3.2 Step 2: Choose the corbel dimensions

To be able to use ACI Appendix A, ACI Sec. 11.9.1 requires a span-to-depth ratio, *a*/*d*, of less than 2. In addition, ACI Sec. 11.9.2. requires that the depth at the outside of the bearing area be at least 0.5*d*. Therefore, select a column face depth of 18 in. (457 mm) and select a depth of 10 in. (254 mm) at the free end of the corbel. The selected dimensions for the corbel are summarized in Fig. (3.2-2).

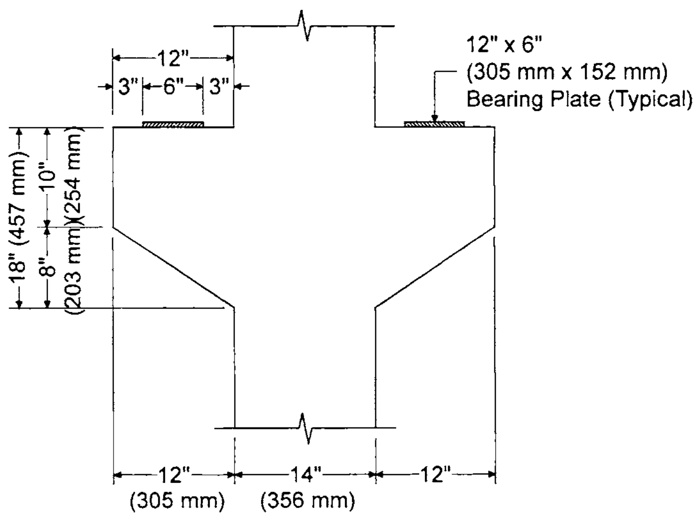

Fig. 3.2-2: Selected corbel dimensions

3.3 Step 3: Establish the strut-and-tie model

To allow for load eccentricities and erection tolerances, consider the reaction force, V_u to be placed 1 in. toward the edge of the corbel from center of bearing plate. Thus, the new position of V_u from the face of column is 3 + 6/2 + 1 = 7 in.

Fig. (3.2-3) shows the geometry of the assumed strut-and-tie model. The location of tie *AA'* is assumed to be 2 in. from the top of the corbel, considering two layers of steel bars and approximately 1 in. of concrete cover. Thus,

$d = 18 - 2 = 16$ in. (406 mm). [ACI Sec. 11.9.1]

The horizontal strut *BB'* is assumed to lie on the horizontal line passing through the sloping end of the corbel.

As shown in Fig. (3.2-3), the column axial load, P_u is resolved into two even loads acting in line with strut *CB*. The location of strut *CB* centerline can be found by calculating its strut width, w_s This width can be obtained from

$$w_s = \frac{F_{u,CB}}{\phi f_{cu} b}, \qquad \text{[ACI Secs. A.3.1 and A.2.6] (3.2-1)}$$

where $F_{u,CB}$ is the required compressive force in strut *CB*, and b = 14 in. is the out-of-plane dimension of the corbel. The strut *CB* force is
$F_{u,CB}$ = 275/2 + 61.8 = 199.3 kips. Because nodal zone *B* is an all-compression (CCC) node and strut *CB* is of prismatic type, the effective compressive strength, f_{cu}, is

$$\begin{aligned} f_{cu} &= 0.85\beta_n f_c' \\ &= 0.85(1.0)(4000) = 3400 \text{ psi.} \end{aligned} \qquad \text{[ACI Sec. A.3.2 eq. (A-3)]}$$

Substituting the above values into eq. (3.2-1) gives w_s = 5.58 in. (142 mm).

This fixes the geometry of the strut-and-tie model.

