

SHEAR IN TWO-WAY SLABS--ACI APPROACH

By W. Gene Corley

Synopsis: This paper describes design provisions for shear in two-way slabs. Specifically, requirements of Chapter 11 of the 1971 ACI Building Code(1) are discussed.

To assist in understanding the design procedure, the history of these provisions and test data that form the basis for some of the requirements are described.

Keywords: building codes; concrete slabs; flat concrete plates; loads (forces); reinforced concrete; two-way slabs; shear properties; shear strength; structural design.

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## BACKGROUND

In the 1950's and early 1960's, several important research programs concerning the flexural strength of slab structures were carried out. At the University of Illinois, several 1/4-scale models of flat slab and flat plate structures were loaded to destruction(2-9). At the PCA Laboratories, the nearly full-size flat plate shown in Fig. 1 was tested to destruction(10). In Australia(11,12) and in Europe(13), several large slabs were also tested.

Test results indicated that nearly complete redistribution of moments can be obtained in flat plates. This, in combination with membrane stresses, provides a considerable reserve capacity in flexure. These findings show that the capacity of a flat plate structure is governed by shear, rather than flexure.

The basis for current provisions of the ACI Building Code for shear strength of two-way slabs can be traced to work done by Richart at the University of Illinois(14). This extensive report of tests on reinforced concrete footings showed that shear failures occurred at relatively low nominal shear stresses.

At about the time that the flexural tests were being completed, the Portland Cement Association carried out a series of tests to determine shear strength of flat plate structures(15).

Results of these tests along with others published in the literature led to development of the 1971 ACI Building Code provisions for shear in slabs and footings(1).

## CONCENTRIC LOADING

In laboratory tests, it has been found that, as shown schematically in Fig. 2, severe overload of a flat plate can lead to development of a failure cone in the slab in the vicinity of the column. Considering the suddenness of this type of failure, it must be avoided by proper design.

The 1963 ACI Building Code(16) recommended that shear stress in the slab in the vicinity of the column be limited to  $4\sqrt{f'_c}$  on a design section at a distance  $d/2$  from the column face. The term,  $d$ , is the effective depth of the slab. This design procedure follows the 1962 recommendations of ACI Committee 326(17).

In the 1971 ACI Building Code(1), methods of providing adequate shear strength are similar to those used earlier, with a few additions. The designer has the following four design procedures he can use to provide the necessary shear strength:

1. Use a thicker slab
2. Use a larger column
3. Use higher strength concrete
4. Use shear reinforcement consisting of bars, rods, wires, or, use shearhead reinforcement consisting of structural steel shapes

When increases in slab thickness, column size, or concrete strength are used to obtain the necessary design strength, Eq. 11-25 of the 1971 ACI Building Code(1) is applicable.

$$v_u = \frac{V_u}{\phi b_o d}$$

Eq. 11-25 can be rearranged and put into terms of the variables  $\sqrt{f'_c}$ , column dimensions  $c_1$  and  $c_2$ , and the slab effective depth,  $d$ .

$$V_u \propto \sqrt{f'_c}, c_1 \text{ \& } c_2, \text{ and } d^2$$

The shear strength for design,  $V_u$ , is proportional to  $\sqrt{f'_c}$ . In practice, concrete strengths used in slabs are often in the range of 3,000 psi (20.7 MPa) to 4,000 psi (27.6 MPa). A change of concrete strength in this range would increase design shear capacity by only about 15%. Consequently, the designer often must resort to other means of providing necessary strength.

Design shear capacity is linearly related to column dimensions. Consequently increased column dimensions significantly increase slab shear capacity.

Shear strength used in design is related to the square of the depth of the slab. Therefore, design strength increases significantly when slab thickness is increased. It should be noted however, that increasing slab thickness also increases dead load. For this reason, where dead load makes up a substantial portion of the total, net gains in strength obtained by increasing slab thickness are approximately linearly related to slab thickness.

### Bars, Rods and Wires

Limited use of shear reinforcement consisting of bars, rods and wires was permitted by the 1963 ACI Code. However, use of this reinforcement was restricted to slabs 10 in. (250 mm) or more in thickness, its efficiency was rated at 50%, and no more than a 50% increase in slab shear strength could be considered in design.

Review of all data on tests of slabs containing shear reinforcement consisting of wires, rods, or bars, similar to those shown in Fig. 3, indicated mixed results. It appeared that when proper anchorage was provided, the shear reinforcement definitely increased strength. However, where anchorage was not adequate, as for example in the "arrowhead" or "lampshade" type of shear reinforcement shown on the left in Fig. 3, very little increase in shear strength was found(15).

As part of a series of tests on very large slab column specimens, both shearhead reinforcement and shear reinforcement consisting of deformed bars were evaluated at the Portland Cement Association(18). Figure 4 shows loads being applied to one of the large specimens tested. Each slab was about 7½-in. (190 mm) thick and was supported by an 18-in. (457 mm) square column. Loads were applied so that conditions near the supports of slabs about 20-ft (6 m) square were represented.

