Simplification of the Testing and Analysis Procedure for the Two Parameter Fracture Model

by D. C. Jansen, W. J. Weiss, and S. H. F. Schleuchardt

Synopsis: This work describes a modification to the two-parameter fracture method's experimental procedure aimed at removing this operator/equipment dependence. With this method, three compliances are used to determine the focal point at which these compliances intersect. This focal point is then used to determine the slope of the unloading compliance that corresponds to the peak of the load vs. CMOD curve. The unloading compliance that corresponds to unloading at the peak load and initial compliance are then used to determine K_{IC} and $CTOD_C$ as normally done with the Two Parameter Fracture Model. Use of this method makes it possible to remove operator and machine dependence, especially if the materials are extremely brittle, such as in pastes or high strength concrete, thereby permitting the loading and unloading to be programmed using testing software removing the need for manual operator loading changes.

Tests on 15 mortar beams with 4 different notch lengths and initial unloading points ranging from 97% to 75% of maximum load are used to validate this approach. The experimental results are typically more consistent and better correlate to results from the peak load test method. These results indicate that utilizing the focal point correction typically reduces K_{IC} and $CTOD_C$ by 12% and 38% respectively for the mortar tested thereby causing the TPFM and peak load method results to coincide even more closely.

Keywords: compliance; concrete; cracking; effective crack; fracture mechanics; post-peak; test methods

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INTRODUCTION

It is widely accepted that the fracture process zone that develops in cement-based materials is not typically negligible in comparison to the overall dimensions of the specimen tested. As a result, analysis using linear elastic fracture mechanics cannot be applied directly to cementitous systems and some modification is needed to account for the region of stable crack growth that occurs prior to the peak load. Over the last three decades several experimental procedures have been developed which provide a minimum of two parameters (i.e., K_{IC} and $CTOD_C$, G_F and c_f, f'_t and G, etc...) which have been used to characterize the behavior of the cementitous system.[1-7] Numerous testing procedures have been proposed for obtaining these parameters and research continues to develop a standard test procedure that is accurate, requires relatively few samples, can be accomplished in a relatively short time of testing, and does not require a great deal of specialty test equipment.[8]

In 1985 Jenq and Shah proposed one such method to account for the precritical crack growth that occurs prior to the maximum load.[2] This method, called the Two-Parameter Fracture Method (TPFM), is based on the simple premise that the change in compliance that occurs when the specimen is unloaded at the peak load can be used to calculate the precritical crack growth (ie., length of an effective crack). While this method is of great value since it can be used to determine fracture properties using one specimen it is logistically difficult to unload the specimens exactly at the peak load since the true peak load of a specimen is unknown before the test. For this reason it was recommended that unloading could be conducted when the specimen load decreased to 95% of its maximum load. This however is a frequently discussed as a point of concern citing that this introduces operator and machine dependence especially if the materials are extremely brittle such as in pastes or high strength concrete. This paper will focus on a modification to the existing two-parameter model, currently a draft recommendation from RILEM,[9] which will provide a systematic method for removing operator dependence and improving overall accuracy and reproducibility.

BACKGROUND

The Two Parameter Fracture Method

The two-parameter fracture method (TPFM), proposed by Jenq and Shah in 1985, is an effective crack model in which the length of the stable crack growth at peak is characterized to provided a method to account for non-linearities that exist in smaller specimen sizes.[2] To perform this test, the specimen is loaded and the compliance of the load versus crack mouth opening displacement (initial compliance, C_i) and initial crack length, a_o, are used to determine the elastic modulus, E, of the concrete as shown in Figure 1. The length of the effective crack length, a_e, can be determined at any time during the test by unloading the specimen and using load-CMOD response (unloading compliance, C_u) in combination with the elastic modulus. Theoretically, the TPFM approach can be used to determine the length of the effective crack at any point along the Load-CMOD curve, however it is of practical interest to determine the length of crack growth exactly at the time the crack would begin to propagate unstably (i.e., the critical crack length, a_c, that occurs at the peak load). The fracture toughness, K_{IC}, and the critical crack tip opening displacement, $CTOD_C$, are determined from the critical crack length and the peak load. More detail of the analysis procedure along with the necessary equations is provided in Appendix A.

