Use three No. 7 bars (A_s = 1.8 in.^{2} > 1.66 in.^{2})

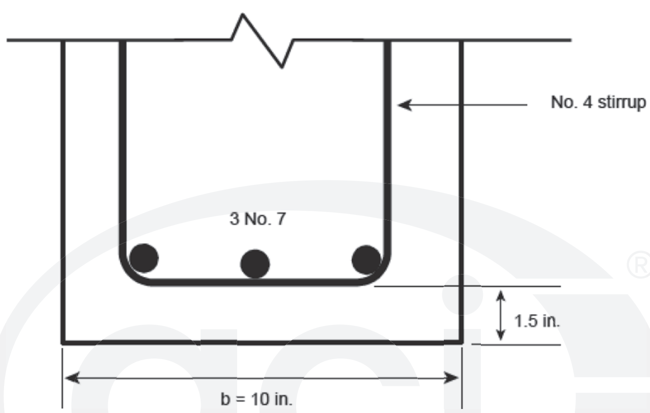
	Tension reinforcement	A_s, in.^{2}
Grade 60, PCA	Three No. 9	3.0 in.^{2}
Grade 100, CS	Three No. 7	1.8 in.^{2}

4. Check minimum required reinforcement.

For f_c' = 4000 psi, the governing requirement is

$$A_{s,min} = \frac{200}{f_y} b_w d = \frac{200}{100,000} \times 10 \times 19 = 0.38 \text{ in.}^2 < A_s \text{ provided} = 1.8 \text{ in.}^2 \quad \textbf{OK}$$

5. Check distribution of reinforcement.



Maximum spacing allowed by ACI 318

$$s = \frac{600}{f_s} - 2.5c_c \leq 12\left(\frac{40}{f_s}\right)$$

c_c = 1.5 + 0.5 = 2.0 in.

Use $f_s = 0.67f_y$ = 67 ksi

$$s = \frac{60}{f_s} - 2.5c_c = \frac{600}{67} - 2.5 \times 2.0 = 3.96 \text{ in. (governs)}$$

$$s = 12\left(\frac{40}{f_s}\right) = 12\left(\frac{40}{67}\right) = 7.16 \text{ in.}$$

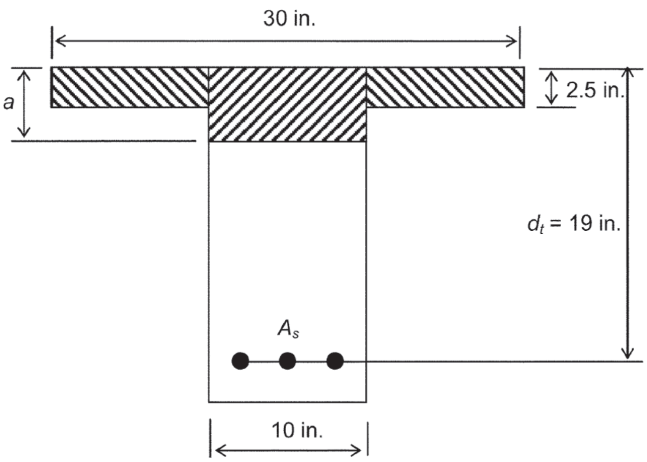
Spacing provided = (1/2)(10 – 2([1.5 + 0.5 + 7/16]) = 2.56 in. < 3.96 in. **OK**

The total area of flexural reinforcement provided by this design is 1.8 in.^{2} The original design in *PCA Notes*, which uses Grade 60 steel, uses 3.0 in.^{2} of reinforcing steel. This represents a 40 percent reduction of the required flexural reinforcement. Flanged sections provide a wide compression block compared with rectangular sections. The wide flange provides adequate compression force to balance the tension force provided by the CS without the need for additional compression reinforcement.

Example 4.6—Design of flanged section with tension reinforcement only

Similar to Example 7.5 in *PCA Notes*, this example illustrates the design procedure for flanged beams reinforced with CS with a neutral axis that lies in the beam web. Select reinforcement for the flanged section shown to carry a factored moment of M_u = 400 ft-kip.

Use f_c' = 4000 psi and f_y = 100,000 psi.



Calculations and discussion

(a) Determine required reinforcement.

