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Flexural Cracking Behavior of Partially Prestressed Pretensioned and Post-Tensioned Beams—State of the Art

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Synopsis: More prestressed beams incorporating mild steel reinforcement are built today, with allowance of tension in the concrete in what is often termed as partial prestressing. Consequently, a study of their cracking behavior and control of the crack width and distribution are becoming more significant.

Available experimental data on the cracking behavior of prestressed beams is limited. Because of the importance of serviceability behavior of these elements, several experimental and analytical investigations have been undertaken and expressions proposed.

Simple mathematical expressions have been developed as a result of the work at Rutgers University corroborated by comparative analysis with other authors. The proposed expressions for evaluating crack widths in partially prestressed beams at working load and overload levels have been developed in terms of the controlling parameters, namely, the variation in the steel reinforcement and percentage of the prestressing tendons and the non-prestressed reinforcement.

The mathematical model is applied to tests on twenty simply supported pretensioned 9-ft. span beams and four 2-span continuous beams of effective 9-ft. span, and twenty-two simply supported post-tensioned beams of 7-ft. 6-in. span. The major controlling parameters were the variation in the steel reinforcement percentage of the prestressed tendons and the non-prestressed reinforcement. The effect of concrete cover was incorporated in the value of the concrete area in tension.

Equations are proposed relating the maximum weighted crack width to the increase in reinforcement stress and the concrete area in tension.

Keywords: beams (supports); cracking (fracturing); crack width and spacing; partial prestressing; post-tensioning; prestressed concrete; pretensioning; serviceability; tensile stress

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INTRODUCTION

As more prestressed beams incorporating mild steel reinforcement are built today, with allowance of tensile strength in the concrete in a range of 0-12 $\sqrt{f'_c}$, study of their cracking behavior and control of the crack width and distribution are becoming more significant.

Available experimental data on the cracking behavior of prestressed beams [1-12] is limited. Because of the importance of serviceability behvior of these elements, several experimental and analytical investigations have been undertaken and expressions proposed.

The expressions that are discussed in this paper are based on the concept advocated by J. Ferry Borges [6], Nawy-Potyondy [7,8], Holmberg [9,10], Nawy-Huang [12], Nawy-Chiang [21], and Nawy [24,25]. This approach is based on the reinforcement stress concept. It is more effective and more reliable than the flexural concrete tensile stress originally advocated in references [3] and [4], or the nominal concrete strain and curvature discussed in ref. [22], because it can accurately evaluate the strains and stresses in the steel reinforcement whereas such an evaluation of strain in the concrete at the beam or at the reinforcement level is very approximate at best.

Simple mathematical equations have been developed as a result of the work at Rutgers University [12,21] and corroborated by comparative analysis in the work of A.E. Naaman [19,23]. The proposed expressions for evaluating crack widths in partially prestressed beams at working load and overload levels have been developed in terms of the controlling parameters.

Tests were conducted on twenty simply supported pretensioned 9-ft. span beams and four 2-span continuous beams of effective 9-ft. span [12,21]. Tests were also conducted on twenty-two simply supported post-tensioned beams of 7-ft. 6-in. span. The major controlling parameters were the variation in the steel reinforcement percentage of the prestressed tendons and the non-prestressed reinforcement. The prestressing tendons were either 1/4" or 1/2" nominal diameter 7-wire strands having an ultimate strength of 250 ksi (1723.5 N/mm^2) for the 1/4" size and 270 ksi (1861.4 N/mm^2) for the 1/2" size. Mild steel reinforcement used was either #3, #4, or #5 deformed high-strength bars of yield strength varying between 60.9 ksi and 84.0 ksi (419.8-579.1 N/mm^2). The effect of concrete cover was incorporated in the value of the concrete area in reinforcement stress and the concrete area in tension.

RESEARCH SIGNIFICANCE

Partial prestressing implies the allowance of some tensile stress in the concrete. The research reported permits control of the flexural cracks caused by the tensile stress through adequate proportioning of the bonded reinforcement percentage. As a consequence, serviceability of the prestressed beams can be maintained under normal and reasonable overload conditions.

TESTING PROGRAM

Materials

The concrete mix was designed for a nominal 28-day compressive strecgth of approximately 4000 psi (127.6 N/mm^2). Water/cement ratio varied between 5.4 and 7.1 gallons/sack of cement. The coarse aggregate used was crushed stone of 3/8 in. (9.35 mm) maximum size, while the fine aggregate was natural local sand. Slump varied between 5 and 7-1/2 in. (12.7 and 19.1 cm) as given in Table 1.

Uncoated, stress-relieved, 250 ksi 7-wire 1/4 in. (6.35 mm) strands were used for prestressing the pretensioned beams. In Series II of the post-tensioned beams, 1/2 in. (12.7 mm) diameter strands were used. The material had a unit elongation of 0.65 percent at 70.0 percent of the ultimate. The material satisfied ASTM A-416 specifications and had a typical stress-strain relationship as shown in Fig. 1.

