PAPER NO. 1 Elements of a theory for the mechanical behavior of concrete are developed. A comparison of the stress-strain curves of sandstone, hardened cement paste, mortar and concrete suggests that the heterogeneity introduced by the aggregates is responsible for the inelastic behavior of concrete. A study of the mortar-aggregate interface behavior indicates that under compressive loading this interface is first to fail and that the cracked interface behaves essentially plastically even though the constituent materials are brittle. This pseudo-plastic behavior of the interface results in a gradually curving stress-strain diagram. An analytic model, based only on observed micro-behavior, measured materials properties, and a plausible statistical distribution of mortar strengths, permits a stress-strain curve to be calculated. This curve is strikingly similar to that of concrete, including a descending branch.

Inelastic Behavior and Fracture of Concrete*

By SURENDRA P. SHAH and GEORGE WINTER

■ THIS STUDY PROVIDES INFORMATION on the nature of inelastic behavior of concrete by examining its micro-behavior under short-time loading. Based on this micro-behavior and on statistical concepts, the study attempts to develop elements of a unified theory for the response of concrete from zero load through the elastic, inelastic and descending ranges of the stress-strain curve to fracture.

ORIGIN OF INELASTIC DEFORMATIONS

Inelastic behavior of concrete is here defined as that behavior in which response of concrete to external loading is non-linear and irre-

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versible. Only the short-time loading in which the deformations due to creep are small compared to the instantaneous deformations caused by loading is considered.

To understand the response of concrete when it is loaded, it is desirable to study the behavior of each of its components. In Figure 1-1 are shown typical stress-strain relations of hardened cement paste, stone, mortar and concrete subjected to uniform compression up to failure of each material. Identical procedures were used for mixing and curing the paste, mortar and concrete specimens, and the methods of testing were the same for all four materials. The curves for paste, mortar and concrete, each represent the average of 8 dial gage readings, two on every specimen for four specimens each. These were prismatic specimens with flared ends and the strains were measured on a 5½ in. length. The curve for stone is the average of 6 one-in. SR-4 gages, two on each of three prismatic specimens. The rate of loading was approximately the same for all four materials. Figure 1-1(a) shows:

1. Considering the two basic constituents of concrete, it is seen that the stress strain curve for stone is linear up to fracture, and that for hardened cement paste is nearly linear.

2. For mortar, the stress-strain relation is definitely curved, much more so than those of the two constituents.

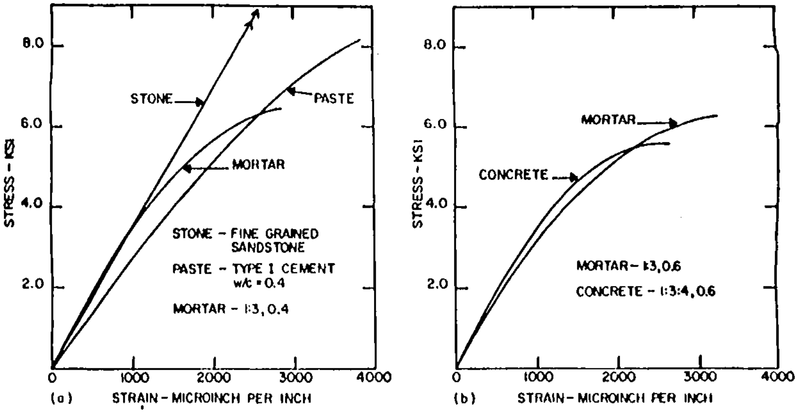

Figure 1-1 Compression stress-strain diagrams for stone, paste, mortar and concrete

3. For concrete, in turn, the diagram is more strongly curved than that for mortar.

4. The compressive strength of mortar is significantly less than that of paste.

5. The compressive strength of concrete in turn is less than that of mortar.

Note that paste and mortar of the same w/c-ratio are compared and, in turn, mortar and concrete of the same w/c-ratio are used.

These observations suggest that the inelasticity in concrete is caused by the introduction of aggregates and by the attendant heterogeneity.

The fact that the stress-strain curve of paste is almost linear has also been observed by Ruetz[1] and Gilkey.[2] Ruetz found that the stress-strain curves for the paste of water-cement ratios of 0.30 and 0.65 are very nearly straight lines. Gilkey plotted the stress-strain curves of concrete, mortar and paste in non-dimensional form and observed that the curve for the paste is nearly linear while those for mortar and concrete are successively more curvilinear.

