STRESS-BLOCK PARAMETERS FOR REINFORCED CONCRETE BEAMS DURING FIRE EVENTS

S.F. El-Fitiany and M.A. Youssef *

Department of Civil and Environmental Engineering, The University of Western Ontario,
London, ON, Canada N6A 5B9

Biography:

ACI student member **S.F. El-Fitiany** is a Ph.D. candidate at The University of Western Ontario, Canada. Mr. El-Fitiany obtained his M.E.Sc. from The University of Western Ontario and his B.Sc. from Alexandria University, Egypt. His research interests include behavior of structures during and after exposure to fire and development of performance based fire design approaches.

ACI member **M.A. Youssef** is an associate professor of Civil Engineering at The University of Western Ontario. Dr. Youssef received his Ph.D. from McMaster University, Hamilton, Ontario. His research interests include: modeling of reinforced concrete structures, fire resilience of concrete structures, use of innovative materials (Fiber Reinforced Polymers, FRP, and Shape Memory Alloys, SMA) to upgrade the capacity of existing structures, using SMA to minimize structural damage during earthquakes, and seismic capacity of existing structures.

* Corresponding authors: E-mail: youssef@uwo.ca, Phone: 519-661-2111 Ext. 88661

ABSTRACT

Fire safety is a critical criterion for designing reinforced concrete (RC) structures. As new design codes are moving towards performance-based design, analytical tools are needed to help engineers satisfy code criteria. These tools are also needed to assess the fire performance of critical structures. As full scale experiments and finite element simulations are usually expensive and time consuming options for designers to achieve specific fire performance, a simplified sectional analysis methodology that tracks the axial and flexural behavior of RC square sections subjected to elevated temperatures from their four sides was previously developed and validated by the authors. In the first part of this paper, the proposed methodology is extended to cover rectangular beams subjected to standard ASTM-E119 fire from three sides. An extensive parametric study is then conducted to study the distribution of the concrete compressive stresses at different ASTM-E119 fire durations. Based on the parametric study, simple equations expressing the equivalent stress-block parameters at elevated temperatures are presented. These equations can be utilized by designers to accurately estimate the flexure capacity of simply supported and continuous beams exposed to fire temperatures.

Keywords: Concrete; Elevated temperatures; Sectional analysis; Fire resistance, Stress-block parameters.

INTRODUCTION

Concrete as well as steel reinforcing bars experience significant deterioration when subjected to elevated temperatures[1]. This deterioration is accompanied by the generation of thermal and transient strains which adds to the complexity of estimating the flexural capacity of a reinforced concrete (RC) section at elevated temperatures. Currently, concrete structures are designed for fire safety using prescribed methods that are based on computational modeling and experimental investigations. These methods usually specify the minimum cross-section dimensions and clear cover to achieve specific fire ratings. As new codes are moving towards performance-based design and conducting experimental tests to satisfy different fire scenarios would be an expensive solution, numerous design tools are needed by design engineers[2]. One of these tools would facilitate the estimation of the flexural behavior of a RC beam at elevated temperatures. These tools are also needed to estimate the fire safety of critical structures.

A simplified method to track the axial and/or the flexural behavior of square column sections subjected to fire at their four sides was previously introduced by El-Fitiany and Youssef[3]. This paper starts by extending the proposed method to be applicable to RC beams exposed to fire from three sides. The overall behavior of RC beams during fire exposure is tracked by constructing the moment-curvature relationships at different fire durations. The unrestrained simply supported beam tested by Lin et al.[4], Fig. 1a, is taken as an illustrative example for the proposed methodology. The tested beam has a normal strength concrete with carbonate aggregate and subjected to ASTM-E119 standard fire.

Civil engineers are familiar with the use of concrete stress-block parameters to calculate the ultimate capacity of RC members at ambient temperatures. These parameters convert the parabolic distribution of concrete compression stresses to an idealized rectangular stress-block. Evaluation of these parameters at elevated temperatures allows designers to easily estimate the flexural capacity of RC beams during fire exposure. The second part of this paper presents a parametric study to evaluate the compressive stresses distribution for different rectangular concrete cross-sections. The effect of different parameters including section dimensions, reinforcement ratio, concrete strength, fire duration, and aggregate type is evaluated. Equations to predict the stress block parameters at elevated temperatures are developed. The use of these equations to predict ultimate moment of concrete beam in fire conditions is explained and their estimates is compared to other standard methods.

