consequence of three effects which the theory did not consider. The first is the Bauschinger effect. The second is the effect of crushing at the beginning of reversal. These two are better explained in Fig. 19 later. These take care of the first half of the diagram, and the deviation in the last half should be the result of the third effect: the effect of previous loading in the same direction.

Comparison of Measured and Computed Strains.—In this section comparisons of measured and computed strains at the levels of steel are given. This will further clarify the behavior of the specimens under reversal and strengthen the findings in the previous section.

Computed strains were taken from the output of the analysis, and measured concrete strains were averaged for all gage lengths in the level of the

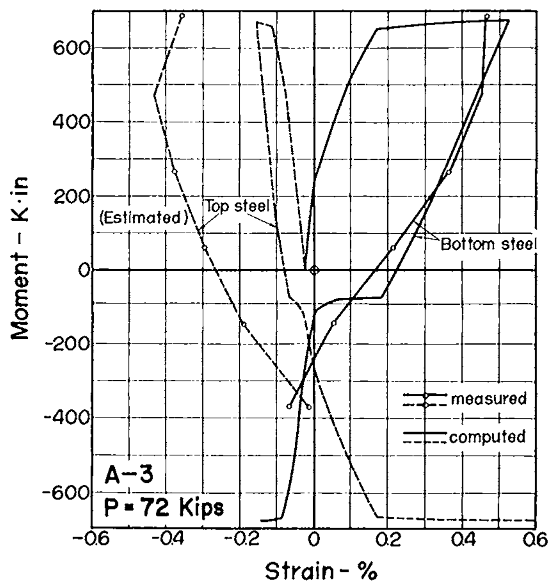

FIG. 19.—MOMENT VS. STRAIN FOR SPECIMEN A-3, P=72 Kips

reinforcement, which gave the average steel strain over the entire constant moment region. Strains in both the top and bottom bars were considered.

Figs. 17, 18 and 19 show the comparison of strains with respect to moment for the second reversal. Since the moment is based on the stresses in the section, this comparison is essentially a check of stress-strain relationship of the steel. The comparison for the first reversal was also made. However this is not included here because it merely indicates the agreement between measured and computed values and the validity of the method of analysis developed previously.

In Figs. 17, 18 and 19 the strain in the top steel is represented by the full lines, and the strain in the bottom steel by the dashed lines. The measured values for the second reversal were inverted, so that direct comparison with computed values could be made.

Fig. 17 indicates clearly that the Bauschinger effect governs the behavior of steel, hence, the behavior of the specimen. It also indicates crack closing by the reversal of strain.

Fig. 18 illustrates two reasons of deviation from the computed values: the Bauschinger effect in the first half of loading and the effect of previous loading in the same direction in the last half of loading. The Bauschinger effect reduces the yield stress, hence the moment. The effect of previous loading is to increase the compressive strain. In addition, there is the effect of lateral bending in this case which shifts the measured value to the compression side.

Fig. 19 shows some interesting points which Fig. 16 did not indicate. In this case the reversal of loading was made after crushing of concrete, whereas the computed curves correspond to reversal from just before crushing. After crushing, the steel strain in tension did not increase very much, and the moment-strain relationship of this bar under reversal is thus not much different from the computed value except for some Bauschinger effect. However, since the steel strain in compression is very large after crushing, the moment-strain relationship is far from the computed values which did not consider crushing. This explains the deviation in the first half of moment-curvature diagram in Fig. 16. The effect of previous loading in the last half cannot be studied in this figure because of the lack of test data. Incidentally, steel strains in compression could not be measured due to concrete crushing, and were estimated from measured strains in tension and measured curvatures.

SUMMARY AND CONCLUSIONS

Behavior.—Since the specimens were subjected to two-point transverse loading with a long constant moment region, every section in that region seemed to have same probability to crack. However, in actual practice cracks formed quite regularly at about 6-in. intervals corresponding to the location of ties. Cracks from the both sides of the specimens due to load cycles tended to meet at mid-height of the section. For the specimen without axial load, it was noticed that the cracks occasionally went through the entire section.

