STATUS AND POTENTIALITIES OF NONLINEAR DESIGN OF CONCRETE FRAMES

Herbert A. Sawyer, Jr.,[1]

With discussion by Milík Tichý and Miloš Vorlíček; and Herbert A. Sawyer, Jr.

SYNOPSIS

Because structural failure generally occurs in successively more severe stages at successively less probable loads, design should ideally account for all stages and be based on comprehensive analysis utilizing a comprehensive, non-linear, force-strain relationship. The criterion for optimum design, using the failure-stage-versus-load profile, is derived. For frames, a method of comprehensive analysis based on a multilinear moment-curvature relationship, using critical moments and "plasticity factors," is presented. Procedures and the relative economics of comprehensive design and its special cases, elastic, plastic, and ultimate strength designs, are compared. A bilinear design procedure for concrete frames, based on two failure stages, is presented.

THE STAGES OF FAILURE AND COMPREHENSIVE DESIGN

Notation.—The symbols adopted for use in this paper are defined where they first appear and are listed alphabetically in Appendix II.

Although asked by the program committee to present a keynote survey paper to the Symposium, it is hardly necessary to state that the views to be presented may not coincide with those of committee members. Rather, what follows will be an effort to establish some basic concepts (flavored by personal biases) by which some of our differences may be clarified, if not reconciled.

As the loads increase on a structure, structural failure may be considered to occur in stages, each more severe than the preceding stage, and each representing a higher relative loss, L, to the owner and users of the structure. However, the probability, p, that the load corresponding to any failure stage will occur in the life of the structure decreases with increase in load. Thus,

[1] Chrm., Symposium Steering Committee; and Prof. of Civ. Engrg., Univ. of Florida, Gainesville, Fla.

a low probability tends to compensate for the high loss of a severe failure stage, and any failure stage may be the most critical in the design of the structure (1).[2]

Therefore, in the ideal, design is a comprehensive procedure by which the resistances of a structure to the various failure stages are correlated to the probabilities of the corresponding loads so that the total cost, including the first cost and the expected losses from all failure stages, is minimized. This comprehensive design procedure necessarily requires a knowledge or determination of the failure-stage-versus-load relationship for any structure assumed by the designer. This relationship must be obtained by some sort of comprehensive analysis—based on a model by which the designer may predict the behavior of the structure with respect to all possible critical failure stages.

Thus, the ultimate concern of the designer is comprehensive design. Elastic design, limit design, or ultimate strength design are but limited aspects of this concern, and so these terms have been purposely eliminated from the title of this symposium.

THE FAILURE-STAGE VERSUS LOAD RELATIONSHIP

Each load on a structure has its own characteristic failure-stage-versus-load relationship. Also, these relationships vary from structure to structure; such relationships for reinforced concrete structures generally differ from those for steel structures, and characteristic differences exist for statically determinate and indeterminate structures.

Fig. 1 shows the writer's concept of this relationship for a possibly typical load on a typical statically indeterminate reinforced concrete building, including an attempted approximation of the losses corresponding to the failure stages, and a plot of these losses versus the probability of the load in the life of the structure on logarithmic scales.

For this case, the first failure stage shown is that caused by the small creep deflections and minor tensile cracking which always occurs and which causes very small loss (i.e., slight loss in corrosion or weathering resistance). With higher load, the yield point of the reinforcement is exceeded at more and progressively longer regions, leading to wide cracking, objectionable deflections, loss of user-confidence, and need for repairs. Then, with further increase in load, the concrete strains begin to exceed the value, approximately 0.004, at which spalling will occur, and deflections become excessive, soon leading to abandonment of the building. The loss at this point will include not only the cost of the building, but the cost of wrecking and the losses from interruption of the affected businesses.