Non-prestressed #3, #4, and #5 deformed bars were used as additional reinforcement at the tension side in all beams except B-1 to B-6. A typical stress-strain diagram for the deformed bars is shown in Fig. 2. Table 2 gives the details of the reinforcement used for the beams in this test program, and Fig. 3 gives typical cross-sections of the test beams.

Fabrication and Testing

Twenty single span and four continuous pretensioned beams were fabricated [1] and tested. The simply supported single span beams were as follows: 1) B-1 to B-6 were T-sections with pretensioned prestressed tendons only; 2) B-7 to B-18 were also T-sections reinforced with both pretensioned and prestressed tendons and non-prestressed mild steel; 3) B-19 and B-20 were I-sections with both prestressed and non-prestressed reinforcement; 4) B-21 through B-24 were I-sections continuous on two spans reinforced with both pretensioned prestressed tendons and mild reinforcement. Straight profile strand patterns were used in all beams. For continuous beams, strands were inserted into plastic tubes in the compression regions to achieve zero bond between concrete and strands. Before testing, those strands in the compression regions were cut through to eliminate the effectiveness of strands in these zones.

Twenty-two post-tensioned T-section beams were tested to failure. The test specimens were in two series and all of them had 7 ft. (2.13 m) spans. Series I: The prestressing tendons were 1/4 in. (6.35 mm) nominal diameter 7-wire strand [250 ksi (1740 N/mm^2)]; Series II: The prestressing tendons were 1/2 in. (12.7 mm) nominal diameter 7-wire strand [270 ksi (1950 N/mm^2)].

Pretensioned beams were 10 in. (25 cm) deep and post-tensioned beams were 11 in. (28 cm) deep as shown in Table 1. They were all overdesigned to resist diagonal tension. No. 3 deformed closed stirrups were used throughout the span.

All reinforcement was instrumented with electric resistive type strain gauges at critical locations. Readings were taken of the change in strain at all the necessary stages of prestressing and loading. In addition, demountable mechanical gauges were used to measure the variation of strain on the concrete faces of the beams.

Mechanical dial gauges with 2 in. (5 cm) travel and .001 in. (0.025 mm) accuracy were used to measure the change in deflection due to loading. Crack widths were measured with illuminated microscopes having a magnification of 25 times and a 0.05 mm accuracy. Crack spacings of all the developing cracks were also recorded as well as the crack penetration of the principal cracks.

For most of the beams, eight to nine increments of load were applied to failure. At the conclusion of each beam test, concrete control cylinders were tested for both compressive and tensile strengths.

FLEXURAL CRACKING TEST RESULTS

Maximum crack widths were measured at the reinforcement level and at the bottom tensile face of the concrete. The spacings of the cracks were measured on both faces of each beam at each loading stage. These spacings were summarized for each test specimen and the mean stabilized crack spacing was calculated. Crack spacing is considered stabilized when no additional cracks develop and the major cracks continue to increase in width with the increase in the applied load. Table 3 gives the mean stabilized crack spacing for load ratios of 50 to 70 percent of the ultimate load. Table 4 gives the measured crack width of the stabilized cracks at the reinforcement levels of the steel closest to outer fibers for the various stress levels.

Mathematical Model Formulation for Serviceability Evaluation

Crack Spacing - Primary cracks form in the region of maximum bending moment when the external load reaches the cracking load. As loading is increased, additional cracks will form and the number of cracks will be stabilized when the stress in the concrete no longer exceeds its tensile strength at further locations regardless of load increase. This condition is important as it essentially produces the absolute minimum crack spacing which can occur at high steel stresses, to be termed the stabilized minimum crack spacing. The maximum possible crack spacing under this stabilized condition is twice the minimum, to be termed the stabilized maximum crack spacing. Hence, the stabilized mean crack spacing a_{cs} is deduced as the mean value of the two extremes.

The total tensile force T transferred from the steel to the concrete over the stabilized mean crack spacing can be defined as

$$T = \gamma\, a_{cs}\, \mu\, \Sigma o \qquad \text{.....(1)}$$

where

γ = a factor reflecting the distribution of bond stress
μ = maximum bond stress which is a function of $\sqrt{f'_c}$
Σo = sum of reinforcing elements' circumferences

The resistance R of the concrete area in tension A_t can be defined as

$$R = A_t f'_t \qquad \text{.......................................(2)}$$

where f_t = tensile splitting strength of the concrete. By equating Eqs. (1) and (2), the following expression for a_{cs} is obtained, where c is a constant to be developed from the tests:

$$a_{cs} = c \frac{A_t \; f'_t}{\Sigma o \sqrt{f'_c}} \qquad \text{....................................(3)}$$