The relation between inelasticity and the presence of aggregates in concrete is also indicated by Kaplan's observation that the higher the percentage of aggregates (with the same w/c ratio), the lower the strain at which the stress-strain curve deviates from a straight line.[3]

Since mortar is weaker than either stone or cement paste alone, and since concrete is weaker than mortar, the bond between cement paste and aggregate is likely to be a weak link in the heterogeneous concrete system. Consequently, one might suspect that the presence of aggregates produces inelasticity in concrete as a result of relative weakness of the interfaces between paste or mortar and aggregates. This idea seems to be supported by the microcrack investigations at Cornell University[4] and later at the Cement and Concrete Association, *London*,[5] by the work of Jones,[6] and Dantu.[7]

The microcrack investigation at Cornell showed that cracks between the aggregate and mortar exist before any load is applied (see also Ref. 5). These interfacial cracks, which are called bond cracks, began to increase noticeably at about 30 to 50 percent of ultimate load and were predominant throughout the entire loading. By studying the ultrasonic-pulse velocity, Jones found that "the onset of cracking arises mainly from spread of microcracks along the boundaries of the coarse aggregate. . . ." Dantu used photo-elastic coatings on the surface of the compression specimens and found that the strain at the mortar-aggregate interface was 4 to 6 times the average strain. All this evidence strongly suggests that the bond between mortar and aggregate is a weak link in the heterogeneous concrete system, and that this interfacial bond is mainly responsible for the inelastic behavior of concrete under short-time loading.

STRESS-STRAIN BEHAVIOR OF THE INTERFACE

Since the behavior of concrete under an external short-time load seems to be significantly influenced by the interfacial bond, the authors decided to study the stress-strain behavior of the interface between mortar and aggregate. In actual concrete, there are innumerable interfaces. To study what happens at the interface by directly loading concrete would be very difficult, but the interface can be isolated and studied by itself.

Experimental Investigation

The behavior of the interface was examined by testing specimens as shown in Figure 1-2. They consist of an inclined stone slab, ½ in. thick, about 3½ in. long and 2 in. wide, and the mortar surrounding the stone. The stone was fine grained sandstone with a compressive strength of 27,000 psi. The mortar had a sand/cement ratio of 3.0 and a water/cement ratio of 0.5. The cement was portland, Type III.

Three different inclinations of the stone, ϕ, were selected, 23, 42 and 50 deg. There was a minimum of three specimens for each angle.

Structurally speaking, the specimen can be divided into zones. The center zone contains the stone slab and represents the influence of the interface. The exterior zones of homogeneous mortar simulate the effect of confinement to which an interface is subjected in the concrete mass. The specimens were loaded to failure and the strains and loads were

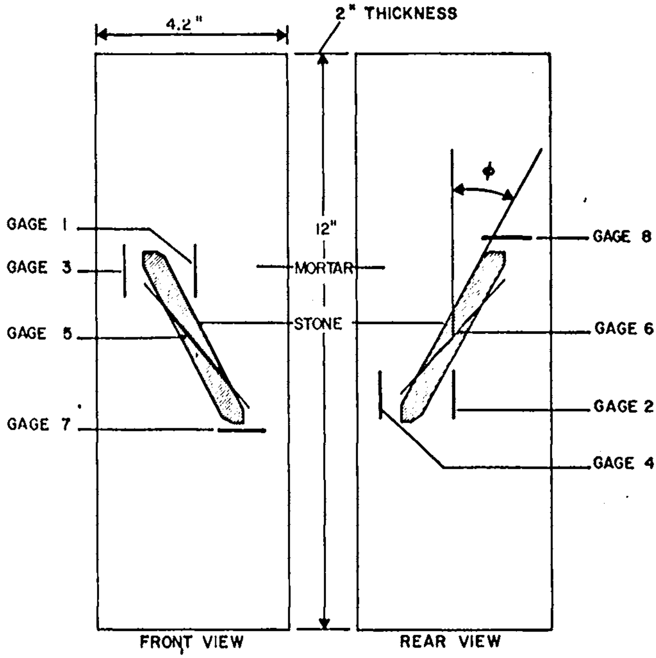

Figure 1-2 The specimen for the interfacial study

recorded at regular intervals. The testing time for each test was about one-half hour. The arrangement of the SR-4 strain gages is shown in Figure 1-2.

RESULTS AND DISCUSSION

The load-strain curves up to failure obtained from the various gages for one each of the specimens with the stone inclined at 23, 42, and 50 deg are shown in Figures 1-3, 1-4, and 1-5 respectively. For the sake of brevity, only the results of the specimens with the stone inclined at 23 deg will be discussed in detail. The following observations are made from Figure 1-3.