The final stages are the actual collapse of portions of the frame, followed by the limit stage of collapse of the entire frame. Both of these stages usually occur at appreciably higher loads than spalling because of: The redistribution of forces that continues after spalling, the increase of resistance of some sections after spalling, and the resistance to collapse of those elements of the building usually neglected in the frame design such as walls, floors and roofs. The loss at total collapse has been estimated so as to include

[2] Numerals in parenthesis refer to corresponding items in the Appendix.

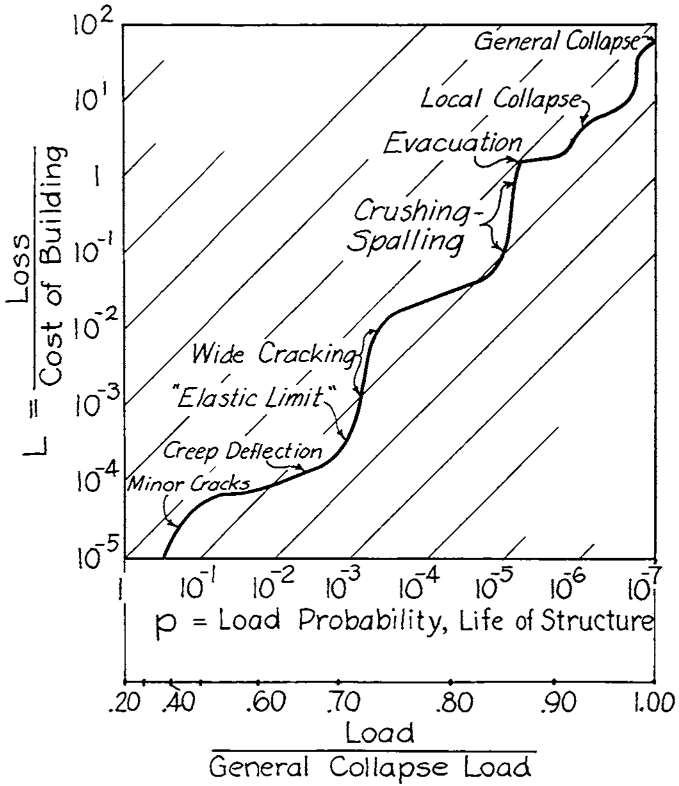
General Collapse
Local Collapse
Evacuation
Crushing-
Spalling
Wide Cracking
"Elastic Limit"
Creep Deflection
Minor Cracks
L = Loss / Cost of Building
10^2
10^1
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
p = Load Probability, Life of Structure
.20
.40
.60
.70
.80
.90
1.00
Load / General Collapse Load

FIG. 1

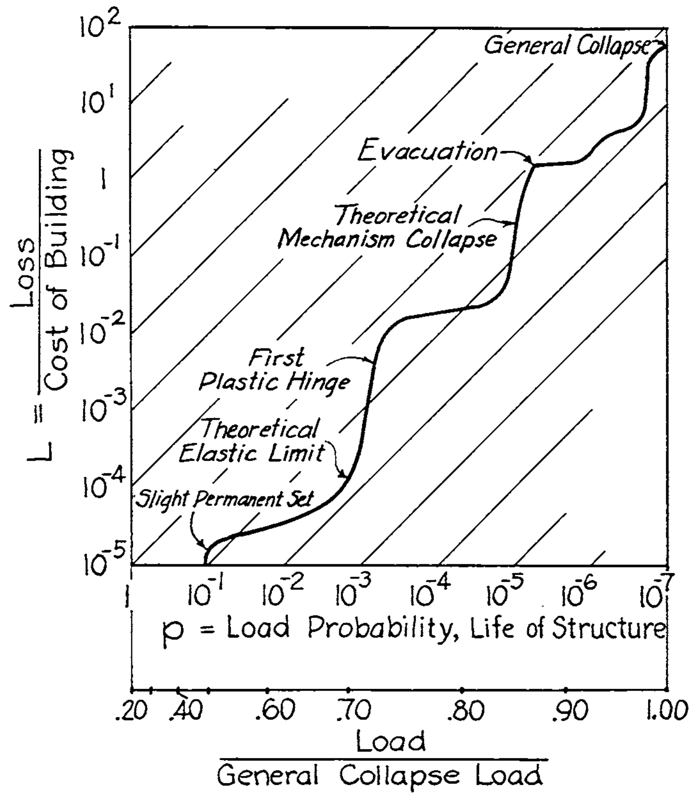
General Collapse
Evacuation
Theoretical
Mechanism Collapse
First
Plastic Hinge
Theoretical
Elastic Limit
Slight Permanent Set
L = Loss / Cost of Building
10^2
10^1
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
p = Load Probability, Life of Structure
.20
.40
.60
.70
.80
.90
1.00
Load / General Collapse Load