Determining the unloading compliance exactly at the peak load is impossible from an experimental viewpoint since one needs to go beyond the peak to know that it has been reached. As a result, the RILEM recommendation[9] suggests that the specimen be unloaded at 95% of the peak load in the post peak region; however this point is still dependent on the operator and the speed at which the unloading can begin by the equipment from the time of the initial unloading request. This is especially critical in highly brittle materials, such as pastes or high strength concretes in which the rapid decrease in strength after the peak makes it extremely difficult to initiate unloading at 95% of the peak load.

More recently, Tang et al.[4], proposed a method for determining K_{IC} and $CTOD_C$ requiring only the peak loads from two or more geometrically dissimilar specimens (ie, different notch lengths or loading conditions). With this method, a K_{IC} is determined which gives the smallest standard deviation of the $CTOD_C$. The elastic modulus can be taken from compression tests, from the initial compliance (if measured), or by estimation. The distinct advantage of the peak load method over the two parameter fracture method is that closed-loop test equipment is not required to determine K_{IC} and $CTOD_C$. The disadvantage is that at least two tests must be performed instead of one and the elastic modulus may not be the same in tension as in compression due to bond between cement paste and aggregates.[10]

Crack Extension During the Loading Process and Its Influence on the TPFM

Numerous attempts have been made to characterize the size of the effective crack length or fracture process zone ahead of the crack experimentally including laser holography, microscopy, acoustic emission, dye penetration, re-cutting, epoxy impregnation, ultrasonic wave measurements, and X-rays. Holography, microscopy, and other surface imaging techniques provide a means of assessing the crack throughout the loading process, however these methods provide a measure only of the length of the crack at the surface of the specimen. Typically surface measurement techniques provide an overestimate of the average length of the crack since the length of the crack along the surface of the beam is longer than at the center of the beam indicating that the cracking profile does not propagate uniformly as the specimen cracks.[11,12]

Alternative non-invasive test methods such as acoustic emission and ultrasonic wave propagation have been used to characterize the length of the crack inside the specimen. Figure 2 shows results from one such investigation where the relative change in compliance for a concrete slab was compared to the length of a crack measured using an elastic wave transmission measurement procedure for assessing crack depth.[13] First it can be noted that the relative length of the crack growth corresponds well with the change in compliance that is observed as one would expect based on the Two Parameter Fracture Method. Second, it can be seen that this curve can be divided into three regions corresponding to load level. At low load levels before peak (i.e., 0%-60%), the change in compliance (and crack length) with load level has a relatively low slope, thus implying that only minimal crack growth is exhibited in this region. At higher loading levels, in both the pre- and post-peak region, the slope of the load level-crack length degradation is steep implying that more significant crack growth is occurring in this region. This is important because it illustrates that a relatively small change in load level from 95% (post-peak load) to 87% (post-peak load) corresponds to a change in crack length from 66 mm to 82 mm. This suggests that the length of the crack can undergo an increase in length between the peak load and the load level at which the specimen is unloaded, resulting in a slightly longer effective crack (i.e., a less brittle material) that is used in assessing the fracture parameters. Finally, at low load levels in the post-peak region the slope once again is reduced presumably due to crack arrest and from overall cross section reduction.

The additional contribution of the crack growth between the peak load and the point of unloading may be of practical interest when results of this test method are compared with the Peak Load approach. In the Peak-Load method, K_{IC} and

$CTOD_C$ are determined based on only the peak loads from two or more beams with different notch lengths and does not use the unloading compliance; thus the results are not subject to effective crack between the point of peak load and unloading. Figure 3 illustrates that the critical stress intensity factor (K_{IC}) and the critical crack tip opening displacement ($CTOD_C$) determined using the conventional TPFM method divided by the parameters determined using the peak load approach for normal and high strength concrete tested at different ages.[14] It can be seen that the ratio of these values is typically slightly greater than one. This result would be consistent with the observation that the conventional TPFM method uses an effective crack length that is slightly longer than the effective crack at peak due to crack growth while reaching load reduction to 95% of peak load. Further confirmation of this can be seen in comparing the values obtained from normal and high strength concrete. It would be expected that the critical effective crack length would be smaller as the strength of the concrete increases.[15] As such, the growth of the length of the crack between peak and unloading loads would have a more significant impact on the determination of properties of more brittle materials, thereby accounting for higher ratio of the $CTOD_C$ values for the higher strength concrete.