1. Determine the depth of equivalent stress block, a, *as for a rectangular section.*

Assume tension-controlled section, $\phi = 0.9$

$M_u = \phi 0.85 f_c' a b_w (d - a/2)$

$400 \times 12 = 0.9 \times 0.85 \times 4 \times a \times 30 \times (19 - a/2)$

$45.9a^2 - 1744.2a + 4800 = 0$

a = 2.98 in. > 2.5 in.

Because the value of a as a rectangular section exceeds the flange thickness, the equivalent stress block extends into the web, and the design should be based on T-section behavior. (Refer to Example 4.5 when a is less than the flange depth.)

2. Compute required reinforcement A_{sf} *and nominal moment strength* M_{nf} *corresponding to the overhanging beam flange in compression.*

Compressive strength of flange

$C_f = 0.85 f_c'(b - b_w)h_f = 0.85 \times 4 \times (30 - 10) \times 2.5 = 170$ kip

Required A_{sf} to equilibrate C_f

$A_{sf} = C_f/f_y = 170/100 = 1.70$ in.^{2}

Nominal moment strength of flange

$$M_{nf} = A_{sf} f_y \left(d - \frac{h_f}{2} \right) = 1.70 \times 100 \times (19 - 1.25) = 3018 \text{ in.-kip} = 252 \text{ ft-kip}$$

3. Required nominal moment strength carried by beam web.

$M_{nw} = M_u/\phi - M_{nf} = 400/0.9 - 252 = 193$ ft-kip

4. Compute reinforcement A_{sw} *required to develop moment strength to be carried by the web.*

$M_{nw} = 0.85f_c'a_wb_w(d - a_w/2)$

$193 \times 12 = 0.85 \times 4 \times a_w \times 10 \times (19 - a_w/2)$

$17a_w^2 - 646a_w + 2316 = 0$

$a_w = 4.01$ in. $> h_f = 2.5$ in.

5. Check to see if section is tension-controlled.

$c_w = a_w/\beta_1 = 4.01/0.85 = 4.72$ in.

$$\varepsilon_s = \left(\frac{d - c_w}{c_w}\right)(0.003) = \left(\frac{19 - 4.72}{4.72}\right)(0.003) = 0.0091 > 0.009$$

Therefore, the section is tension-controlled and $\phi = 0.9$

(Refer to Section 4.4 of this guide.)

$$A_{sw} = \frac{0.85f_c'a_wb_w}{f_y} = \frac{0.85 \times 4 \times 4.01 \times 10}{100} = 1.36 \text{ in.}^2$$

6. Total reinforcement required to carry factored moment M_u *= 400 ft-kip.*

$A_s = A_{sf} + A_{sw} = 1.70 + 1.36 = 3.06$ in.^{2}

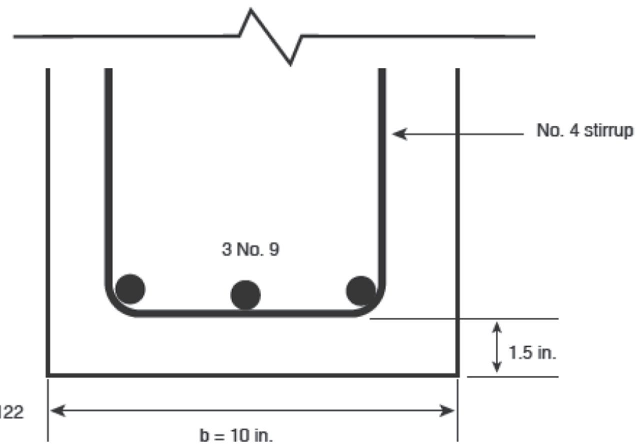
	A_{sf}	A_{sw}
Grade 60, PCA (5.10 in.^{2})	2.83 in.^{2}	2.27 in.^{2}
Grade 100, CS (3.06 in.^{2})	1.70 in.^{2}	1.36 in.^{2}

7. Check moment capacity.

$$\phi M_n = \phi\left(A_{sf}f_y\left[d - \frac{h_f}{2}\right] + A_{sw}f_y\left[d - \frac{a_w}{2}\right]\right) = 0.9\left(1.70 \times 100\left[19 - \frac{2.5}{2}\right] + 1.36 \times 100\left[19 - \frac{4.01}{2}\right]\right)$$

$= 4796$ in.-kip $= 400$ ft-kip **OK**

(b) Select reinforcement to satisfy crack control criteria.



122

Use three No. 9 bars (A_s = 3.0 in.^{2}, 2 percent less than required, assumed sufficient)

Maximum spacing allowed by ACI 318

c_c = 1.5 + 0.5 = 2.0 in.