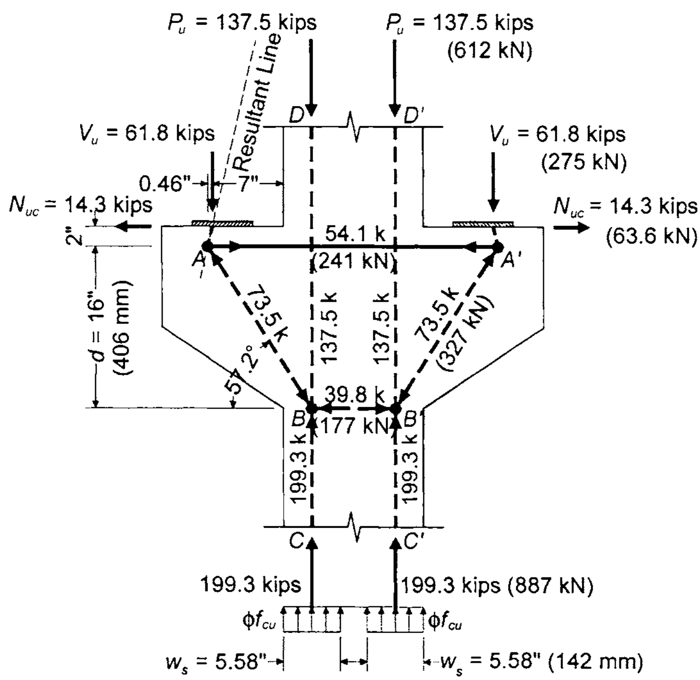

Fig. 3.2-3: Selected strut-and-tie model

3.4 Step 4: Determine the required truss forces

The required forces in all the members of the truss are determined by statics and are shown in Table (3.2-1). Note that positive sign indicates tension; negative sign indicates compression.

Table 3.2-1: Truss forces

Member	*AA'*	*AB= A'B'*	*BB'*	*CB = C'B'*	*BD = B'D'*
Force (kips)	+54.1 (+241 kN)	-73.5 (-327 kN)	-39.8 (-177 kN)	-199.3 (-887 kN)	-137.5 (-612 kN)

3.5 Step 5: Select the tie reinforcement

The required area of reinforcement for tie *AA'* is

$$\frac{F_{u,AA'}}{\phi f_y} = \frac{54.1}{0.75(60)} = 1.20\,\text{in.}^2 \qquad \text{[ACI Secs. A.2.6 and A.4.1]}$$

Further, the provided steel area must be at least

$$0.04\frac{f_c'}{f_y}bd = \frac{0.04(4)}{60}(14)(16) = 0.60\,\text{in.}^2 \qquad \text{[ACI Sec. 11.9.5]}$$

Choose 6 #4 (#13 mm) bars, $A_{st} = 6(0.20) = 1.20\,\text{in.}^2$ (774 mm^2). These bars are arranged in two layers as shown in Fig. (3.2-5).

3.6 Step 6: Design the nodal zones and check the anchorages

The width w_s of nodal zone *B* was determined in Sec. 3.3 to satisfy the stress limit on the nodal zone. Therefore, only nodal zone *A* is checked in this section.

To satisfy the stress limit of nodal zone *A*, the tie reinforcement must engage an effective depth of concrete, w_t at least equal to

$$\frac{F_{u,AA'}}{\phi f_{cu} b} = \frac{54.1(1000)}{2040(14)} = 1.89\,\text{in. (48 mm).} \qquad \text{[ACI Secs. A.2.6 and A.5.1]}$$

As shown in Fig. (3.2-4), this limit is easily satisfied because the nodal zone available is 2(2) = 4 in. (102 mm).

To anchor tie *AA'*, weld the 6 #4 bars to a steel angle of 4 in. × 4 in. × ½ in. (102 mm × 102 mm × 13 mm). The details are shown in Fig. (3.2-5).

3.7 Step 7: Check the struts

Strut *AB* will be checked based on the sizes determined by nodal zones *A* and *B*. Other struts will be checked by computing the strut widths and checked whether they will fit within the space available.

ACI defines the nominal strength of strut *AB* as

$$F_{ns} = f_{cu} A_c, \qquad \text{[ACI Sec. A.3.1 eq. (A-2)]}$$

where

$$\begin{aligned} f_{cu} &= 0.85\beta_s f_c' \\ &= 0.85(0.75)(4000) = 2550 \text{ psi} \end{aligned} \qquad \text{[ACI Sec. A.3.2 eq. (A-3)]}$$

and A_c is the smaller area at the two ends of the strut. From Fig. (3.2-4), $A_c = 14(4.88) = 68.32 \text{ in.}^2$ Thus, $F_{ns} = 2550(68.32)/1000 = 174$ kips. From Table 3.2-1, the factored load of strut *AB* is 73.5 kips (327 kN). Because this is less than the limit, i.e., ϕF_{ns} = 0.75(174) = 131 kips (583 kN), strut *AB* is adequate. Because β_s is assumed to be 0.75, minimum reinforcement has to be provided and is described in the next section.