1. Up to a load of 12 kips, the strains in gage 1, representing the center zone, and gage 3, representing mortar in the exterior zones, are nearly alike and linear. This means that up to 12 kips, the interface was intact and as a result the specimen behaved homogeneously and elastically.

2. For loads beyond 12 kips, and up to failure, the center strain gage 1 did not show any appreciable increase in strain at all. This

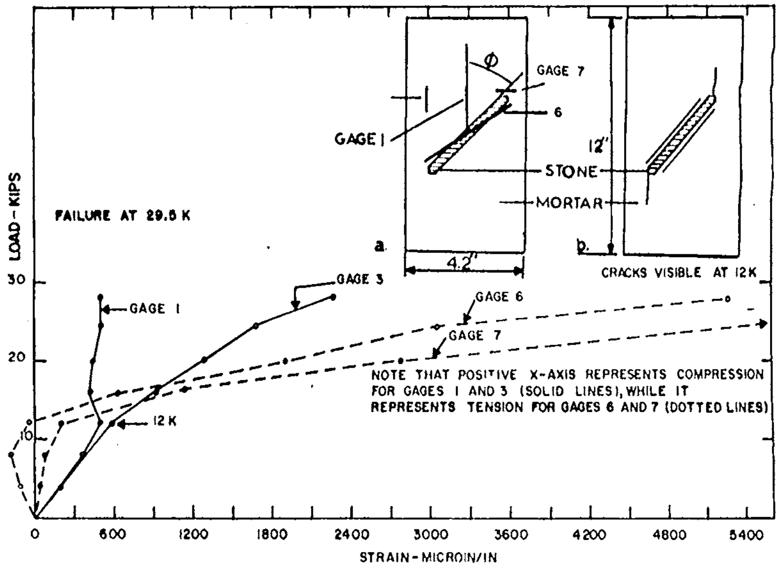

Figure 1-3 Behavior of restrained interface. Specimen, cracks and strains for ϕ = 23 deg

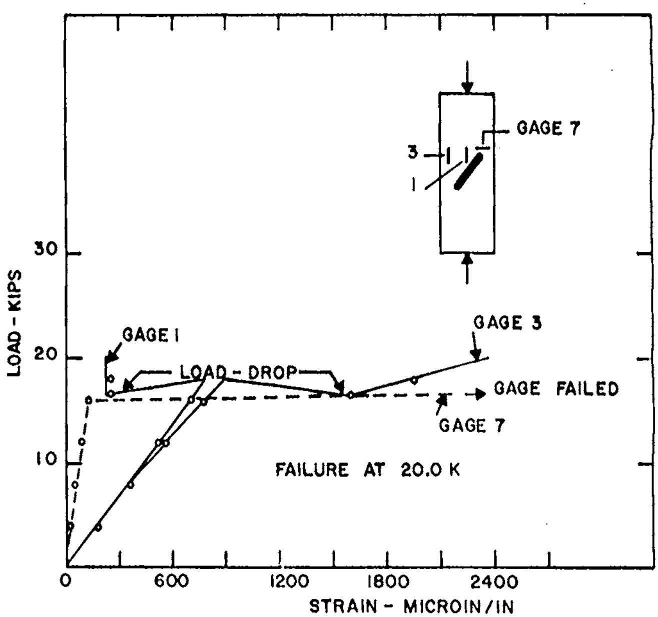

Figure 1-4 Load-strain relations for specimen W-8, ϕ = 42 deg

means that the center portion, the one containing the aggregate, did not take any additional stress beyond that which was present at a load of 12 kips. The increase in load above 12 kips, therefore, must have been resisted by the mortar in the exterior zones. This is clearly indicated by the increased rate of strain of gage 3 at 12 kips.

3. At about 12 kips, cracks at the interface or bond cracks were observed visually. This indicates that at that load, the unconfined bond strength of this interface was exceeded. Since these inclined bond cracks did not open until after final failure, it seems likely that the contact between the stone and the mortar was maintained. This means that the cracked interface can continue to carry a share of the total load through friction. Thus, at 12 kips, only the cohesion between the stone and mortar was lost.

4. Inclined, bond cracks were shortly followed by vertical cracks in the mortar at the top and bottom corners of the stone slab (see Fig. 1-6). As the loading proceeded, these vertical cracks did not perceptibly increase in length until final failure.