Use f_s = 0.67, f_y = 67 ksi

$$s = \frac{600}{f_s} - 2.5c_c = \frac{600}{67} - 2.5 \times 2.0 = 3.96 \text{ in. (governs)}$$

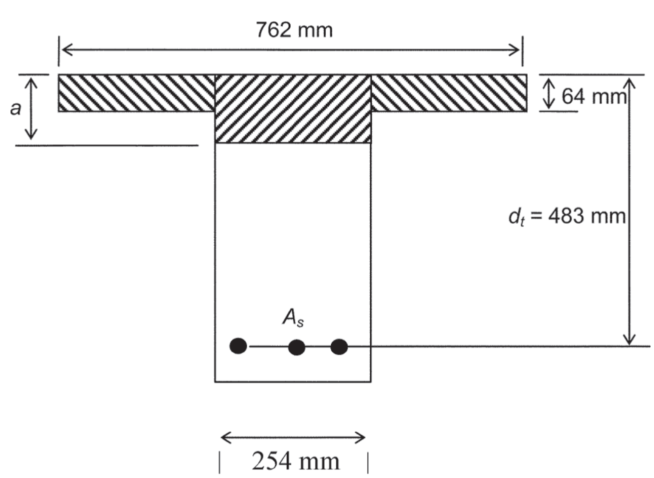
$$s = 12\left(\frac{40}{f_s}\right) = 12\left(\frac{40}{67}\right) = 7.16 \text{ in.}$$

Spacing provided = (1/2)(10 – 2[1.5 + 0.5 + 9/16]) = 2.44 in. < 3.96 in. **OK**

The total area of flexural reinforcement provided in this design is 3.0 in.^{2} The original design in *PCA Notes*, which uses Grade 60 steel, uses 5.0 in.^{2} of reinforcing steel. This represents a 40 percent reduction of the required flexural reinforcement. For members exhibiting T-beam behavior at nominal strength, reduce the required flexural reinforcement by using CS with a suitable design approach.

Example 4.6 (SI)—Design of flanged section with tension reinforcement only (SI units)

This example is identical to Example 4.6 but presented in SI units. Select reinforcement for the flanged section shown, to carry a factored moment of M_u = 542.4 kN-m. Use f_c' = 27.6 MPa, and f_y = 690 MPa.



Calculations and discussion

(a) Determine required reinforcement.

1. Determine the depth of equivalent stress block, a, *as for a rectangular section.*

Assume tension-controlled section, $\phi = 0.9$

$$M_u = \phi 0.85 f_c' a b_w (d - a/2)$$

$$542.4 \times 1000 = 0.9 \times 0.85 \times 27.6 \times a \times 0.762 \times (483 - a/2)$$

$$8.04a^2 - 7770a + 542{,}400 = 0$$

$$a = 75.8 \text{ mm} > 64 \text{ mm}$$

Because the value of a as a rectangular section exceeds the flange thickness, the equivalent stress block extends into the web, and the design should be based on T-section behavior. (Refer to Example 4.5 when a is less than the flange depth.)

2. Compute required reinforcement A_{sf} *and nominal moment strength* M_{nf} *corresponding to the overhanging beam flange in compression.*

Compressive strength of flange

$$C_f = 0.85 f_c'(b - b_w)h_f = 0.85 \times 27.6 \times (762 - 254) \times 64 = 762{,}730 \text{ N}$$

Required A_{sf} to equilibrate C_f

$$A_{sf} = C_f/f_y = 762{,}730/690 = 1105 \text{ mm}^2$$

Nominal moment strength of flange

$$M_{nf} = A_{sf} f_y \left(d_t - \frac{h_f}{2}\right) = 1105 \times 690 \times (483 - 32) = 343{,}865{,}000 \text{ N-mm} = 343.9 \text{ kN-m}$$

3. Required nominal moment strength carried by beam web.

$$M_{nw} = M_u/\phi - M_{nf} = 542.4/0.9 - 343.9 = 258.8 \text{ kN-m}$$

4. Compute reinforcement A_{sw} *required to develop moment strength to be carried by the web.*

$$M_{nw} = 0.85 f_c' a_w b_w (d - a_w/2)$$

$$258.8 \times 1000 = 0.85 \times 27.6 \times a_w \times 0.254 \times (483 - a_w/2)$$

$$2.98a_w^2 - 966a_w + 86{,}846 = 0$$

$$a_w = 100 \text{ mm} > h_f = 64 \text{ mm}$$

5. Check to see if section is tension-controlled.

$$c_w = a_w/\beta_1 = 100/0.85 = 118 \text{ mm}$$

$$\varepsilon_s = \left(\frac{d - c_w}{c_w}\right)(0.003) = \left(\frac{483 - 118}{118}\right)(0.003) = 0.0093 > 0.009$$