One of the specimens contained shear reinforcement consisting of deformed bars as shown in Fig. 5. This reinforcement met all requirements for stirrups including those for anchorage. The shear reinforcement was effective. In fact, it increased the capacity so that the final mode of failure was flexure rather than shear. Close inspection after completion of the test showed that there were no signs of bond failure or of impending shear failure.

As a result of the review of past tests and of evaluating the results of this test, the 1971 ACI Building Code was revised to permit the use of bars, rods and wires in slabs of any thickness(1). This reinforcement may be used to increase the shear strength by as much as 50% if the anchorage requirements for stirrups are met. When bars, rods or wires are used, the Code requires that all shear be carried by the shear reinforcement when the nominal stress  $v_c$  exceeds  $2\sqrt{f'_c}$ :

### Shearheads

Shearhead reinforcement consisting of structural shapes has been used extensively by its originator W. H. Wheeler since the mid 1930's. To gain acceptance by building officials prior to 1971, several structures containing the Wheeler shearhead, shown in Fig. 6, were load tested in the field.

In the design of lift slabs, collars are used to connect the slab to the column. These collars serve the same function as a shearhead. Its similarity to the Wheeler type of shearhead reinforcement can be seen in Fig. 7.

In the mid 1960's, tests to develop a design procedure for shearheads were carried out at the Portland Cement Association(19).

Two types of shearheads were used as shown in Fig. 8. One was made with two pairs of crossed channels. The other type was fabricated from single crossed beams. Single arm shearheads are considerably less expensive to fabricate than are those made with parallel arms.

From the test data, the design procedure currently in the ACI Code was formulated. This procedure requires the following:

1. Provide a minimum flexural capacity for the shearhead reinforcement to assure that the shear capacity is reached before its moment capacity is exceeded.
2. Limit the nominal stress in the slab at the end of the shearhead reinforcement.

After these two requirements are satisfied, the designer can:

3. Determine the contribution of the shearhead reinforcement to the moment capacity of the slab. This will permit a slight reduction in the negative column strip reinforcement.

## UNBALANCED LOADING

The transfer of unbalanced moments from slabs to columns has long been a concern of the designer. Prior to 1971, the ACI Building Code did not give specific recommendations concerning moment transfer.

A thorough evaluation of design procedures and test data concerning transfer of moment from slabs to columns was reported by N.W. Hanson and J.M. Hanson(20). With the help of Mr. M. P. Van Buren, the authors developed an expression that defines the moment transfer that must be considered in design.

Figure 9 shows a slab column junction in the vicinity of an interior square column supporting a structure. As described earlier, the 1971 ACI Building Code limits the design strength of this junction to  $4\sqrt{f'_c}$  calculated on a design section located at a distance  $d/2$  from the face of the column.

When the slab column junction is subjected to loadings that produce an unbalanced moment, as shown in Fig. 9, the design strength is limited to a maximum shear stress of  $4\sqrt{f'_c}$  on one face. The total stress is made up of that portion due to vertical loads only, plus a portion added to one face and attributed to the moment. The shear attributed to unbalanced moment is calculated by Section 11.13.2 using the total moment transferred(1). The portion of this moment causing shear stresses ranges from 0.4 to 1.0.

## ADDITIONS TO 1977 CODE

For changes that will be effective in January 1977, ACI Committee 318 has changed the ultimate shear stress considered in design for rectangular columns. As the result of recent work done by Professor Neil Hawkins of the University of Washington, nominal shear stress on rectangular columns is as shown in Fig. 10. For columns with an aspect ratio of 2 or less,  $v_c$  is taken as  $4\sqrt{f'_c}$ .

For columns with an aspect ratio greater than 2,  $v_c$  is reduced by the equation shown, where the term  $\beta$  is the ratio of the long side of the column to the short side. For walls,  $\beta$  becomes very large and this equation reduces to  $2\sqrt{f'_c}$ , the current shear stress permitted on slabs supported by walls.

## SUMMARY

This paper describes current ACI Building Code shear design provisions for flat slab and flat plate structures.

At interior columns where unbalanced moments are small, a limiting shear stress of  $4\sqrt{f'_c}$  on the design section is specified. Shear strength depends on slab thickness, column size, concrete strength, and bar or shear-head reinforcement. The ACI Code specifies methods for determining the amount of moment that is considered to contribute to shear stress when large moments are transferred. Maximum shear stress on one face of a column is limited to  $4\sqrt{f'_c}$ .

In designs following provisions to be incorporated in future versions of the Code, reductions in shear stress will be required for slabs supported on rectangular columns.

As indicated in this paper, the ACI design procedure is simple in concept and easy to apply. In more than a dozen years of use, the design procedure has been proven to give safe, economical, and satisfactory structures.