If the interface had not been confined by the mortar in the exterior zones, there would have been sliding along the interface when the load in the central zone was carried by friction alone. This is because the angle of inclination of the interface from the horizontal, *67 deg*, is greater than the angle of friction which is 30 to 40 deg at the interface. The confinement, or prevention of sliding provided by mortar produces tensile stresses in the mortar as shown in Figure 1-7(a). This is evident by the readings of gage 7 in Figure 1-3. When these tensile

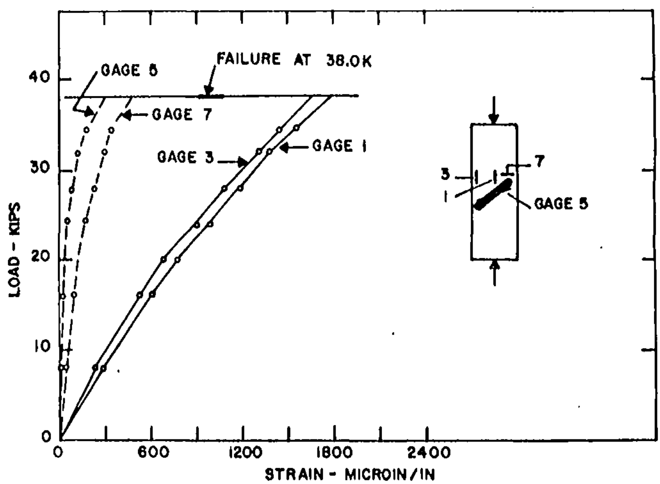

Figure 1-5 Load-strain relations for specimen W-11, ϕ = 50 deg

stresses exceed the strength of mortar, local vertical mortar cracks form.

5. After the formation of local vertical mortar cracks, the central zone can be strained longitudinally without necessarily *increasing the* stresses in that zone. The opening of local vertical mortar cracks permits sliding at the interface so that the central zone deforms longitudinally. Readings of gages 6 and 7 show that there was sliding at the interface and that the local vertical mortar cracks did widen (Fig. 1-3).