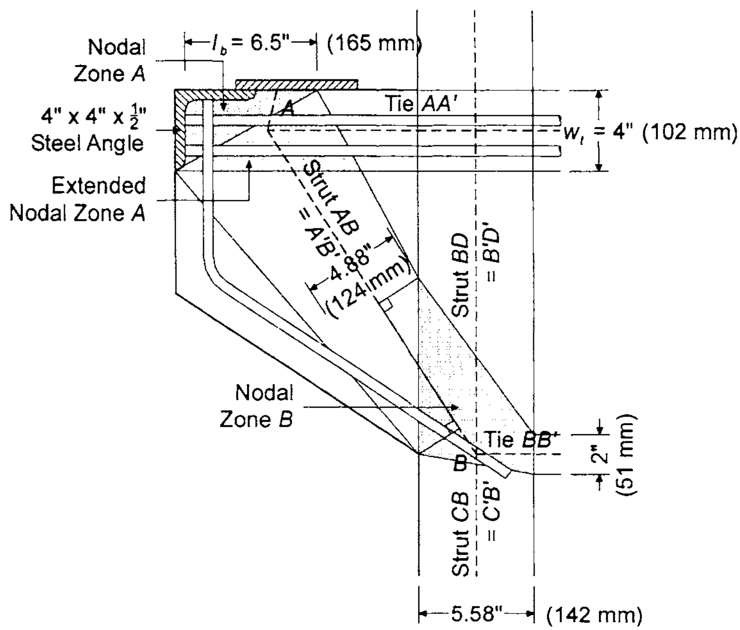

Fig. 3.2-4: Dimensions of strut-and-tie model components

The stress of the horizontal strut *BB'* is limited to

$$\begin{aligned}\phi f_{cu} &= 0.75\left(0.85\beta_s f_c^{'}\right)\\ &= 0.75[0.85(1.0)(4000)] = 2550 \text{ psi.}\end{aligned}$$ [ACI Secs. A.2.6 and A.3.2]

Hence, the required width for strut *BB'* is

$$\frac{F_{u,BB'}}{\phi f_{cu} b} = \frac{39.8(1000)}{2550(14)} = 1.11 \text{ in.}$$ [ACI Secs. A.2.6 and A.3.1]

The stress of the vertical strut *BD* is limited to

$$\begin{aligned}\phi f_{cu} &= 0.75\left(0.85\beta_s f_c^{'}\right)\\ &= 0.75[0.85(0.6)(4000)] = 1800 \text{ psi.}\end{aligned}$$ [ACI Secs. A.2.6 and A.3.2]

Hence, the required width for strut *BD* is

$$\frac{F_{u,BD}}{\phi f_{cu} b} = \frac{137.5(1000)}{1800(14)} = 5.46 \text{ in.}$$ [ACI Secs. A.2.6 and A.3.1]

Choose 2 in. (51 mm) width for strut *BB'*, and set the width of strut *BD* equals the width of strut *CB*. The required width for strut *CB* has been computed in Sec. 3.2, i.e., 5.58 in. (142 mm).

As shown in Fig. (3.2-4), all the strut widths fit into the outline of the corbel region. Thus, this solution is accepted.

3.8 Step 8: Calculate the minimum reinforcement required for crack control

ACI Sec. 11.9.4 requires closed stirrups or ties parallel to the reinforcement required for tie *AA'* to be uniformly distributed within 2/3 of the effective depth adjacent to tie *CB*, i.e., 2/3 (16) = 10.7 in. Use 10.5 in. In addition, the area of these ties must exceed

$$A_h = 0.5(A_{st} - A_n), \qquad \text{[ACI Sec. 11.9.4]}$$

where A_n is the area of reinforcement resisting the tensile force N_{uc} and $A_{st} \equiv A_s$ of ACI Sec. 11.9. Hence, the minimum area required is

$$0.5(A_{st} - A_n) = 0.5\left(A_{st} - \frac{N_{uc}}{\phi f_y}\right) = 0.5\left(1.40 - \frac{14.3}{0.85(60)}\right) = 0.56\,\text{in.}^2$$

Try 3 #3 closed stirrups, $A_v = 3(2)(0.11) = 0.66\,\text{in.}^2$, with average spacing of 10.5/3 = 3.5 in.