The Focal Point for Multiple Unloading Curves

Recently, Lee and Willam[16] re-illustrated the idea that if multiple lines, which define unloading stress strain curves, are extended, then they will meet at a unique focal point. Lee and Willam's tests were performed on concrete cylinders tested in compression, and the identified focal point could be used for assessing the change in stiffness using this point and the load displacement curve. Using these results, Tasdemir et al.[17] demonstrated that multiple unloading compliances from notched beam tests also meet at a mutual focal point similar to that as shown in Figure 4. This implies that changes in stiffness (alternatively compliance) can be determined for a specimen at any point thereby implying that the length of the effective crack can be determined at any point along a load-deflection curve provided the focal point of the material is known.

PROPOSED MODEL

This section outlines a procedure for determining the focal point and how this focal point is used to determine the fracture properties of a given material. The proposed method is to find the focal point based on the unloading compliances for the Load-CMOD response. A line is extended from the focal point back to the peak load. This new line provides the 'true' unloading compliance corresponding to the peak load, C_{uc}. If three unloading compliances, shown in Figure 4, are used and the lines are defined by[18]

$$P = \frac{1}{C_{u1}} \mathrm{CMOD} + B_1 \qquad (1)$$

$$P = \frac{1}{C_{u2}} \mathrm{CMOD} + B_2 \qquad (2)$$

$$P = \frac{1}{C_{u3}} \mathrm{CMOD} + B_3 \qquad (3)$$

Using error minimization to find the closest point to the lines defined by equations 1-3, the coordinates of the focal point, as shown in Figure 5, can be determined:

$$\mathrm{CMOD}_f = \frac{1}{3}\left[\frac{(B_1 - B_3)}{\left(\frac{1}{C_{u3}} - \frac{1}{C_{u1}}\right)} + \frac{(B_2 - B_1)}{\left(\frac{1}{C_{u1}} - \frac{1}{C_{u2}}\right)} + \frac{(B_3 - B_2)}{\left(\frac{1}{C_{u2}} - \frac{1}{C_{u3}}\right)}\right] \qquad (4)$$

$$P_f = \frac{1}{3}\left[\frac{1}{C_{u3}}\frac{(B_1 - B_3)}{\left(\frac{1}{C_{u3}} - \frac{1}{C_{u1}}\right)} + \frac{1}{C_{u1}}\frac{(B_2 - B_1)}{\left(\frac{1}{C_{u1}} - \frac{1}{C_{u2}}\right)} + \frac{1}{C_{u2}}\frac{(B_3 - B_2)}{\left(\frac{1}{C_{u2}} - \frac{1}{C_{u3}}\right)} + B_1 + B_2 + B_3\right] \qquad (5)$$

The unloading compliance which corresponds to the peak load, C_{uc}, is defined by the inverse of the slope of the line which extends from the focal point (CMOD_f, P_f) to the critical (peak) load (CMOD_c, P_c):

$$C_{uc} = \frac{\mathrm{CMOD}_c - \mathrm{CMOD}_f}{P_c - P_f} \qquad (6)$$

In the same manner as the TPFM,[2,9] the elastic modulus is determined from the initial compliance and initial notch length. With the focal point method, the critical crack length, a_c, which truly corresponds to the peak load, is found using the new compliance, C_{uc}, and the elastic modulus. Using the new a_c and the load at peak, the toughness, K_{IC}, and critical crack tip opening displacement, CTOD_c, can be determined using the same equations as with the TPFM.

EXPERIMENTAL VERIFICATION

Experimental Program

To verify the model, 15 mortar notched beam specimens were tested. In addition, 5 cylinders, 102 mm diameter by 203 mm length, were tested in compression. The water:cement:sand ratio for the mortar was 0.50 : 1.00 : 3.00. All specimens were tested between 32 and 33 days after casting. From the cylinder tests, the compressive strength of the mortar was determined to be 41.4±1.4 MPa and the modulus of elasticity, E_C, was 25.4±0.7 GPa.

The beams which were tested to evaluate the fracture model were 559 mm long by 127 mm high by 50.4 mm thick. Notches were cast into the beams, and 4 different initial notch lengths were used (12.7 mm, 25.4 mm, 43.1 mm, and 50.8 mm) corresponding to 10% of the beam depth ($\alpha = a_0/d = 0.1$), 20% of the beam depth ($\alpha = 0.2$), 30% of the beam depth ($\alpha = 0.3$), and 40% of the beam depth

(α = 0.4), respectively. The notches were 2.38 mm thick and were tapered to a point at the tip of the notch.