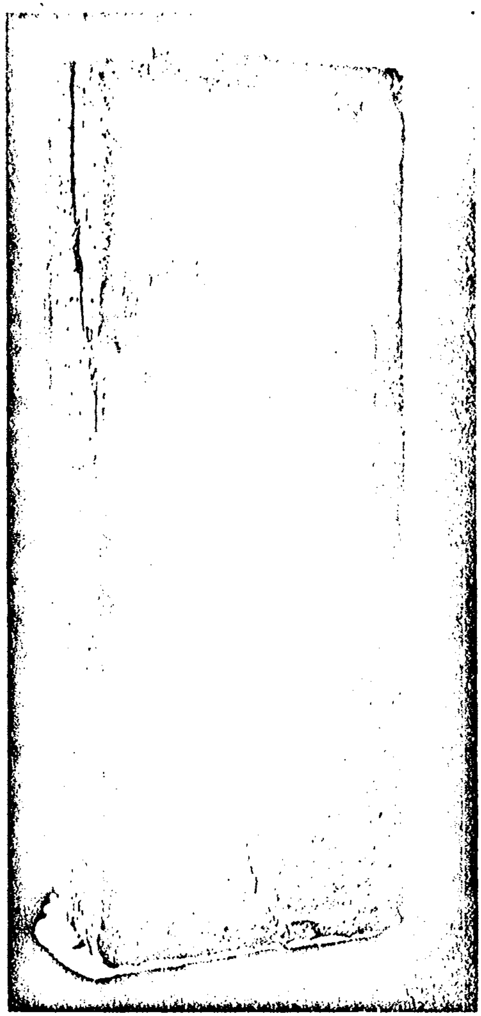

Figure 1-6 Photograph of failed specimen W-5

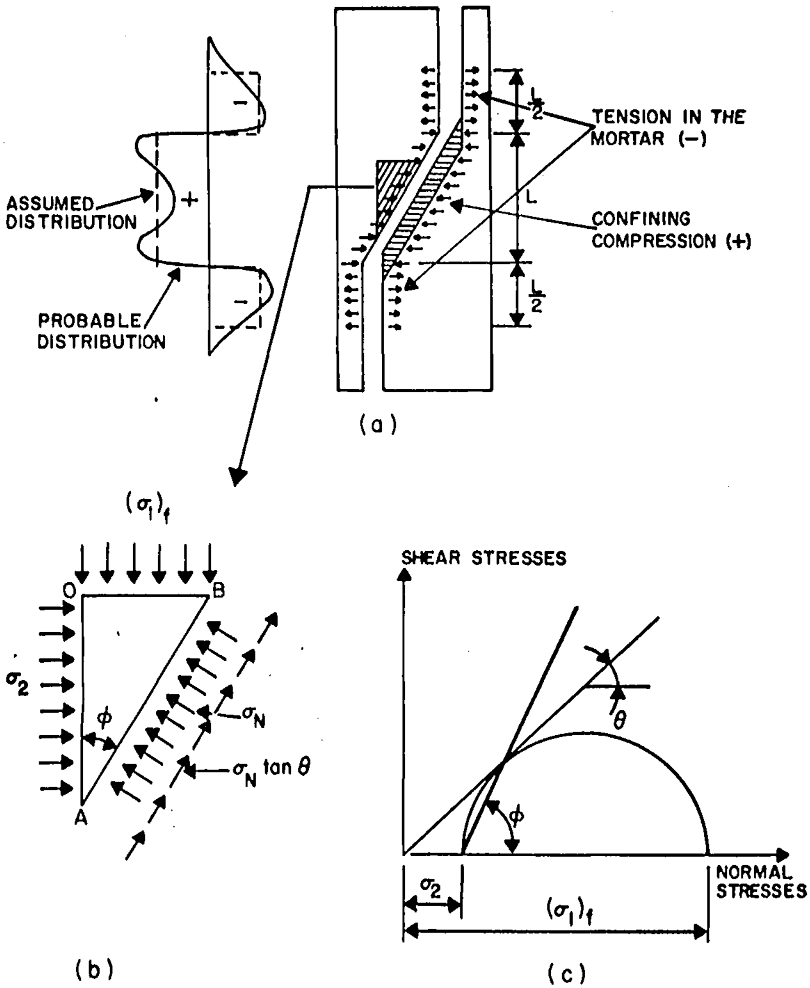

Figure 1-7 Stress-distribution in the interfacial specimen after the cohesion between the stone and the mortar is lost

Thus, the formation of mortar cracks provided the interface with a yielding type of confinement, i.e., a confinement which furnished a reasonably constant horizontal restraining force as further limited sliding proceeded.

The net effect was that for loads above 12 kips, the center zone or the cracked interface behaved essentially plastically (gage 1 on Fig. 1-3), even though there is no flow process involved in these brittle materials.

6. At the final failure of the specimen, crushing of the mortar in the exterior zones was observed (Fig. 1-6). This is because beyond the load of 12 kips the increase in load was carried entirely by the exterior zones. Therefore, the specimen failed when the mortar in these zones could not carry any additional load.

Numerical confirmation

When a compressive load is applied to the specimen, there are normal (σ) and shearing (τ) stresses acting on the plane of the interface between stone and mortar. If σ_1 is the applied vertical compressive stress on the specimen, then the stresses acting on the interface inclined at an angle ϕ to the vertical are given by:

$$\tau = \sigma_1 \sin 2\phi/2 \tag{1}$$

$$\sigma = \sigma_1 \sin^2\phi \tag{2}$$

The relation between the normal and shear stresses acting on the interface at failure is given by the Coulomb-Mohr theory:

$$\tau = (\tau)_{\text{lim}} = c + \sigma\tan\theta \tag{3}$$

where c is the cohesion and θ the angle of friction for the interface (Ref. 8). By combining (1), (2), and (3), the maximum vertical compression that an unconfined interface inclined at an angle ϕ can support is given by:

$$(\sigma_1)_{\text{u.b.}} = c[\cot\phi + \tan(\theta + \phi)] \tag{4}$$

$(\sigma_1)_{\text{u.b.}}$ is called the unconfined bond strength of the interface. The calculated values of $(\sigma_1)_{\text{u.b.}}$ in terms of load for all three angles of the interface tested is given in the first row of Table 1-1. The values of c and θ correspond to the type of mortar and stone used and are taken from a prior and independent investigation (Ref. 8). Since these values can be determined only approximately, two sets of the values of c and θ, differing from each other by about 10 percent, are used. The calculated values of $(\sigma_1)_{\text{u.b.}}$ in terms of load agree well with the observed values for the appearance of bond cracks.

After the appearance of bond crack, the interface carries its share of total load entirely through friction. $(\sigma_1)_f$ represents the maximum stress the interface can carry through friction so that the necessary horizontal restraint is no greater than the tensile strength of the mortar. $(\sigma_1)_f$ was calculated approximately by assuming the stress distributions of Figure 1-7. That is, the confining compression (σ_2) which is necessary to keep the interface from sliding is assumed to be uniformly distributed and is equal in magnitude to uniformly distributed tensile stresses in the mortar. Then $(\sigma_1)_f$ can be approximately calculated from equilibrium,