Because β_s equal to 0.75 is used to calculate the strength of strut *AB*, minimum reinforcement provided must also satisfy

$$\sum \frac{A_{si}}{bs_i}\sin\gamma_i \geq 0.0030, \qquad \text{[ACI Sec. A.3.3.1 eq. (A-4)]}$$

where γ_i is the angle between the axis of minimum reinforcement and the axis of strut. In this design, γ_i has to be greater than 40° per ACI Sec. A.3.3.2 because only horizontal reinforcement is provided. Based on the provided reinforcement,

$$\sum \frac{A_{si}}{bs_i}\sin\gamma_i = \frac{2(0.11)}{14(3.5)}\sin 57.2^\circ = 0.0038 > 0.0030.$$

Because this amount of reinforcement satisfies both requirements, provide 3 #3 (#10 mm) closed stirrups at 3.5 in. (89 mm) spacing, distributed over a depth of 10.5 in. (267 mm) from tie *AA'*.

3.9 Step 9: Arrange the reinforcement

The reinforcement details are shown in Fig. (3.2-5).

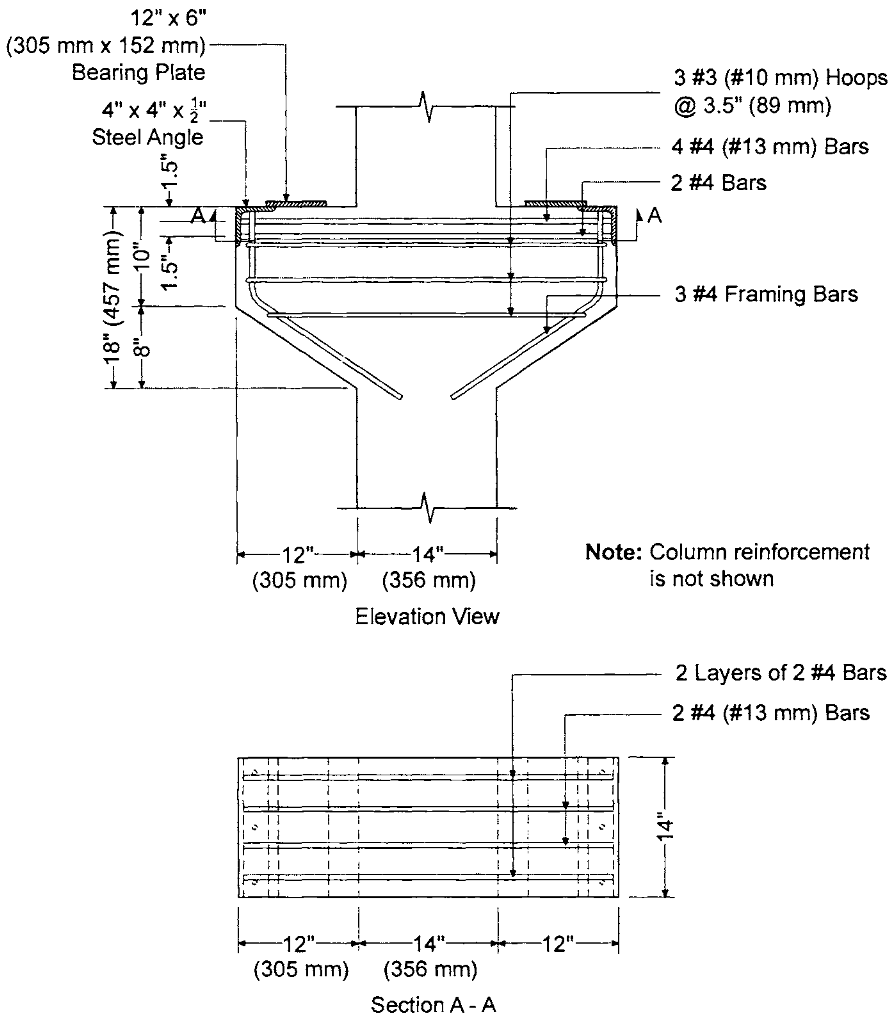

Fig. 3.2-5: Reinforcement details

4 Summary

The design of a double corbel has been presented. This design was completed using ACI 318-02 Appendix A "Strut-and-Tie Models" and the ACI 318-02 Sec. 11.9 "Special Provisions for Brackets and Corbels."