The notched beams were tested in three point bending with an overall span length of 508 mm using a 100 kN closed-loop servo-hydraulic test machine. To measure the crack mouth opening displacement, two ±0.5 mm LVDTs were attached to the sides of the specimen level to the crack mouth. The average signal from the two LVDTs was used as the feedback control and recorded as CMOD. The tests were performed at a CMOD rate of 0.0004 mm/second, and the points at which unloading occurred were manually signaled to the test machine.

Each specimen was unloaded 3 times. For each notch length, 2 specimens were unloaded at 95%, 85%, and 75% of the peak load (specimens M*A and M*D), one was unloaded at 85%, 75%, and 65% of the peak load (specimen M*B), and one was unloaded at 75%, 65%, and 55% of the peak load (specimens M*C). A summary of the specimens and notch lengths is provided in Table 1. Data was recorded at a rate of 1 Hz throughout the test.

Data Analysis and Results

The data was analyzed to determine K_{IC} and $CTOD_C$ by three methods: 1) the two parameter fracture method[2,9], 2) the peak load method[4], and 3) the focal point method. With the two parameter fracture method, only the compliance from the first unloading was used, and the actual equations along with a more detailed description of the analysis method can be found in Appendix A. The results of the analysis are given in Table 2; the mean and standard deviations were calculated only from the results of specimens where the first unloading compliance started near 95% of the peak load (specimens M*A and M*D). With the peak load method, all specimens were included in the analysis, and K_{IC} and $CTOD_C$ were determined to be 23.9 N/mm$^{3/2}$ and 0.0057 mm, respectively. With the focal point method, for each specimen the focal point of three unloading compliances was determined, from which the unloading compliance corresponding to the line extending from the focal point to the peak load was calculated. The coordinates of the focal point and the unloading compliance from the peak load are given in Table 1. K_{IC} and $CTOD_C$ for each specimen determined from the focal point method are given in Table 2.

Comparison of Results

To further illustrate that K_{IC} and $CTOD_C$ increase significantly as the point at which the initial unloading continues along the post-peak of the Load vs. CMOD curve with the two parameter fracture method, Figures 6a and 7a show K_{IC} and $CTOD_C$ versus percent of peak load at which first unloading took place.

It should be understood that unloading should be done at least at 95% of the peak load and the tests where unloading took place at loads less than this value should not be considered as correct.[2,9] The mean and standard deviation for K_{IC} and $CTOD_C$ given in Table 2 for the TPFM are determined only using specimens M*A and M*D. This is not unexpected, as discussed earlier, the crack extends significantly during the post-peak, and the TPFM uses this effective crack length when unloading takes place, a_e, as the critical crack length in calculating K_{IC} and $CTOD_C$. The longer crack lengths result in high reported values for K_{IC} and $CTOD_C$. With the focal point correction, K_{IC} and $CTOD_C$ are as significantly affected by the point at which the initial unloading takes place as shown in Figures 6b and 7b, although a consistent and decreasing trend is observed for the specimens unloaded at 75% of peak load (specimens M*C). Furthermore, K_{IC} and $CTOD_C$ determined by the focal point method are reduced by 12% and 38%, respectively, in comparison to the TPFM. The results from the focal point method also correspond more closely to the results from the peak load method than do the results obtained using the two parameter fracture method, as illustrated in Figure 8.

DISCUSSION AND CONCLUSIONS

This paper has illustrated that a focal point can be used to determine the fracture properties of concrete correcting for crack extension which occurs after the peak load. Although in this work the focal point method was determined from the unloading at 3 points along the postpeak, the focal point can also be found from the intersection of two unloadings or even from a single unloading compliance and the initial compliance.

The focal point method has the distinct advantage over the TPFM in that the first unloading does not need to take place while at least 95% of the load carrying capacity of the beam remains, but can take place at nearly any point along the postpeak. By implementing this type of approach, determining the fracture properties of cementitous materials can be automated since a operator controlled unloading is no longer needed, instead preset points of CMOD can be used to signal when unloading is to occur. This implies that operator/equipment response dependence can be eliminated.

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