Therefore, the section is tension-controlled and $\phi = 0.9$

(Refer to Section 4.4 of this guide.)

$$A_{sw} = \frac{0.85 f_c' a_w b_w}{f_y} = \frac{0.85 \times 27.6 \times 100 \times 254}{690} = 864 \text{ mm}^2$$

6. Total reinforcement required to carry factored moment $M_u = 542.4$ *kN-m.*

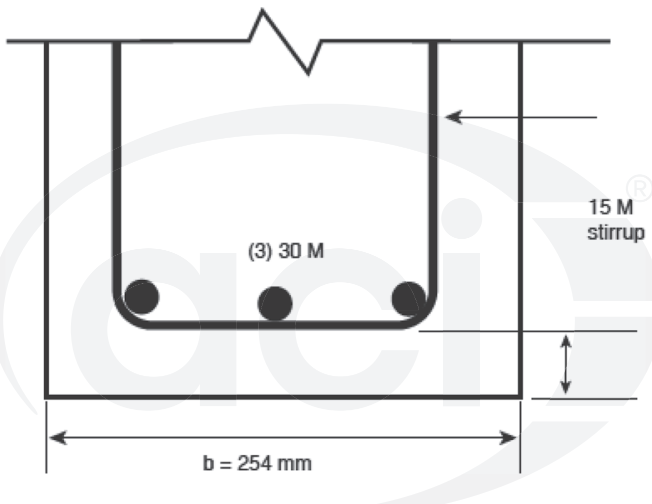
$A_s = A_{sf} + A_{sw} = 1105 + 864 = 1969 \text{ mm}^2$

7. Check moment capacity.

$$\phi M_n = \phi\left(A_{sf} f_y \left[d - \frac{h_f}{2}\right] + A_{sw} f_y \left[d - \frac{a_w}{2}\right]\right) = 0.9\left(1105 \times 690\left[483 - \frac{64}{2}\right] + 864 \times 690\left[483 - \frac{100}{2}\right]\right)$$

$$= 541{,}802{,}000 \text{ N·mm} = 542 \text{ kN·m} \quad \textbf{OK}$$

(b) Select reinforcement to satisfy crack control criteria.



Use three 30M bars ($A_s = 2100 \text{ mm}^2 > 1969 \text{ mm}^2$ required).

Maximum spacing allowed by ACI 318M

$$s = 380\left(\frac{280}{f_s}\right) - 2.5c_c \leq 300\left(\frac{280}{f_s}\right)$$

$c_c = 38 + 16 = 54$ mm

Use $f_s = 0.67 f_y = 462$ MPa

$$s = \frac{106{,}400}{f_s} - 2.5c_c = \frac{106{,}400}{462} - 2.5 \times 54 = 95 \text{ mm (governs)}$$

$$s = 300\left(\frac{280}{f_s}\right) = 300\left(\frac{280}{462}\right) = 182 \text{ mm}$$