The concrete stretched area, namely the concrete area in tension A_t for both the evenly distributed and non-evenly distributed reinforcing elements, is illustrated in Fig. 4. With a mean value of $f'_t/\sqrt{f'_c} = 7.95$ in this investigation, a regression analysis of the test data resulted in the following expression for the mean stabilized crack spacing

$$a_{cs} = 1.20 \; A_t/\Sigma o \qquad \text{....................................(4)}$$

<u>Crack Width</u> - If Δf_s is the net stress in the prestressed tendon or the magnitude of the tensile stress in the normal steel at any crack width load level in which the decompression load (decompression here means $f_c = 0$ at the level of the reinforcing steel) is taken as the reference point [3,5] then for the prestressed tendon

$$\Delta f_s = f_{nt} - f_d \qquad \text{....................................(5)}$$

where

f_{nt} = stress in the prestressing steel at any load beyond the decompression load

f_d = stress in the prestressing steel corresponding to the decompression load

The unit strain $\epsilon_s = \Delta f_s/E_s$, and it is logical to disregard as insignificant the unit strains in the concrete due to the effects of temperature, shrinkage and elastic shortening [4,5]. Hence, the maximum crack width can be defined as

$$w_{max} = k\ a_{cs}\ \epsilon_s^{\alpha} \qquad \text{.................................(6)}$$

where k and α are constants to be established by tests, or

$$w_{max} = k'\ a_{cs}(\Delta f_s)^{\alpha} \qquad \text{.................................(6a)}$$

where k' is a constant in terms of constant k.

<u>Expressions for Pretensioned Beams</u>

Eq. 6a is rewritten in terms of Δf_s so that analysis of the test data of all the simply supported test beams in Table 4a leads to the following expression at the reinforcement level:

$$w_{max} = 1.4 \times 10^{-5}\ a_{cs}\ (\Delta f_s)^{1.31} \qquad \text{...................(7)}$$

A 25 percent scatter band envelops all the data for the expression in Eq. 7 for Δf_s = 20 ksi to 80 ksi.

Linearizing Eq. 7 for easier use by the design engineer leads to the following simplified expression of the maximum crack width at the reinforcing steel level:

$$w_{max} = 5.85 \times 10^{-5}\ \frac{A_t}{\Sigma o}\ (\Delta f_s) \qquad \text{........................(8a)}$$

and a maximum crack width (in.) at the tensile face of the concrete:

$$w'_{max} = 5.85 \times 10^{-5}\ R_i\ \frac{A_t}{\Sigma o}\ (\Delta f_s) \qquad \text{...................(8b)}$$

where R_i is the distance ratio as defined in the notations.

A plot of the data and the best fit expression for Eq. 8a is given in Fig. 5 with a 40 percent spread, which is reasonable in view of the randomness of crack development and the linearization of the original expression of Eq. 7.

<u>Expressions for Post-Tensioned Beams</u>

The expression developed for the crack width in post-tensioned bonded beams which contain mild steel reinforcement is based on the test results given in Table 4b

$$w_{max} = 6.51 \times 10^{-5}\ \frac{A_t}{\Sigma o}\ (\Delta f_s) \qquad \text{........................(9a)}$$

for the width reinforcement level closest to the tensile face, and

$$w'_{max} = 6.51 \times 10^{-5} R_i \frac{A_t}{\Sigma o} (\Delta f_s) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9b)$$

at the tensile face of the concrete lower fibers.

For non-bonded beams, the factor 6.51 in Eqs. 9a and 9b becomes 6.83. A regression plot of Eq. 9a is given in Figure 6. It shows a ± 40 percent scatter band, which is not unexpected in the randomness of flexural cracking behavior.

DISCUSSION OF TEST RESULTS

It is observed from this investigation that the initial flexural cracks started at a relatively low net steel stress Δf_s level between 3 and 8 ksi. These initial cracks formed in a rather random manner and with an irregular spacing. All major cracks usually devloped at a net steel stress level of 25 to 30 ksi. At higher stresses the existing cracks widened and new cracks of narrow width usually formed between major cracks. Visible stabilized crack spacing condition was generally reached at 0.5 to 0.7 of the ultimate load.

This investigation established that the maximum crack spacings after stabilization were close to twice the minimum possible spacings, having a mean value of 2.02 and a standard deviation of 0.29. The effect of the variation of percentages of the non-prestressed steel was significant both on the crack spacing and crack width. For beams with non-prestressed steel, the number of flexural cracks was almost twice as many as those for the case of no mild steel. <u>These cracks were more evenly distributed, with considerably closer spacing and finer widths</u>. This behavior can be attributed to the fact that the bond of the mild steel to the surrounding concrete played a pronounced role in crack control. A typical plot of the effect of the various steel percentages on the crack spacing at the various stress levels Δf_s is given in Fig. 7. It is seen from this plot that crack spacing stabilized at a net stress level of 36 to 40 ksi.