FIG. 2

the losses from destruction of the tools and records of the businesses and the loss from a 0.5 probability of loss of life of the occupants of the building. Evaluating the loss of a life is difficult and even controversial, and the loss has here been assumed at twice the total future earnings of the occupant, with one occupant assumed for each 250 sq ft of an average-cost building.

A comparable and somewhat typical relationship for a building with continuous steel framing is shown in Fig. 2. Here, the successive failure stages for successively higher loads involve very small, and then larger, amounts of permanent set and deflection, with abandonment of the building following soon after the formation of sufficient hinges for the theoretical collapse mechanism, and with actual collapse occurring at an appreciably higher load because of the strength reserves from strain hardening, the moment gradients at the hinges, and the elements of the building neglected in the frame design.

A significant feature of relationships similar to those of Figs. 1 and 2 is that the relative total expected loss in the life of the building from all failure stages for the particular loading is closely given by the area under the curve:

$$\frac{\text{Total Expected Loss}}{\text{Cost of Building}} = \int_{p=0}^{p=1} L\,dp = \int_{L=0}^{L=L_c} p\,dL \quad \ldots\ldots (1)$$

in which L_c is the value of L at collapse. Mathematically, the error in this relationship becomes negligible for low probabilities, and this error may be neglected for probabilities of magnitude corresponding to the failure stages shown.

Because both scales of Figs. 1 and 2 are logarithmic, the areas of these plots are of variable density. The parallel lines inclined at 45° shown on these figures are contours of constant density, with each successive line indicating a change in density by a factor of ten. Thus, the expected loss is very sensitive to horizontal displacement of any "hump" on the curve representing a failure stage on the curve, and the horizontal placement of these humps is the prerogative of the structural designer.

Two goals of comprehensive design, as distinguished from design for one failure stage, may now be stated in terms of the loss-failure-load relationship. The major objective is the optimum horizontal placement of the failure-stage profile. A minor objective is the adjustment of this profile within the rather severe limitations imposed by statics and other design requirements, toward the optimum shape, for which the failure stages are located close to a common 45°-inclined line.

OPTIMUM PLACEMENT OF FAILURE-STAGE PROFILE

The optimum horizontal placement of the failure-stage profile will be considered first. For an infinitesimal increase in strength or load factor of the structure all failure stages will occur at an infinitesimally higher load with corresponding decreases in probability of occurrence in the life of the structure, dp. That is, each point of the curves of Figs. 1 or 2 will move to the right a distance dp. (The scale of load relative to true collapse load will move likewise.) The corresponding change in expected loss will be the change in area under the curve of the figure plus the changes in areas of all other similar curves representing all other modes of failure and loading:

$$\frac{d\text{ (expected loss)}}{C_T} = \sum^{\text{All Modes}} \int_{L_1}^{Lc} (dp)\, dL$$

in which C_T is total cost of the building, and L_1 is the L-axis intercept (a more significant value of L_1 will be established later). Because, mathematically

$$dp = p\, d(\log_e p)$$

the change in expected loss becomes:

$$\frac{d\text{(expected loss)}}{C_T} = \sum^{\text{All Modes}} \int_{L_1}^{L_c} d(\log_e p)\; p\, dL \;. . . (2)$$

The optimum or most economical horizontal position will occur when the sum of the change in expected loss and the change in the cost of the building is zero. The change in the cost of the building may be obtained from the curve of frame cost versus collapse load of which Fig. 3 is an example. Obviously,

$$\frac{d\text{ (cost frame)}}{C_T} = \frac{\partial\left(\frac{C_F}{C_T}\right)}{\partial(\log_{10} p_c)}\, d(\log_{10} p_c)$$

$$= 0.434\, \frac{\partial\left(\frac{C_F}{C_T}\right)}{\partial(\log_{10} p_c)}\, d(\log_e p_e) \;. . . . . (3)$$

Adding Eqs. 2 and 3 and equating to zero

$$0.434\, \frac{\partial\left(\frac{C_F}{C_T}\right)}{\partial(\log_{10} p_c)}\, d\,(\log_e p_c)$$

$$+ \sum^{\text{All Modes}} \int_{L_1}^{L_c} d(\log_e p)\; p\, dL = 0 \;. . . . . (4)$$

In general, the horizontal movements dp, actually d(ln p), of the points on a curve such as Figs. 1 or 2 will not be equal. Instead, the increase in load for any given failure condition will tend to be proportional to the load. Furthermore, the load scale corresponding to the logarithmic probability scale will likely be somewhat irregular and nonlinear, as shown in Figs. 1 and 2. The value of load corresponding to a probability of unity will usually be the dead load; the value of load corresponding to a probability of 10^{-1} will usually be greater than the service load; and the collapse load probability is usually not less than 10^{-6}. Therefore, the load-scale tends to expand from left to right.

Both of these factors cause the values of d(ln p) for $1 > p > 10^{-1}$ or even $1 > p > 10^{-2}$ to be much smaller than the values for the remainder of the scale. Therefore, a good approximation of the integral of Eq. 4 may be obtained by ignoring values of L equal to or less than those for the lowest plateaus of Figs. 1 and 2, and the limit L_1 may then be redefined as the value of L at the top of this lowest plateau, roughly corresponding to the theoretical elastic limit for either steel or reinforced concrete.

A second approximation, involving more error than the one considered above, but nevertheless potentially useful, now becomes apparent. Between L_c and the redefined L_1, corresponding to values of p on Figs. 1 and 2 between approximately 10^{-7} and 10^{-3}, the load scale is almost linear. If it is assumed that for this range the value of $d(\log_e p)$ is constant, Eq. 4 becomes

$$-0.434 \frac{\partial\left(\frac{C_F}{C_T}\right)}{\partial(\log_{10} p_c)} = \sum_{\substack{\text{All}\\\text{Modes}}} \int_{L_1}^{L_c} p \, dL \quad \ldots\ldots\ldots(5)$$

This basic relationship may be written thus: For optimum placement of the failure-stage profile, Fig. 1, the slope of the frame-cost versus collapse-load-probability curve, Fig. 3, is numerically equal to the expected loss from all modes of failure for L exceeding the redefined L_1.

Eq. 5 has more potential significance in the quantitative, rational determination of load factors than is indicated by the rigor of some of its assumptions because of the extremely large changes in probability corresponding to small changes in load.

VARIATION IN SHAPE OF PROFILE

The shape of the failure stage profile is subject to great variation from causes that may or may not be controllable by the designer. At one extreme there is the practically vertical line profile corresponding to an elastic-range instability, fortunately rare, and at the other extreme, there is the structure with broad steps for successive failure stages representing great reserves of strength beyond the elastic limit.

Factors in this variation over which the designer has some control are: (a) The configuration of the cross section of members—for reinforced concrete the choice of tee-beam, flat slab, or folded plate is significant; (b) the ductility of the materials in the members, even considering only reinforced concrete, the choice is first whether or not to prestress; the designer then knows that ductility varies inversely with the net reinforcing index, q - q', over a wide range; and (c) finally, the interaction with flexure of "secondary" effects, often critical, including shear, bond, and the participation of floors, walls, and roofs in flexural action, over which the designer may exert limited control.