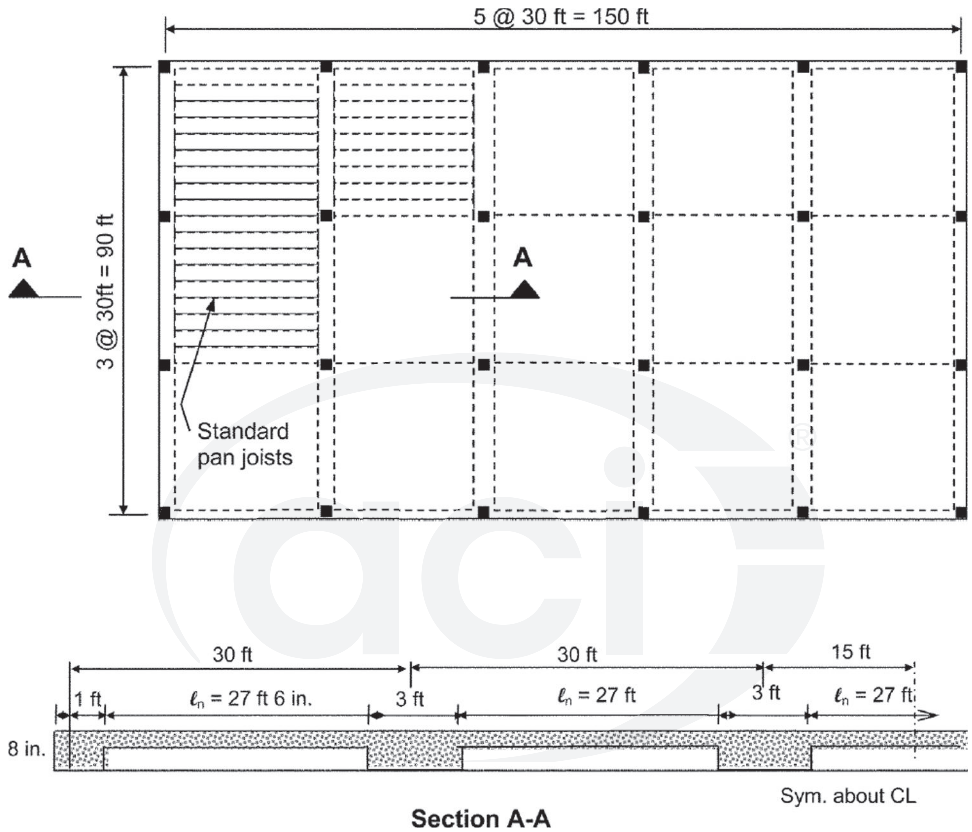
Spacing provided = (1/2)(254 – 2[38 + 16 + 29.9/2]) = 58 mm < 95 mm **OK**

The total area of flexural reinforcement provided in this design is 1969 mm^2. The original design in the *PCA Notes*, which uses Grade 420 steel, uses 3226 mm^2 of reinforcing steel. This represents a 39 percent reduction of the required flexural reinforcement. For members exhibiting T-beam behavior at nominal strength, reduce the required flexural reinforcement by using CS with a suitable design approach.

Example 6.2—Design of one-way joist

Similar to Example 7.6 in *PCA Notes*, this example illustrates the design procedure for continuous one-way joist systems reinforced with CS. Determine the required depth and reinforcement for the one-way joist system shown in the following. The joists are 6 in. wide and are spaced 36 in. on center. The slab is 3.5 in. thick. Use f_c' = 4000 psi and f_y = 100,000 psi.

Service DL = 130 lb/ft^2 (assumed total for joists and beams plus superimposed dead loads)
Service LL = 60 lb/ft^2
Width of spandrel beam = 20 in., width of interior beams = 36 in.
Columns: interior = 18 x 18 in., exterior = 16 x 16 in.
Story height (typ.) = 13 ft



Calculations and discussion

(a) Compute the factored moments at the faces of the supports and determine the depth of the joists.

w_u = [(1.2 × 0.13) + (1.6 × 0.06)] × 3 = 0.756 ft-kip

Using the approximate coefficients, the following table summarizes the factored moments along the span.

Location	M_u, ft-kip
End span	
Exterior negative	$w_u\ell_n^2/24 = 0.756 \times 27.5^2/24 = 23.8$
Positive	$w_u\ell_n^2/14 = 0.756 \times 27.5^2/14 = 40.8$
Interior negative	$w_u\ell_n^2/10 = 0.756 \times 27.25^2/10 = 56.1$
Interior span	
Positive	$w_u\ell_n^2/16 = 0.756 \times 27^2/16 = 34.4$
Negative	$w_u\ell_n^2/11 = 0.756 \times 27^2/11 = 50.1$

For reasonable deflection control, choose a reinforcement ratio ρ equal to approximately one-half ρ_t.

$\rho_t = 0.002125\beta_1 f_c' = 0.002125 \times 0.85 \times 4 = 0.00723$

(Refer to Table 4.3.4 of this guide.)

Set $\rho = 0.5 \times 0.00723 = 0.00362$

Determine the required depth of the joist based on M_u = 56.1 ft-kip

$$\omega = \frac{\rho f_y}{f_c'} = \frac{0.00362 \times 100}{4} = 0.0905$$

From Table 7-1 in the *PCA Notes*, $M_u/\phi f_c' bd^2 = 0.0855$

$$d = \sqrt{\frac{M_u}{\phi f_c' b_w (0.0855)}} = \sqrt{\frac{56.1 \times 12}{0.9 \times 4 \times 6 \times 0.0855}} = 19.1 \text{ in.}$$

Allowing 1.25 in. for concrete cover and half bar diameter, then $h \approx 19.1 + 1.25 = 20.4$ in.

