a\, I_a = F\,(z–e) / E_b\, I_b \qquad (10)$$

whence

$$e = z / (1 + E_b I_b/E_a I_a) \qquad (11)$$

$$z–e = z / (1 + E_a I_a/E_b I_b) \qquad (12)$$

ε_n is the net contraction = ε_a - ε_b, ie the difference in contraction between the overlay and the base in the time period in question (see below). In many cases ε_b can be taken as zero, including all concrete (including prestressed) over about one year old. Conversely, if the base is relatively young and the overlay thin (so that early thermal contraction is minimal), ε_a and ε_b will rapidly converge so that ε_n can be taken as zero, ie the differential contraction is not significant.

The expression for F can be clarified by writing $E_b I_b/E_a A_a = m_c\, I_b/A_a = q^2$ and $I_b/A_b = r^2$; q is a measure of the influence of the overlay on the base and r is the radius of gyration of the base. Then

$$F = \varepsilon_n\, E_b\, I_b / [q^2 + r^2 + z\,(z-e)] \qquad (13)$$

The force F is applied as an external force to the base section (not to the composite section), enabling any property to be calculated; most important will usually be the curvature from which the deflection is then calculated. The tensile stresses induced may also be significant.

The method can be used over any period of time. For instance, the effects of early age contraction are virtually immediate, so that short-term values would be used to give the immediate deflection. Long-term values can be used to give the long-term effect, although splitting the behaviour into steps with a spreadsheet will be more accurate as *F* will decline with time. The same spreadsheet approach may be necessary to find the point at which ε_b overtakes ε_a if the base is prestressed.

For thin overlays in which cracking would be visible and undesirable, it could be controlled with reinforcement. Where the substrate is less than about 4-6 months old and is expected to be above 15 C for the first month, it will probably be enough to provide ρ_{imm}. In other cases, it will be advisable to provide ρ_{mat} (ρ_{imm} and ρ_{mat} are defined below).

CONTRACTION OF INSITU SLABS IN COMPOSITE CONSTRUCTION

In composite construction, contraction of the insitu concrete slab can be significant. The theory for insitu overlays in the previous section applies, with the simplification that as the slab is usually thin relative to the steel beam the eccentricity *e* can be ignored. The average slab thickness should be used, not the minimum.

Some assumptions need to be clarified. The first is that because the metal decking is bonded continuously to the concrete, it can be treated in the same way as embedded reinforcement. And because the slab is attached to the steel beam which forces it to contract linearly it does not matter if the decking - or the reinforcement – is not concentric in the section. For typical slab profiles with steel decking 0.9-1.2 mm [0.035-0.047 in] thick and light fabric reinforcement, ρ_n ranges from 1.0 to 1.4 %, ie higher than is normal in conventional reinforced concrete slabs. Using the typical value of the modular ratio *m* of 17.5 shows that the effect of the restraint is a factor of 0.80-0.85.

The force *F* is applied as an external force to the steel beam (not to the composite section) enabling the curvature and thence the deflection to be calculated. The curvature κ is given by *M*/*EI*, so here

$$\kappa = F\,z \,/\, E_b\, I_b = \varepsilon_{cr}\, z \,/\, (q^2 + r^2 + z^2) \qquad (14)$$

The principal current UK guide is the code of practice for design of composite beams BS 5950-3-1 (9). However, shrinkage of the concrete slab is not mentioned as a contribution to deflection. BS 5950-3-1 will be supplanted by EN 1994-1-1 (10), currently available in draft. This states that “calculation of stresses and deformations at the serviceability limit state shall take into account the effects of [*inter alia*] creep and shrinkage of concrete”, and further that “the effect of curvature due to shrinkage of concrete should be included when the ratio of span to overall depth of the beam exceeds 20 and the predicted free shrinkage strain of the concrete exceeds 400 x 10^{-6}” (presumably not when below these limits).

The author has carried out calculations for typical arrangements which show that the shrinkage deflection is significant but that there is relatively little variation between widely different steel beams; all approach span/750. A simple rule would therefore be to assume shrinkage deflection is equal to span/750 unless it is estimated by a more accurate calculation – in spite of silence in BS 5950-3-1 and the let-off in EN 1994-1-1.