RESEARCH SIGNIFICANCE

The proposed simplified method extends the use of sectional analysis to be applicable at elevated temperatures. Designers are familiar with this method at ambient temperature, which will allow them to use it in their fire calculations. The ultimate/nominal flexural capacity of RC beams can be evaluated at ambient temperature using equivalent stress-block parameters. The second part of this paper presents an extensive parametric study on the non-linear distribution of compression stresses for a number of rectangular cross-sections at different ASTM-E119 fire durations up to 2.5 hr. It ends by providing simplified equations for designers to allow them to estimate the stress-block parameters at elevated temperatures.

SECTIONAL ANALYSIS AT AMBIENT TEMPERATURE

At ambient temperature, RC sections can be analyzed using the well-known sectional analysis approach[5]. For cases of single curvature, i.e. bending about horizontal axis, the concrete section is divided into horizontal discrete layers. Utilizing the uniaxial stress-strain relationship for each layer and taking into account equilibrium and kinematics, the mechanical behavior of the section is analyzed. To simplify the analysis, two variables can be assumed; incremental centroidal axial strain, $\Delta\varepsilon_c$, and incremental curvature, $\Delta\psi$. Assuming a linear strain distribution, the incremental moment and axial force are obtained using Eq. (1).

$$\begin{pmatrix} \Delta M \\ \\ \Delta P \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{n} E_i \times A_i \times y_i^2 & -\sum_{i=1}^{n} E_i \times A_i \times y_i \\ -\sum_{i=1}^{n} E_i \times A_i \times y_i & \sum_{i=1}^{n} E_i \times A_i \end{pmatrix} \times \begin{pmatrix} \Delta \psi \\ \\ \Delta \varepsilon_c \end{pmatrix} \qquad (1)$$

Where E_i is the modulus of elasticity of layer i, A_i is the area of layer i, y_i is the distance between the center of area of layer i and center of area of the cross-section.

For a given axial load, the moment-curvature behavior is obtained in two stages. In the first stage, the axial strain is increased incrementally while curvature is kept equal to zero until reaching the required axial load. In the second stage, the axial load is kept constant and the applied curvature is increased. The corresponding change in the axial strain and the moment are calculated using Eq. (1). This process is repeated until reaching the required curvature value.

SECTIONAL ANALYSIS AT ELEVATED TEMPERATURES

To apply sectional analysis at elevated temperatures, a number of modifications were proposed and validated by El-Fitiany and Youssef[3]. These modifications account for the two dimensional temperature gradient within the concrete cross section, which affects its homogeneity and increase the nonlinearity of the mechanical strain distribution. The following sections generalize the previously developed method to be applicable to rectangular sections exposed to fire temperature at a number of or all of their sides. Beam B1, shown in Fig. 1, is used to illustrate the concepts. The beam is exposed to ASTM-E119 fire at three of its sides for duration of an hour.

Concrete and steel constitutive models

The constitutive models proposed by Youssef and Moftah[6] are adopted and their application to beam B1 is presented in the following sub-sections.

Concrete compressive strength

Hertz model[7], Eq. (2), is used to predict the reduced concrete compressive strength (f'_{cT}) at elevated temperatures. If concrete is loaded prior to fire, f'_{cT} should be increased by 25%[7].

$$f'_{cT} = R \times f'_c \tag{2.a}$$

$$R = \frac{1}{1 + \frac{T}{15{,}000} + \left(\frac{T}{800}\right)^2 + \left(\frac{T}{570}\right)^8 + \left(\frac{T}{100{,}000}\right)^{64}}$$, for concrete with siliceous aggregate (2.b)

$$R = \frac{1}{1 + \frac{T}{100{,}000} + \left(\frac{T}{1080}\right)^2 + \left(\frac{T}{690}\right)^8 + \left(\frac{T}{1000}\right)^{64}}$$, for concrete with carbonate aggregate (2.c)

Where R is a reduction factor, T is the temperature in degree Celsius [1 $^oF = 1.8\ ^oC + 32$], and f'_c is the concrete compressive strength at ambient temperature.