The main steps in this design involve defining the D-region and the boundary forces acting on the region, selecting a strut-and-tie model carrying the boundary forces in the D-region, solving for the member forces of the strut-and-tie model, providing reinforcement to serve as the steel ties, dimensioning the struts and nodes, and providing distributed reinforcement for crack control and ductility.

The entire corbel is the D-region because there exist statical discontinuities, i.e., the concentrated forces, and geometrical discontinuities within one section flexural depth of the corbel on either side of the discontinuity. A simple strut-and-tie model was employed in the design. This strut-and-tie model resulted in the use of 6 #4 (#13 mm) bars for the main tie. Particular attention was given to the anchorage of the main tie to ensure that it can carry the required force without having anchorage failure. To satisfy the anchorage requirements, all of the main bars are welded to a structural steel angle that is provided at each end.

References

ACI 318-02: *Building Code Requirements for Structural Concrete and Commentary*. ACI Committee 318, American Concrete Institute, Detroit, Michigan, 2002, 443 pp.

Example 4: Deep beam with opening

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Synopsis:

The example problem of a deep beam with a rectangular opening represents a strong example of the application of Strut-and-Tie modeling of reinforced concrete structures. Since the entire beam constitutes a D-region, this example demonstrates the principles and methods that can be utilized to solve a wide range of problems. Example #4 has been fully evaluated per the requirements of Appendix A of ACI 318-02.

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1 System

The deep beam with an opening (Fig. 4.1) has been designed according to Appendix A of ACI 318-02 – Strut-and-Tie Models. The system as a whole is considered a D-Region because of force and geometric discontinuity.

For simplicity, the self-weight of the structure has been accounted for by an appropriate increase in the applied point load.

Materials:

Concrete – specified compression strength of concrete

$$f'_c = 4{,}500\,psi \qquad (31\frac{N}{mm^2})$$

Steel – specified yield strength of nonprestressed reinforcement

$$f_y = 60{,}000\,psi \qquad (414\frac{N}{mm^2})$$

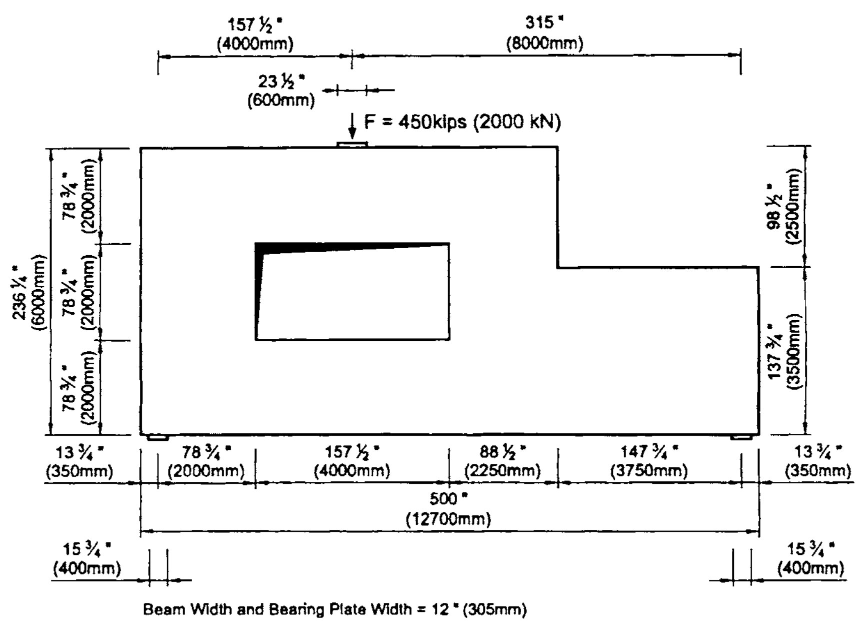

Fig. 4-1: