Moisture Transport and Shrinkage Stress in Polymer Based Repair Materials

by M.K. Rahman

Synopsis: Moisture transport, shrinkage and creep in repair material have a profound influence on long-term durability and serviceability of patch repair in concrete structures. In the process of drying of a newly cast repair mortar, the moisture diffuses within the domain and convects at the bounding surfaces resulting in hygral gradient across the depth and coupled drying shrinkage. This leads to the generation of tensile stresses in the repair layer, and cracks are developed at the surface of patch repair and at its interface with the parent concrete. Polymer-based repair materials also undergo significant shrinkage and tensile creep, which influences the stresses and cracking in the repair overlays. Experimental investigations including strength tests, moisture loss, shrinkage, and creep measurements were conducted on four selected repair mortar including two polymer-based repair mortars. These tests provide the parameters required for computation of stresses and prediction of cracking in repair overlays. Nonlinear finite element based diffusion analysis and experimentally obtained drying curves can be used to develop the empirical moisture diffusivity law and quantification of the surface transfer coefficient which is used for prediction of moisture loss in a repair layer. Multiphysics finite element software COMSOL provides a convenient tool for computation of moisture transport and associated evolution of stresses due to shrinkage in a composite system in which a polymer based repair mortar is used for patch repair of a concrete member.

Keywords: Polymer-based repair mortar, moisture diffusion, drying, shrinkage, moisture diffusivity, finite element, COMSOL, repair layer, shrinkage stress.

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INTRODUCTION

The arid and harsh environment in the Arabian Gulf, with high temperature and ambient salinity, coupled with low humidity and high wind, causes corrosion of reinforcement and cracking and delamination of concrete cover, necessitating repairs in beams, columns and slabs. Repair of concrete structural components forms a significant fraction of construction activities in the region. Repair of a repair is also rampant due to failure of the patch repair in a short span of time (Fig. 1). The durability in repair systems is quite different from that of concrete. The dimensional change in concrete due to shrinkage is low and the deterioration of the concrete is a slow process and takes several years to materialize. In a repair system, on the other hand, a relatively thin repair layer is cast over a hardened concrete substrate, and tensile stresses are generated in the repair layer at an early age due to restrained drying shrinkage that may cause a failure in the patch repair. For durability of a repair system it is important to have a repair material with a low cracking potential and/or delamination, and having a strong bond to the host concrete (Rahman 1999). The generation of stresses in a repair system is complex with several physical and chemical processes in action. These processes include:

- Diffusion of Moisture from repair domain and convective mass transfer at interface
- Evolution of strength and stiffness of the repair layer due to evolution of hydration
- Expansion in the repair matrix due to time-state activated expansive agents
- Drying shrinkage evolving as a consequence of moisture desorption from the repair layer.
- Stress buildup in the repair layer due to boundary and strain gradients restraint
- Tensile creep relief in the repair layer due to shrinkage induced stresses
- Cracking in zones of high stresses where the evolving tensile strength is surpassed.

In polymer-based repair mortars, polymer modification results in the formation of dual matrices, a cement hydration matrix and a polymerized skeleton which interpenetrate into each other. Unhydrated cement particles are enveloped by closely packed layer of polymer particles. The polymer films envelope cement hydrates, and the co-matrix has a strong influence on mechanical properties of a polymer-based repair mortar.

When a repair layer is cast over a hardened concrete substrate, with proper curing, most of the moisture remains entrapped in the repair layer. Exposure of the repair layer to the environment, which is characterized by a low humidity, results in a hygral shock on the repair system. The potential generated due to differential in moisture content of the repair layer and the environment and also between the repair layer and the host (if unsealed/unsaturated) gives rise to the process of moisture diffusion. Moisture diffusion is a process in which moisture is dispersed from the repair layer into the atmosphere at the repair-fluid (air) interface by a convective process. Simultaneously, a moisture differential is also created between the surface and the core resulting in domain moisture transport towards the surface. The moisture diffusion is high initially, and it decays with time as more and more moisture is lost from the repair layer. The diffusion of moisture from the repair domain is a key physical process, which is important for repair system integrity and durability. The principal physical process that is generated as the moisture is lost from the repair layer is drying shrinkage which results in the development of tensile, peeling and shear stresses in the repair domain. These stresses are, however, relieved due to tensile creep in the repair layer. The net tensile stresses in the repair layer at the top surface, if they exceed the evolving tensile strength, will result in cracks emanating from surface and progressing inwards. Also, the shear and peeling stresses at the interface result in delamination and cracks at the interface if they exceed the bond/adhesive strength between the concrete and the repair layer (Bazant and Najjar 1971; Granger et al. 1997; Sakata 1983; Rahman et al. 1999).

The engendering of stresses and development of cracks in the repair layer is a consequence of a several coupled processes as stated earlier. For example, the stresses and microcracks are believed to accelerate the diffusion process, the creep behavior is coupled with the shrinkage process and evolving material properties are coupled with other processes. In a numerical model of a repair system for predicting stresses, consideration of these coupling effects is difficult and complicates the macroscopic model for prediction of deformational and stress response in a repair system. In a simple phenomenological model for prediction of stress response of a repair system under environmental loading, it is desirable to neglect the coupling effects of various processes and incorporate these processes as independent action in the system and subsequently superpose the stresses due to these actions.

In this paper we couple the physics of diffusion and structural mechanics in the COMSOL Multiphysics software (COMSOL 3.5, 2008) so as to develop a numerical model for predicting moisture diffusion and associated stress developed in a repair layer under drying shrinkage loading. An experimental-numerical approach is used to establish the material parameters governing the diffusion of moisture in selected repair mortars. A 2-D composite system with a polymer-based repair layer laid over an existing hardened concrete substrate is simulated using the multiphysics capability of COMSOL to predict evolving moisture loss, deformation and stresses in the repair layer.

MOISTURE DIFFUSION IN CEMENTITIOUS MATERIALS

Moisture diffusion in cementitious materials is governed by nonlinear diffusion theory. Two parameters, the moisture diffusivity *D(C)*, which is dependent on the spatial moisture content in the domain of the structural element, and the convective moisture transfer coefficient h_f at the surface, control the moisture transport in a newly cast repair mortar. The quantum and rate of moisture loss from repair layer also depends on the moisture content in concrete and the ambient humidity (Bazant and Najjar 1971)

The mechanism of domain moisture transport in cementitious materials involves several mechanisms at various levels. A phenomenological approach in which various transport mechanisms are lumped together by defining "moisture dependent diffusivity law" of the cementitious material at a macroscopic level provides a convenient way to address the problem using nonlinear diffusion theory. Diffusivity in this paper is consequently defined as the rate of vapor and liquid transport in the domain of the system without distinction of different moisture flow mechanisms, which are considered in an integral form.

The governing differential equation (Fick's Second law) for 3-D moisture diffusion in a cementitious material and the associated, initial and boundary conditions are as follows:

$$\frac{\partial C(x_k,t)}{\partial t} = \frac{\partial}{\partial x_i}\left[D(C)\frac{\partial C(x_k,t)}{\partial x_i}\right] \quad \begin{matrix} k=1,3 \\ i=1,3 \end{matrix} \tag{1}$$

Initial Condition:

$$C(x_k,0) = C_o \text{ @ } t=0 \; k=1,3 \text{ for 3-D} \tag{2}$$

Boundary Condition:

$$D(C)\frac{\partial C(x_i,t)}{\partial x_i}n_i - h_f\left(C_e - C_s\right) = 0 \quad x_i \in \Gamma q \tag{3}$$

$C=C\ (x_i,\ t),\ i=1,3$ is the moisture content varying in domain with time, *D(C)* =Isotropic moisture diffusivity coefficient which is function of moisture content C, n_i is the outward normal at the boundary, h_f is the convective moisture transfer coefficient, C_e is the moisture of external environment, C_s is the surface moisture content of the solid, and $\partial C/\partial n$ is the moisture flux normal to the exposed surface.

FUNCTIONAL FORM OF MOISTURE DEPENDENT DIFFUSIVITY LAW

The solution of the governing differential equation (Eq. 1) requires an empirical moisture diffusivity law for computations of time history of moisture content over the spatial domain of the repair layer and the substrate. Different forms of dependence of diffusivity D on the moisture content/humidity has been used by various researchers. Some of the moisture diffusivity law for cementitious materials used in the literature includes:

S-shaped curve for diffusivity coefficient as a function of pore humidity (Bazant and Najjar 1972):

$$D(h) = D_1\,\alpha + \left(1-\alpha\right)\Big/1 + \left[\left(1-h\right)/\left(1-h_c\right)\right]^n \tag{4}$$

where D_1, α, h_c and n are the parameters of the model.

Power function:

$$D(C) = p_1 + p_2 C^{P_3} \qquad \text{(Philajavaara 1965)} \tag{5}$$

Exponential form:

$$D(C) = p_1 \exp(p_2 C) \qquad \text{(Mensi et al. 1988)} \tag{6}$$

Hyperbolic expressions:

$$D(C) = p_o + p_1 \left(\frac{C}{1-C} \right)^{p_2} \text{(Penev and Kawamura 1991)} \tag{7}$$

Polynomials:

$$D(C) = p_1 + p_2 C + p_3 C^2 + p_4 C^3 \quad \text{(Pleinert et al. 1988)} \tag{8}$$

p_1, p_2, p_3, p_4 are the regression parameters

In this paper a three-parameter trigonometric tangent function form of diffusivity law proposed by Rahman (1999) with three unknown parameters was used for computing concrete diffusivity.

$$D(C) = b_o \tan(b_1 C^n) \tag{9}$$

where b_o, b_1 and n are parameters to be evaluated for best fit.

Determination of moisture diffusivity law of cementitious materials is contingent upon precise measurement of moisture loss as a function of space and time. No simple experimental technique exists currently for determination of moisture diffusivity of cementitious materials. Sophisticated methods such as nuclear magnetic resonance and computer tomography have been developed but are not readily available (Pleinert et al. 1988). Several methods have been reported in literature for determination of diffusivity of cementitious mortar. Drying tests on specimen of various sizes is most commonly used. The Boltzmann-Matano method has been used in several studies (Sakata 1983; Penev and Kawamura 1991). A combined experimental-finite element based approach was used in this study for computing moisture diffusivity (Rahman 1999; Wittmann et al.1989). A non-linear finite element method using multiphysics software COMSOL was used for the formulation of the diffusivity law for selected repair materials in this paper.

EXPERIMENTAL INVESTIGATIONS

The experimental investigations were carried out on four types of commercially available repair mortars including two types of polymer based repair mortars. These include:

- Polymer modified cementitious material (PMC), which is a single component shrinkage compensated mortar containing cement, filler and acrylic-based polymer with 5% silica fume,
- Polymeric fiber reinforced cementitious material (FRM), which is a shrinkage compensated, fiber reinforced, thixotropic repair mortar. It contains acrylic polymer fiber,
- Flowing micro concrete (FMC), which is a repair concrete with graded aggregates and time-state controlled expansive agent which provides a flowing, self-compacting, rheoplastic, non-segregating, thixotropic mortar, and
- Modified cementitious material (MCM), which is a single component modified cementitious repair material formulated for use in hot climates.

The experimental program included strength tests, shrinkage tests, creep tests and drying tests. The shrinkage tests were conducted on 25 mm x 25 mm x 285 mm (1in x 1in x 11.4 in) prisms as per ASTM standard C-157 and 40 mm x 40 mm x 300 mm (1.6 in x 1.6 in x 12 in) prisms. Tensile creep tests were conducted on dog bone specimen 40 mm x 40mm (1.6 in x 1.6 in) in cross section. The curing regimes of samples included curing in water after demoulding, curing in air after 7 to10 days at room temp and humidity, and sealing in aluminum foil after demoulding. Embedded strain gauges and surface gauges were used for measuring strain in the samples.

Table 1 show results of compressive, tensile, and flexural strength and tensile modulus of elasticity obtained for the four repair materials from the experimental program. Table 2 show the results of expansion, shrinkage and creep strains for these repair materials. Figures 2, 3 and 4 show typical results for polymer-based repair mortar, PMC. The polymer-modified repair mortar PMC had a low compressive and tensile strength as compared to the polymeric fiber reinforced repair mortar and self compacting repair mortar. PMC also had a very low tensile modulus of elasticity as compared to other repair mortars. The effect of air curing which aids in the development of polymeric skeleton in polymer based repair mortar is evident in Figs. 2 and 3. The polymer-based repair mortar PMC also showed a high initial expansion strain and the gross shrinkage strain at 60 days. It also had a high tensile creep strain which helps in the relaxation of tensile stresses in the repair layer.

Drying tests were conducted on repair mortar specimens with varying length of path of unidirectional moisture movement for computing diffusivity law parameters b_0, b_1 and n in Eq. 9 for the selected repair mortars. Specimens of three different widths 40 mm, 80 mm, and 120 mm (1.6 in, 3.2 in, and 4.8 in), but similar cross sectional area (200 mm x 20 mm or 8 in x 0.8 in) were used for 1-D drying test. The samples were cured in water for 28 days and drying tests were carried out inside the environmental chamber with controlled humidity and temperature as shown in Fig. 5. The tests were performed at a temperature of 30±1°C (86±1.8°F) and the environmental humidity was kept constant at 65% RH.

COMPUTATION OF DIFFUSIVITY LAW PARAMETERS USING COMSOL

A 3-D finite element analysis was used for computing the diffusivity law parameters (b_0, b_1 & n) and convective transfer coefficient (h_f) for the selected repair mortars. Numerical computation of moisture loss over the 3-D domain of the experimental sample, using the Diffusion Module in COMSOL, with an assumed set of values for the parameters b_0, b_1, n and h_f was first carried out. The specimen was insulated from four sides and the moisture diffuses only from the 200 mm x 20 mm (8 in x 0.8 in) surfaces. The evolution of mean moisture loss in the 3-D sample using COMSOL was compared with the experimentally obtained moisture loss curves and the values of the parameters are varied to achieve a close correspondence between the computed and experimental moisture loss curves. Iterative runs were first carried out for the 200 mm x 20 mm x 40 mm (8 in x 0.8 in x 1.6 in) specimen to ascertain the values of b_0, b_1, n & h_f., which best fits the experimental moisture loss data. The computed values of constants b_0, b_1, n and h_f. for 40 mm wide specimen were subsequently used for 3-D models in COMSOL for 200 mm x 20mm (8 in x 0.8 in) specimen of width 40 mm (1.6 in) and 120 mm (4.8 in). The correct value of these constants will show a good match between experimental and numerical results for the three specimens with different length of path of moisture transport.

The moisture content surface in the 3-D sample subjected to drying for three specimens of the polymer based repair mortar PMC at the end of 60 days is shown in Fig. 6. It can be observed from this figure that the moisture content in the outer layer of the sample was in equilibrium with environmental humidity (65%), whereas, the core of the sample had a moisture content of about 73, 82 and 88%. The mean total moisture loss in these samples was computed as 28, 19 and 16% respectively with environmental moisture content being 65%.

Evolution of mean moisture loss for specimen of various sizes obtained from 3-D COMSOL diffusion analysis and those obtained experimentally, for the repair mortar PMC, from drying test is shown in Fig. 7. The experimental and computed moisture loss curves show a very good agreement. The parameters b_0, b_1, n and h_f. determined for 200 mm x 20 mm x 40 mm wide (8 in x 0.8 in x 1.6 in) specimen is also valid for specimens 80 mm (3.2 in). and 120 mm (4.8 in). wide. The values of constants for diffusivity law (Eq. 9) for the four repair materials are shown in Table 3. Typical values of constants and subdomain expressions for diffusivity law used in COMSOL for the polymer-based repair mortar PMC are shown in Table 4.

Once the diffusivity law for the selected repair materials is defined, it can be used to compute moisture loss in the repair matrix and any size and for one, two or three dimensional diffusion of moisture. In the sections that follow the multiphysics problem of moisture diffusion and shrinkage stresses in a repair system is presented using the diffusivity law parameters determined above for repair material PMC.

MULTIPHYSICS COMPUTATION OF STRESSES IN A CONCRETE REPAIR OVERLAY

Multiphysics software provide a convenient platform for simultaneous modeling of the physics of diffusion and stress generation in patch repair and overlays and visualizing the evolution of stresses in a composite repair system with time. A simple approach for computation of stresses in concrete repair overlay presented. A composite system consisting of a thin layer of polymeric repair mortar placed over an existing old concrete substrate was analyzed using COMSOL Multiphysics software to compute the stresses generated in the repair layer due to moisture diffusion and associated shrinkage deformation. The composite system 500 mm (20 in) in width consists of a 75 mm (3 in) thick polymer-based layer PMC placed over a 200 mm (8 in) thick dried concrete host layer. The interface of the substrate for this analysis is considered to be mechanically isolated by the application of epoxy bonding layer which precludes the movement of the moisture from the repair layer into the dried concrete substrate and provides high bond strength between the repair overlay and the parent concrete. A 2-D finite element model of the composite system is shown in Fig. 8. Finite element discretization of the PMC repair layer and the interface zone has a fine mesh of quadratic Lagrangian triangular elements. A relatively coarser finite element mesh with triangular elements was used for the host concrete away from the interface. The repair layer and the substrate have common nodes at the interface. A perfect bond between the two layers was assumed.

The diffusion module in COMSOL is used to compute the evolution of moisture loss with time in the domain of the PMC repair layer. The moisture loss from diffusion analysis is simultaneously used for plane stress analysis in structural mechanics module to compute the shrinkage strain and stress. Experimental tests conducted on various repair materials show that a linear relationship exists between moisture loss and shrinkage. A typical shrinkage – moisture loss curve for the repair material FMC is shown in Fig. 9.The linear relationship between moisture loss and free shrinkage strain is given as:

$$\Delta\varepsilon_{sh} = \alpha_{hygro}\,\Delta C \tag{10}$$

Where $\Delta\varepsilon_{sh}$ is the increase in free shrinkage strain due to the increment in moisture loss ΔC and α_{hygro} is the coefficient of hygral contraction similar to the coefficient of thermal expansion in a thermal a stress analysis problem (Baluch et al. 2008). The coefficient of hygral contraction, α_{hygro} for the repair material PMC used to obtain the incremental values of the shrinkage strain was taken as 15×10^{-6} mm/mm*C% (in/in*C%).

The subdomain expressions, and subdomain settings used for computing the moisture diffusion and strain and stress fields in the composite system are shown in Table 5. The initial moisture content in the fresh repair layer domain was considered as 99%, whereas, the moisture content in the old concrete substrate was kept at 65%. The environmental moisture concentration was also specified as 65%. The diffusivity law shown in subdomain expression in Table 5 was obtained for the repair material PMC, whereas, D_c is kept zero for the host concrete simulating the condition that moisture diffusion in the old concrete had ceased. A flux boundary condition was assumed at the three boundaries where moisture diffusion takes place as shown in Fig. 8. All other boundaries were specified as insulated.

For computing the evolution of stresses in the repair layer due to restrained shrinkage, with restraint being provided by the host concrete layer, plane stress linear elastic analysis in the structural mechanics module in COMSOL was used. The thermal expansion provision in the structural mechanics module was activated for simulating the moisture loss-induced shrinkage deformation in the repair layer. The strain temperature was specified as moisture concentration (c%) and the reference strain temperature as the initial moisture content (99%) in the domain. The subdomain settings including the elastic modulus, Poisson's ratio, and coefficient of hygral contraction α_{hygro} is shown in Table 5. The elastic modulus of the repair layer was decreased to account for the creep effect. In a more sophisticated model the law for the evolution of tensile creep with time as shown in Fig. 4 could be incorporated in the model to compute the relaxation of shrinkage induced stresses in the repair overlay. All edges of the composite system except the base of the parent concrete, which was fully restrained, were free to deform for stress analysis of the composite system. A perfect bond between the overlay and the parent concrete was assumed. Incorporation of an interface layer would be more appropriate, but it requires determination of the mechanical properties of the interfacial zone for various repair materials.

MOISTURE DIFFUSION IN THE REPAIR LAYER

In the composite system, the moisture loss is two dimensional with the moisture being lost from the repair layer from the top and the sides as shown in Fig. 8 (Flux boundary condition). The moisture content surface after an exposure of the patch repair for a period of 60 days is shown in Fig. 10. The moisture content in the core near the interface at the end of 60 days is about 90% whereas in the outer skin the moisture content is about 65% which is the environmental moisture content assumed for the problem. The average moisture loss in the repair layer after 60 days is about 14.5%. Under the stipulated environmental moisture content of 65%, a potential for further loss in moisture from the core of this layer exists. A simulation for a period of one year showed that average moisture loss in the repair layer is 25.5%. The diffusive flux surface in the repair layer at 60 days is shown in Fig. 11.

The evolution of moisture loss across the depth of the repair layer at the center line at various ages is shown in Fig. 12. The moisture content decrease sharply in the outer layers of the repair layer. The moisture content at the interior point near the interface however, decrease at a very slow rate. The moisture loss history at six selected points shown in the Fig. 8 in the repair layer is shown in Fig. 13. It can be observed from these figures that there is a sharp drop in moisture content at points located close to the diffusing boundaries, whereas, the rate of moisture loss is very slow at an interior point.

SHRINKAGE STRESSES IN THE COMPOSITE SYSTEM

The normal stress (σ_{xx}) surface in the composite system at an age of 60 days is shown in Fig. 14 and the variation of stress σ_{xx} across the depth of the repair layer PMC at various ages is shown in Fig. 15. It can be observed from Fig. 15 that as the process of shrinkage commences, the region near the top boundary in the repair layer, goes into tensile stress state immediately. At the surface of the repair layer it can be seen that a tensile stress of about 2.5 MPa (362.5 psi) is generated at 1 day which increases to 3.4 MPa (493 psi) at 7 days and 4 MPa (580 psi) at 60 days. The stresses beyond this boundary layer at mid of the repair layer is very low initially, but at an age of 60 days the tensile stress at this location increases to about 1 MPa (145 psi). The tensile stress in the repair layer at the interface is about 0.75 MPa (108.75 psi) at 60 days and the corresponding stress at interface in the host concrete is about 1.5 MPa (217.5 psi) compressive. The compressive stress in the host concrete decreases nonlinearly from 1.5 MPa (217.5 psi) at the interface to zero at the base of the composite system. The state of stress σ_{xx} at 60 days is evident in Fig. 15. The normal stress σ_{yy} and the y-displacement in the composite system are shown in Figs. 16 and 17 and the shear stress state in the composite system is shown in Fig. 18.

It can be observed from these figures that tensile, peeling and shear stresses are developed at the interface of the composite system. If the tensile and peeling stresses generated at the interface exceed the allowable bond strength, it would result in cracking and delamination of the repair layer. The *y*-displacement surface shows the peeling tendency of the concrete overlay at the interface near the edges. For a crack free repair layer, it is essential that the tensile stresses developed in the repair layer are lower than tensile strength. The peeling stress developed at the interface must be lower than the bond strength to prevent a delamination of the concrete repair layer.