However, the designer has very little control over other important causes of variation, most important of which are the configuration of the loads and

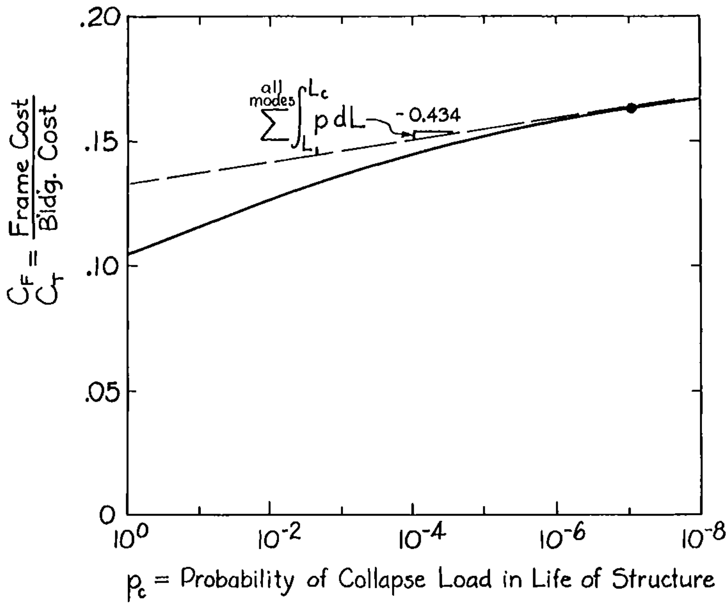

FIG. 3.—FRAME COST VS. COLLAPSE LOAD

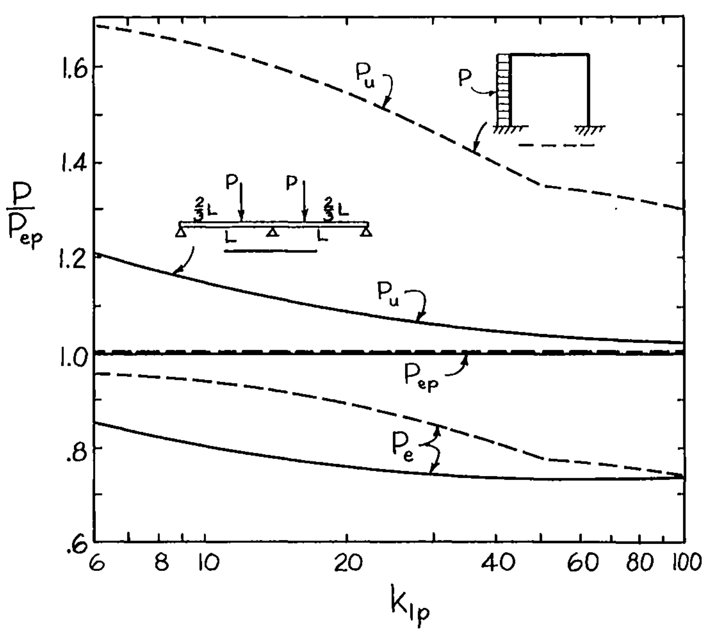

FIG. 4.—VARIATION IN FAILURE-STAGE LOADS WITH DUCTILITY AND STRUCTURE-LOAD CONFIGURATION

related configuration of the axes of members placed to support these loads. An example of variation from these causes is shown in Fig. 4, which illustrates the theoretical relative load levels for the two statically indeterminate structure-load configurations shown over a range of ductility indicated by the "plasticity factors," k_1p, characteristic of bilinear moment-curvature relationships for reinforced concrete (defined in Fig. 8). The load levels are those given by an elastic distribution of moments, P_e, an inelastic distribution of moments limited by an ultimate compressive concrete strain of 0.0038, P_{ep}, and a complete rigid-plastic redistribution distribution of moments, P_u. These levels represent approximately a wide-cracking, a crushing-spalling, and a collapse failure stage, respectively (the last neglecting the restrictions of limited ductility on a full redistribution of moments). Variations in maximum moment at a section for these failure stages have been neglected; therefore, the variations in strength shown are only those caused by differing redistributions of moment. Obviously, the three curves for any statically determinate structure would all coincide on the straight line, $P/P_{ep} = 1$.

Thus, Fig. 4 shows, for only three parameters (configuration of structure and load and ductility) of the several mentioned: First, a wide variation in horizontal spread in failure stage profile (vertical spread on Fig. 4), and second, a wide variation in relative position of the failure stages with respect to each other.

TRADITIONAL VERSUS COMPREHENSIVE DESIGN

Whereas Eq. 5 determines the optimum horizontal placement of any failure-stage profile regardless of its shape or spread, the traditional design procedure of the past has been based on a single failure stage, and placement of the entire profile has depended on a fixed (constant load factor) placement of one hump of the profile. The results for elastic moment design are shown in Fig. 5. In order to assure a permissible horizontal position for the collapse hump for any possible structure (here the statically-determinate structure profile is usually critical), the design loads or load factor for the elastic analysis must be set very high, resulting in overly safe and uneconomical designs for those structures and loadings with the highest reserves of strength from redistribution of moments.

As Fig. 6 shows, a similar situation exists for the plastic design of steel, except that control of an intermediate hump probably provides slightly better control over the humps at either extreme. Here, to insure proper position of permanent-set and over-all collapse humps, the loads for mechanism collapse (probably closest to actual local collapse loads) must be set uneconomically high.

Thus, the primary advantage of comprehensive design is that it leads to the use of lower, more economical, load factors.

COMPREHENSIVE ANALYSIS

Of course, the prerequisite to the optimum placement of the load-failure-load profile is the determination of this profile through a comprehensive analysis, and the solution to this problem will now be considered.

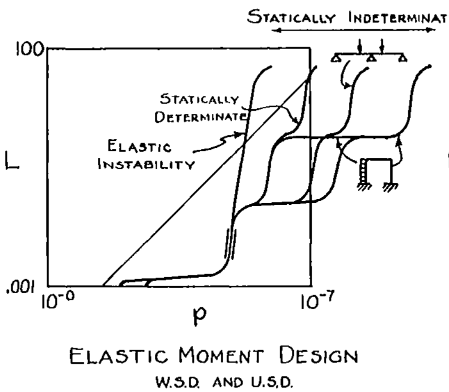

FIG. 5

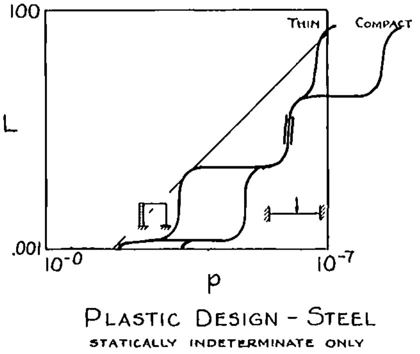

FIG. 6

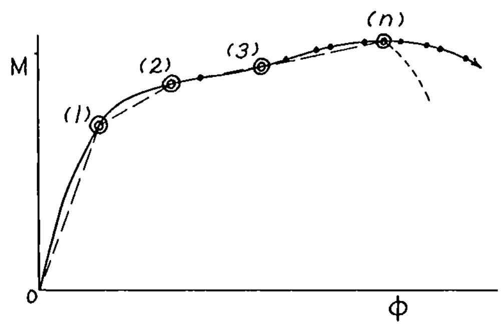

FIG. 7.—TYPICAL M-Ø RELATIONSHIP, REINFORCED CONCRETE

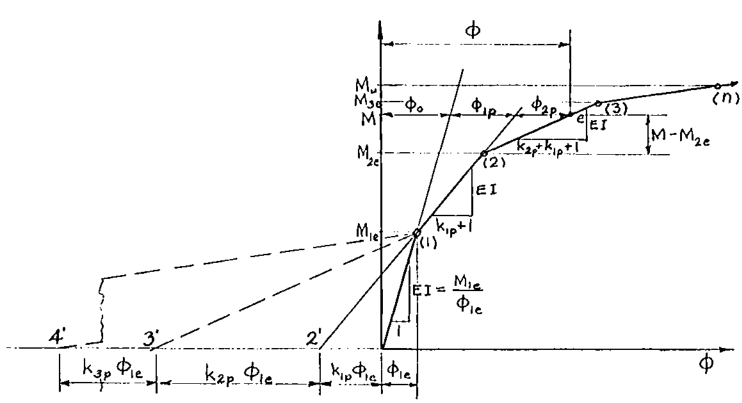

FIG. 8.—MULTI-LINEAR M-Ø RELATIONSHIP

For analytical purposes, the basic interrelationship between behavior and failure-stages at a section and for the entire structure is best defined and accounted for through the moment-curvature relationship for the section and the curvature diagram for the structure.

Fig. 7 shows a typical moment-curvature (M- ϕ) relationship for a member at one of its cross sections. All failure stages associated with a limiting value of compressive or tensile strain (cracking, permanent set, crushing-spalling) can be represented by reasonably definite points (1), (2), (3) . . ., on this relationship. However, the criterion for both local collapse and general collapse is not a limiting strain, but the condition that the derivative of load on the structure with respect to deflection is zero: $\partial W/\partial \Delta = 0$. Therefore, referring to Fig. 7, collapse does not occur at a particular moment on any M- ϕ (or M- θ) relationship. Rather, it occurs when roughly half of the moments at the inelastic or "hinging" regions are on the ascending portions of their respective M-ϕ curves to the left of the point of maximum moment, and the remainder are on the descending portions to the right of the maximum point, all as shown schematically by the black dots of Fig. 7.

To the writer's knowledge, no serious attempt at accurate evaluation of the corresponding "true" collapse load has been made, perhaps more because of the inherent uncertainties and errors in determining M-ϕ relationships and in translating them into M-rotation relationships for the descending range (2) than because of the analytical complexities. Instead, two approximate, simpler solutions have been proposed with accuracy perhaps more compatible with the inherent uncertainties.

One solution, by far the better known, is the nondeformational, rigid-plastic solution equivalent to the assumption that the maximum moments occur at all sections simultaneously. Obviously, this assumption always theoretically results in an upper bound solution, but in practice, this solution is often conservative because of the conservative values usually assumed for ultimate moment (3) and the neglect of nonframe portions of the structure which also resist collapse. However, if, as sometimes occurs with reinforced concrete, a sudden fracture at maximum moment causes a precipitous loss in strength such as that shown by the dotted line of Fig. 7, the moments at practically all other inelastic (often called "hinging") regions will be on the ascending curve, and the upper bound solution may be grossly high. (In connection with the rigid-plastic solution it is suggested that the terms "hinge" and "lower bound," although useful for the plastic theory involving a perfectly plastic material, can be dangerously misleading and should be avoided for a material of limited ductility such as reinforced concrete. The concept of a hinge, involving indefinite rotation capacity at constant moment, does not apply with any precision to reinforced concrete. And the "lower bound" for a perfectly plastic material is not at all the lower bound for any material of limited ductility; in fact, for such a material, a statically admissible "lower bound" may far exceed the actual ultimate strength.)

The second approximate solution involves the assumption that the collapse load is adequately given by a deformational analysis, using M-ϕ relationships, with no deformation or curvature to exceed the maximum-moment curvature at any section. Thus, for this solution, no moments would be on the descending portion of the M-ϕ relationships. Obviously, this solution would always be a lower bound solution, even for the case of sudden fracture,