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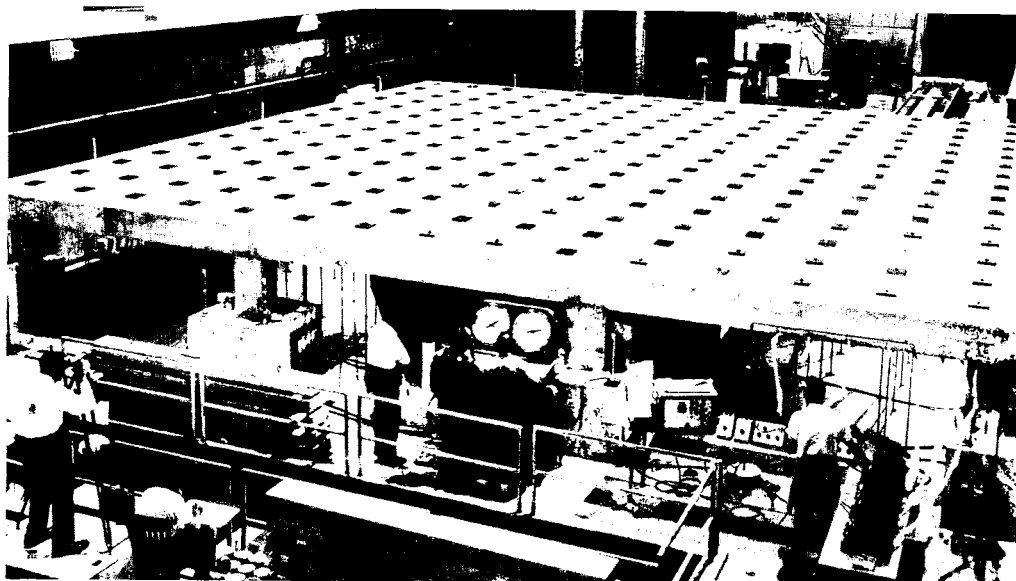


Fig. 1--Test of flat plate structure

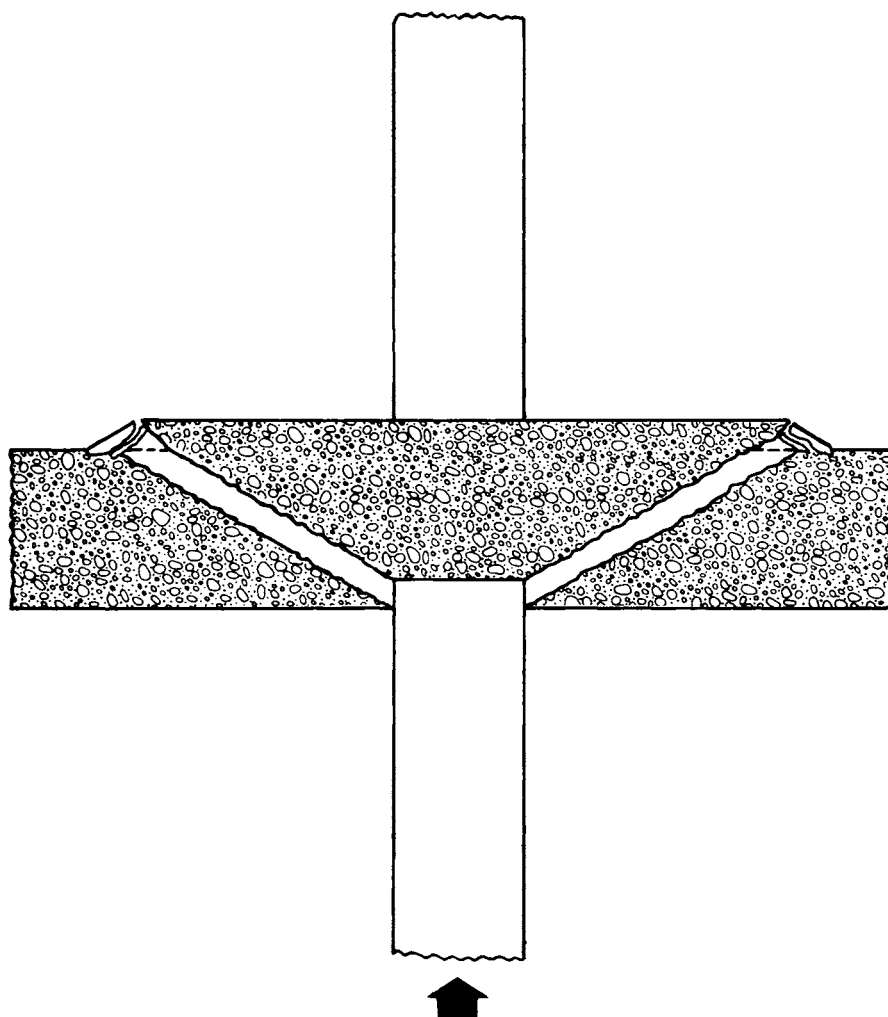


Fig. 2--Failure cone for flat plate