Fire induced strains

Total concrete strain at elevated temperatures (ε_{tot}) is composed of three terms[6]: instantaneous stress related strain (ε_{fT}), unrestrained thermal strain (ε_{th}), and transient creep strain (ε_{tr}). The value of ε_{fT} at the peak stress (ε_{oT}) defines the stress-strain relationship during the heating stage and can be predicted using the model proposed by Terro[8], Eq. (3). Flexural RC elements have different stress values within concrete compression zone which implies different preloading levels λ_L for each layer.

$$\varepsilon_{oT} = (50\lambda_L^2 - 15\lambda_L + 1)\,\varepsilon_{o1} + 20(\lambda_L - 5\lambda_L^2)\,\varepsilon_{o2} + 5(10\lambda_L^2 - \lambda_L) \times 0.002 \tag{3}$$

where $\varepsilon_{o1} = 2.05\times10^{-3} + 3.08\times10^{-6}T + 6.17\times10^{-9}T^2 + 6.58\times10^{-12}T^3$

$\varepsilon_{o2} = 2.03\times10^{-3} + 1.27\times10^{-6}T + 2.17\times10^{-9}T^2 + 1.64\times10^{-12}T^3$

ε_{th} is the free thermal strain resulting from fire temperature and can be predicted using the Eurocode model[6], Eq. (4).

$$\varepsilon_{th} = -1.8\times10^{-4} + 9\times10^{-6}(T-20) + 2.3\times10^{-11}(T-20)^3 \quad \le 14\times10^{-3}$$

, for concrete with siliceous aggregate (4.a)

$$\varepsilon_{th} = -1.2\times10^{-4} + 6\times10^{-6}(T-20) + 1.4\times10^{-11}(T-20)^3 \quad \le 12\times10^{-3}$$

, for concrete with carbonate aggregate (4.b)

ε_{tr} is induced during the first heating cycle of loaded concrete and is considered the largest component of the total strain. Its value can be estimated using Terro's model[8].

$$\varepsilon_{tr} = \varepsilon_{0.3} \times \left(0.032 + 3.226\frac{f_c}{f'_c}\right)\frac{V_a}{0.65} \qquad (5)$$

Where

V_a is the volume fraction of aggregates

$\varepsilon_{0.3}$ is the transient creep strain for initial axial stress of $0.3\, f'_c$, and is given by Eq. (6)

$$\varepsilon_{0.3} = 43.87\times10^{-6} - 2.73\times10^{-8}\,T - 6.35\times10^{-8}\,T^2 + 2.19\times10^{-10}\,T^3 - 2.77\times10^{-13}\,T^4 \qquad (6)$$

Reinforcing steel tensile stress-strain relationship

Lie's model[1] is used to predict the reduced yield strength of reinforcing bars f_{yT}, Eq. (7).

$$f_{yT} = \left(1 + \frac{T}{900\times\ln(T/1750)}\right)\times f_y \qquad 0 < T \le 600\ ^oC \quad [32 < T \le 1112\ ^oF] \qquad (7.a)$$

$$f_{yT} = \left(\frac{340 - 0.34\times T}{T-240}\right)\times f_y \qquad 600 < T \le 1000\ ^oC \quad [32 < T \le 1112\ ^oF] \qquad (7.b)$$

Lie[1] has also proposed another model representing a general stress-strain $(f_{sT} - \varepsilon_{sT})$ relationship of reinforcing bars at elevated temperatures, Eq. (8). The effect of creep of steel bars is found to have a minor effect on the behavior of RC sections during fire exposure[9] and, thus is not included in this study.

$$f_{sT} = \frac{f(T,0.001)}{0.001} \times \varepsilon_{sT} \qquad \varepsilon_{sT} \le \varepsilon_p \qquad (8.a)$$

$$f_{sT} = \frac{f(T,0.001)}{0.001} \times \varepsilon_p + f(T,[\varepsilon_{sT} - \varepsilon_p + 0.001]) - f(T,0.001) \qquad \varepsilon_{sT} > \varepsilon_p \qquad (8.b)$$

$$\varepsilon_p = 4 \times 10^{-6} f_y \qquad (8.c)$$

$$f(T,0.001) = (50 - 0.04T) \times [1 - e^{(-30+0.03T)\sqrt{0.001}}] \times 6.9 \qquad (8.d)$$

Concrete compressive stress-strain relationship

The model proposed by Youssef and Moftah[6] is adopted in this study. The model includes simplified representation of transient creep strains. The relationship between the compressive stress, f_{cT}, and the corresponding compressive strain, ε_{cT}, is given by Eq. (9).