CONCLUSIONS

The experimental investigations conducted showed that polymer-based repair has low tensile, compressive and flexural strength and tensile modulus of elasticity as compared to polymeric fiber reinforced and self consolidating repair mortar. A low tensile modulus with a high specific creep strain would result in lower tensile stress in the polymeric patch repair. The strength and modulus of polymer-based repair mortar increases significantly when specimens are cured in air which aids in the development of a polymeric matrix. Combined experimental-finite element based approach using the Multiphysics software COMSOL for 3-D nonlinear moisture diffusion simulation of experimental drying specimen was used for establishing the non-linear moisture dependent diffusivity law for the selected repair materials. Finite element simulation of moisture diffusion and stress analysis in a composite system in which the polymeric repair overlay is placed over an existing old concrete is presented for a selected repair material. COMSOL Multiphysics provides a viable tool for simulation of moisture transport and quantification of stresses evolving with time in a patch repair layer under environmental loading.

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Table 1—Strength and modulus of repair mortars at 28 days

Repair Material	Compress. Strength MPa(psi)		Tensile Strength MPa(psi)		Flexural Strength MPa(psi)		Tensile Modulus GPa(ksi)	
	Water cured	Air cured	Water cured	Air cured	Water cured	Air cured	Water cured	Air cured
PMC	24(3480)	34(4930)	2.5(362.5)	4.0(580)	2.5(362.5)	3.5(507.5)	7.0(1015)	15(2175)
FRM	56(8120)	69(10005)	4.6(667)	5.0(725)	4.6(667)	5.0(725)	40(5800)	45(6525)
FMC	56(81200	65(9425)	3.6(522)	3.8(551)	6.5(942.5)	9.8(1421)	38(5510)	37(5365)
MCM	35(5075)	44(6380)	3.2(464)	3.2(464)	6.0(870)	7.5(1087.5)	24(3480)	30(4350)

Table 2—Expansion, shrinkage and creep in repair mortars

Repair Material	Expansion strain 25x25x285 mm (1x1x11.4 in) (με)	Shrinkage strain 25x25x285 mm (1x1x11.4 in) 60 days (με)	Shrinkage strain 40x40x300 mm (1.6x1.6x12 in) 60 days (με)	Specific creep strain 40x40x300 mm (1.6x1.6x12 in) 30 days με/MPa(με/psi)
PMC	+470	-1300	-1040	130(0.9)
FRM	+250	-1075	-825	75(0.52)
FMC	+220	-770	-580	150(1.03)
MCM	+240	-1160	-700	Not available

Table 3—Values of constants for moisture diffusivity law $D(C) = b_o \tan(b_1 C^n)$

Repair Material	b_0 $mm^2(in^2)$	b_1	n	h_f *mm/day(in/day)*
PMC	75(0.12)	1.3	10	4(0.16)
FRM	40(0.0640	1.2	20	3(0.12)
FMC	50(0.08)	1.2	20	2(0.08)
MCM	65(0.104)	1.2	10	2(0.080

Note: Diffusivity units mm^2/day (in^2/day)

Table 4—Constants and subdomain expressions for diffusivity computations for PMC

Name	Constants
h_f	4 mm/day (0.16 in/day)
c_e	65%
b_0	75 mm^2 (0.12 in^2)
b_1	1.3
n	10

Name	Subdomain Expression
D_c	$b_0*tan[b_1*(C/100)^n]$ mm^2/day (in^2/day)

Table 5—Subdomain expressions and subdomain settings for stress analysis of composite repair system

Name	**Subdomain Expression**
Dc (PMC)	b_0*tan[b_1*(c/100)^n] mm^2/day(in^2/day)
Dc (Host Conc)	0.0

Name	**Subdomain Setting –PMC repair layer**
E	15000 MPa (2175 ksi)
v	0.1
α_{hygro}	15e-6 mm/mm*c% (in/in*c%)

Name	**Subdomain Setting –Concrete layer**
E	25000 MPa (3625 ksi)
v	0.1
α_{hygro}	12e-6 mm/mm*c% (in/in*c%)