To satisfy the requirements for joist construction in ACI 318, $h_{max} = 3.5 \times b_w = 3.5 \times 6 = 21$ in.

These calculations indicate a 21 in. joist depth is adequate. ACI 318, however, indicates a minimum thickness of $(\ell/18.5) \times (0.4 + f_y/100{,}000)$ = 27 in., unless deflections are computed. This is applicable only to "members in one-way construction not supporting or attached to partitions or other construction likely to be damaged by large deflections." Otherwise, deflections must be computed.

For purposes of illustration, compute the required reinforcement for a 21 in. deep joist without considering deflection further. Assume d = 21 in. – 1.25 in. = 19.75 in.

(b) Compute required reinforcement.

1. End span, exterior negative.

$$\frac{M_u}{\phi f_c' bd^2} = \frac{23.8 \times 12}{0.9 \times 4 \times 6 \times 19.75^2} = 0.0339$$

From Table 7-1 in *PCA Notes,* $\omega \approx 0.0346$

$$A_s = \frac{\omega bd\, f_c'}{f_y} = \frac{0.0346 \times 6 \times 19.75 \times 4}{100} = 0.16 \text{ in.}^2$$

For f_c' = 4000 psi, use

$$A_{s,min} = \frac{200 b_w d}{f_y} = \frac{200 \times 6 \times 19.75}{100{,}000} = 0.24 \text{ in.}^2 > 0.16 \text{ in.}^2$$

Use $A_s = 0.24$ in.^{2}

Distribute bars uniformly in top of slab

$A_s = 0.24/3 = 0.08$ in.^{2}/ft

Maximum spacing allowed by ACI 318

$$s = \frac{600}{f_s} - 2.5c_c \le 12\left(\frac{40}{f_s}\right)$$

Assuming No. 3 bars

$c_c = h - d - 3/16 = 21 - 19.75 - 3/16 = 1.1$ in.

Use $f_s = 0.67f_y = 67$ ksi

$$s = \frac{600}{f_s} - 2.5c_c = \frac{600}{67} - 2.5 \times 1.1 = 6.21 \text{ in. (governs)}$$

$$s = 12\left(\frac{40}{f_s}\right) = 12\left(\frac{40}{67}\right) = 7.16 \text{ in.}$$

Use No. 3 at 6 in. ($A_s = 0.22$ in.^{2}/ft). $A_s = 6 \times 0.11$ in.^{2} = 0.66 in.^{2} in 36 in. for each joist.

(*PCA Notes*: use No. 3 at 10 in.^{2})

Check if the joist is tension-controlled

$$\rho = \frac{A_s}{b_w d} = \frac{0.66}{6 \times 19.75} = 0.0056 < \rho_t = 0.00723 \quad \textbf{OK}$$

2. *End span, positive.*

$$\frac{M_u}{\phi f_c' bd^2} = \frac{40.8 \times 12}{0.9 \times 4 \times 36 \times 19.75^2} = 0.0097$$

From Table 7-1 in *PCA Notes*, $\omega \approx 0.01$

$$A_s = \frac{\omega bd f_c'}{f_y} = \frac{0.01 \times 36 \times 19.75 \times 4}{100} = 0.28 \text{ in.}^2$$

Check rectangular section behavior

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{0.28 \times 100}{0.85 \times 4 \times 36} = 0.22 < 3.5 \text{ in.} \quad \textbf{OK}$$

Use one No. 5 bar ($A_s = 0.31$ in.^{2}) (*PCA Notes*: use two No. 5 bars)

3. *End span, interior negative.*

$$\frac{M_u}{f_c' bd^2} = \frac{56.1 \times 12}{0.9 \times 4 \times 6 \times 19.75^2} = 0.0799$$

From Table 7-1 in *PCA Notes*, $\omega \approx 0.084$

$$A_s = \frac{\omega bd\ f_c'}{f_y} = \frac{0.084 \times 6 \times 19.75 \times 4}{100} = 0.40 \text{ in.}^2$$

Distribute bars uniformly in top of slab

$A_s = 0.40/3 = 0.13$ in.^{2}/ft