$$f_{cT} = K_{hT} \times f'_{cT} \left[2 \times \left(\frac{\varepsilon_{cT}}{\varepsilon_{oT} + \varepsilon_{tr}} \right) - \left(\frac{\varepsilon_{cT}}{\varepsilon_{oT} + \varepsilon_{tr}} \right)^2 \right] \qquad \varepsilon_{cT} \le \varepsilon_{oT} + \varepsilon_{tr} \qquad (9.a)$$

$$f_{cT} = K_{hT} \times f'_{cT} [1 - Z(\varepsilon_{cT} - \varepsilon_{oT} - \varepsilon_{tr})] \qquad \ge 0.2 \times f'_{cT} \qquad \varepsilon_{oT} \ge \varepsilon_{oT}\varepsilon_{tr} \qquad (9.b)$$

where,

$$K_{hT} \text{ (confinment factor)} = 1 + \frac{\rho_s \times f_{yT}}{f'_{cT}} \qquad (9.c)$$

$$\rho_s = \frac{\text{volume of transverse reinforcement}}{\text{volume of concrete core measured to their perimeter}}$$

f_{yT} is the reduced yield strength for the stirrups at elevated temperature

Z is the slope of the descending branch of the concrete stress-strain relationship and is given by Eq. (9.d)

$$Z = \frac{0.5}{\frac{3+0.29f'_c}{145f'_c - 1000} \times \frac{\varepsilon_{oT}}{\varepsilon_o} - \varepsilon_{oT}} \tag{9.d}$$

The ultimate compressive strain at failure can be assumed 0.0035 in ambient conditions according to the Canadian standards CSA A23.3-04[10]. Due to the limited literature on the failure compressive strain at elevated temperature, this value is increased by the transient strain ε_{tr} as proposed by El-Fitiany and Youssef[3].

$$\varepsilon_{cuT} = \varepsilon_{cu} + \varepsilon_{tr} \tag{10}$$

Heat transfer model

Several methods were developed to predict the temperature distribution in a concrete section during fire exposure[1]. The Finite Difference Method (FDM) is chosen in this research because of its ability to account for irregular shapes, its accuracy, and the ease of implementation in any programming code. A detailed description of the FDM, in the form of prescribed equations, is given by Lie et al.[1].

For beam B1, a 45 degree heat transfer mesh of 5.4 mm by 5.4 mm [0.21 in] square elements is generated as shown in Fig. 2. Based on the size of the elements, the total fire duration (τ_f = 1 hour), is divided into time steps $\Delta\tau_f$ of 4.2 seconds to accurately predict the temperature of the elements. Concrete initial moisture content is assumed to be zero due to its negligible effect on the temperature predictions[1]. A heat analysis based on the FDM is then conducted and the temperatures for steel bars were found to range from 302 ^{o}C [576 ^{o}F] to 513 ^{o}C [955 ^{o}F]. Fig. 3 shows a comparison between the average measured temperatures of bottom steel bars by Lin et al.[4] and the FDM predictions at different fire durations. To allow using sectional analysis, the 45 degree mesh elements are converted to horizontal square mesh elements[1,3]. The temperature at the center of each square element, Fig. 2b, is taken as the average temperature of the adjacent 45 degree mesh elements.

Average layer temperature

The methodology proposed by El-Fitiany and Youssef[3] is adopted. The square mesh elements are grouped into horizontal layers to simplify the use of sectional analysis. Therefore, an equivalent temperature T_i has to be assigned for each layer to allow estimating the concrete compressive strength, its modulus of elasticity, transient creep, and thermal strains. To accurately predict the behavior using sectional analysis, El-Fitiany and Youssef[3] suggested the use of two different T_i's, one for estimating stresses and the other for strain values. It is clear from Eq. (1) that the tangential modulus of elasticity is the most important factor, thus the first average layer temperature is estimated such that it produces the average modulus of elasticity for the square elements within the layer. At elevated temperatures, initial modulus of elasticity of loaded concrete is proportional to its reduced compressive strength[6]. Therefore, the first average temperature distribution for each layer is based on the average strength of the square mesh elements, Fig. 2d, composing this layer. The second average temperature distribution is used to estimate the thermal and transient creep strains. Eqs. (4) and (5) show that these variables are proportional to the fire temperature. Thus, the second temperature is equal to the algebraic average of the square mesh elements composing the layer. Fig. 4 shows the two distributions for the analyzed beam section after one hour of ASTM-E119 standard fire exposure. The temperature of steel bars can be assumed to be the same as the temperature of the square mesh element within which they are located[1].