Cracking in composite concrete slabs

Tension is induced in the slab in three ways. First is the internal restraint of the reinforcement and the metal deck. Taking typical values of $\varepsilon_{cs} = 400$ με, $E_b = 200$ GPa (29 000 ksi) and $m = 17.5$ gives $f_{ct1} = 0.7$ MPa (100 psi) for $\rho = 1.0$ % and 0.9 MPa (130 psi) for ρ = 1.4 %.

Second is the restraint of the steel beam itself. This produces a tensile stress

$$f_{ct2} = \varepsilon_{cr} E_a / [1 + (r^2 + z^2) / q^2] \quad (15)$$

The term $(r^2 + z^2) / q^2$ varies from about 3 for a heavy beam to as much as 15 for a light one. Taking typical values of $\varepsilon_{cr} = 325$ με and $E_b = 11$ GPa (1670 ksi) gives $f_{ct2} = 0.25$-0.9 MPa (35-130 psi).

Third is the restraint of the surrounding structure. This will usually be at least 0.25 MPa (35 psi).

This suggests that the resulting tensile stress will be at least 1.2 MPa (175 psi) and could be over 2.0 MPa (290 psi). The tensile strength of concrete under sustained loading is probably in the range 1.5-2.1 MPa (220-300 psi) for the grades of concrete normally used in composite construction. It is clear that the possibility of the concrete cracking is very real and should be considered, particularly where composite slabs are to be left exposed to view. This could be controlled by providing a reinforcement content of ρ_{mat}. It might be thought that the metal deck would perform this function, at least at right angles to the corrugations, but evidence from sites suggests otherwise.

EXTERNAL RESTRAINT

This section uses a rational approach to give guidance on providing sufficient reinforcement to control cracking in direct tension. It is particularly relevant to walls and ground slabs in basements. These are also structures which often need to be water-resisting. It is therefore on the safe side to assume they will be restrained and determine the reinforcement on this basis.

Controlled cracks

If increasing axial tension is applied to a section it will eventually crack across its full depth and the force will be transferred to the reinforcement. If the tensile force (or the strain) continues to increase, provided the reinforcement bridging the crack is stronger than the uncracked section a new crack will form in a different place. And so on. This is controlled cracking. However, if the reinforcement is not strong enough it will yield, and the first crack will be uncontrolled and will just get wider.

Design criteria for controlling cracking

Before showing how this understanding can be used to determine the amount of reinforcement to be provided, it is important to consider the criteria to be applied to cracking in these circumstances. A crack which is not controlled has an unlimited width, and will therefore be unsightly at best and will leak if water is present. The target should therefore be that uncontrolled cracks are unlikely to occur. A reasonable interpretation of this would be to limit the probability of an uncontrolled crack occurring to 5 % or 1 in 20. This would require the upper characteristic axial tensile strength $f_{ctm,0.95}$ of concrete to be used.

In order to maintain the 5 % probability, $f_{ctm,0.95}$ should be used with the mean strength of the reinforcement. US deformed high yield steel currently has design strength 414 MPa (60 000 psi) and mean yield strength 490 MPa (71 000 psi). When EN 1992-1-1 comes into use, the design strength is expected to be 435 MPa (63 000 psi) and the mean to be at least 550 MPa (80 000 psi).

Minimum reinforcement in direct tension

The role of reinforcement in controlling cracking can be derived from first principles by making the strength of the reinforcement greater than the strength of the concrete in tension. Applying this principle, the reinforced axial capacity is:

$$F_s = A_s f_s \tag{16}$$

and the uncracked capacity is:

$$F_c = f_{ct} A_c + (m - 1) A_s \tag{17}$$

The second term in (17) is quite small and is neglected at this point, although a correction for it is applied later. Putting $F_s \geq F_c$ gives:

$$\rho \geq 1.0 \, (f_{ct} / f_s) \tag{18}$$

Two cases need to be considered, early age contractions in immature concrete and total long-term contraction in mature concrete.

Early age contractions

If the contractions are restrained (eg by pouring against an earlier pour), prediction of the contractions is so unreliable that it is safer to assume that cracking will occur and to provide sufficient reinforcement to control it.