It is also observed that it is advantageous to locate the non-prestressed steel below the prestressed tendons. This is due to the fact that mild steel has larger diameters than the prestressed reinforcement, hence larger bond area of contact with the surrounding concrete. Also, by placing the mild steel close to the tensile concrete face, cracks will be more evenly distributed, hence crack spacing and consequently width will be smaller.

It should be noted that the areas of the bonded steel used in this research program were larger than the minimum required by the ACI 318-83 code. The code stipulates a minimum $A_s = 0.004\ A$, where A is the area of the part of the concrete section between the flexural tension face and the center of gravity of the gross section considered. For the T-sections used in most of the test beams, $0.004\ A \simeq 0.07$ in.^{2} whereas the smallest area of bonded steel used as in beam B-1 was 0.108 in.^{2} (Table 2a). The majority of the test beams had bonded longitudinal steel areas ranging between 0.144 in.^{2} and 1.01 in.^{2} with corresponding steel percentages ρ = 0.17 to 1.01 percent and a total steel percentage at the tensile side of ρ = 1.24 percent. Fig. 7 demonstrates that the crack spacing is better controlled using higher steel percentages than the minimum required by the ACI code. A range of not less than ρ = 0.60 to 0.70 percent for the bonded reinforcement is recommended to achieve a crack spacing of about 5 inches. As a result, acceptable crack widths can be achieved in partially prestressed beams.

The effect of spacing of the stirrups on the crack spacing was not pronounced. It was found that the final crack spacing and crack pattern did not necessarily follow the vertical shear reinforcement. Even though the first few cracks usually started at the stirrups, the vertical legs of the stirrups served only as initial weak areas of stress concentration. In most cases, the stabilized mean crack spacings were smaller than the spacings of stirrups.

Based on these discussions and the results of this investigation, it can be stated that the proposed expressions for crack control can be reasonably applied by the design engineer for maintaining the serviceability of pretensioned and post-tensioned partially prestressed beams under working load and overload conditions. Once the allowable crack width is established for the prevailing environmental conditions, the proper percentage of non-prestressed reinforcement can be determined to ensure serviceable behavior.

Discussion of Other Work

Based on the analysis of results of various investigators, Naaman produced the following modified expression for partially prestressed pretensioned members

$$(w_{max})\text{in.} = [42+5.58\ \frac{A_t}{\Sigma o}\ (\Delta f_s)] \times 10^{-5} \quad \ldots\ldots\ldots\ldots\ldots(10)$$

This regression expression is very close to Eq. 8a by the author. If plotted against the experimental results of the various investigators it gives a best fit as shown in Fig. 8.

The concrete cover has not been substantially varied in the reported Nawy's work since the magnitude of clear cover in prestressed beams is essentially standardized and is accounted for in the term A_t describing the concrete area in tension.

Additional tests being presently conducted by the author where the variation in the concrete cover is the major parameter have further verified that such variation does not affect the parameters of the expressions in Eqs. 8a and 9a for pretensioned and post-tensioned beams.

The study reported in Ref. [22] concentrating on the area of concrete in tension and the nominal strain in the concrete at the tensile face would not give a reliable prediction of the crack width. In particular it does not account for the actual stress in the steel reinforcement and depends on measuerements of strain at the concrete surface which are difficult to reliably evaluate.

A comparison is made between the author's equations and the CEB-FIP equations using similar notations.

Nawy: $$w_{max} = \xi \left(\frac{A_t}{\Sigma o}\right) \Delta\sigma_p$$

CEB-FIP: $$w_{max} = \left[K\frac{\phi}{\rho_t} + \alpha_o \right] \frac{\Delta\phi_p}{E_s} \left[1 - \beta\left(\frac{\Delta\sigma_{pr}}{\Delta\sigma_p}\right)^2 \right]$$

where

$$\Sigma o \simeq \lambda\left(\frac{\phi}{\rho_t}\right)$$ with λ being a multiplier

K, β are variables,

and α_o, $\left(\frac{\Delta\sigma_{pr}}{\Delta\sigma_p}\right)^2$ are terms in the CEB-FIP expression not of major significance and accounted for in the $\xi = 5.85$ for pretensioned beams and $\xi = 6.51$ for the post-tensioned beams in the author's expressions.

Fig. 9 shows a typical test set-up. Fig. 10 shows the cracking pattern at failure of one of the beams tested in this program, while Fig. 11 gives a close-up of the flexural cracks at failure of another beam at rupture in this investigation. One can recognize in these photogtraphs the relatively even distribution of crack spacing and their extension to the compression flange at top.