Strain gages had been mounted at several locations on the bars between ties, where no cracks formed. The uncracked concrete around the reinforcement restricted the stretching of steel, and the measured steel strain after yielding could not be used as an effective measure of the behavior of the specimens. Mechanical strain readings were taken extensively on the concrete surface using 6-in. gage lengths. The readings at the level of the reinforcement were believed to be the average strain of the steel in the particular gage length. They gave a very good measure of the behavior of the specimen.

The load-carrying capacity for a specimen under initial and first reversed loadings was in good agreement with computed value. Under the later cycles it showed some decrease, which was again what the analysis had predicted, except for Specimen A-3 which lost a considerable portion of its load-carrying capacity by concrete crushing.

The measured load-deflection or moment-curvature relationships showed a remarkable effect of axial load on the behavior under reversal of bending.

When there was no axial load, the hysteresis loop was more or less spindle-shaped. When an axial load of 36 kips was applied, which was about one quarter of the balance load, the hysteresis curve was constricted at mid-height, although this phenomenon was not clearly observed in later cycles. For the specimen subjected to an axial load of 72 kips, which was about 55 percent of the balance load, the hysteresis curve was again spindle-shaped without any significant pinching.

Analysis.—The analysis was based on three assumptions: a linear strain distribution over the section; elasto-plastic stress-strain relationship of steel neglecting the Bauschinger effect; elasto-plastic stress-strain relationship of concrete replacing the real curve and neglecting any stress in tension.

The effects of variables were studied using this analysis. Reinforcement ratio and concrete strength had minor effects on the shape of moment-curvature diagram. The amount of axial load and the plastic deformation under previous loading made drastic changes. When the member had no axial load, the diagram had a shape like a parallelogram and larger the plastic deformation, the wider the diagram was in the horizontal direction. When the amount of axial load was small, the moment-curvature diagram had a step in the way of reversal, and the larger plastic deformation made this step larger. When the axial load was higher, this step became smaller and for higher axial load it vanished completely, making a spindle-shaped diagram.

When the outcome of the analysis was compared with test results, it was found that the agreement was almost perfect for initial loading and for the first reversal which was made slightly after the yielding in the initial loading. The only discrepancy found was that the stiffness at the moment close to yield was overestimated. It could be improved by a more realistic assumption regarding the stress-strain curve of concrete. However, the agreement was not very good for reversal after the first reversal from a point considerably beyond the yielding. The discrepancies are best seen in Figs. 14, 15 and 16. Study of this discrepancy showed that it was the result of several reasons. One was that the Bauschinger effect was not considered in the second assumption. Another was that the analysis for the second cycle was not made, whereas the actual behavior under the second reversal was slightly different from the first reversal. Still another was that the analysis beyond crushing was not made, whereas one of the actual specimens had crushing before second reversal. If these are all taken into account in the analysis, the agreement should be much better.

DISCUSSION

KURT H. GERSTLE,[8] AND LEONARD G. TULIN,[9]—This discussion concerns itself mainly with the assumption made by the author regarding the

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stress-strain relations of the component materials. Computations and tests similar to those reported in Aoyama's paper have been performed on beams in pure bending at the University of Colorado. Particular attention was paid to the question of the validity of the simplified bi-linear stress-strain diagram to represent the response of concrete, and the neglect of the Bauschinger Effect in steel under reversed loading.

Fig. 20 shows an exact stress-strain curve obtained from repeated loading tests on 3 in. x 6 in. cylinders,[10] along with a bi-linear approximate curve similar to the one used by Aoyama. Fig. 21 shows[11] three moment-curvature curves for a singly reinforced beam section, of steel ratio .0219, in pure bending. The solid curve represents the experimental results, the dashed curve is an analytical curve (obtained by computer methods similar to Aoyama's) based on the exact, curvilinear stress-strain curves of the concrete, and the dash-dot curve is an analytical curve based on the bi-linear approximation. We note that in these simply reinforced beams the Bauschinger Effect plays no role.

A comparison of these results shows that use of the bi-linear approximation furnishes excellent results. It should also be noted that due to the presence of a strain gradient in the beam, the rotation capacity of the beam is considerably greater than is to be expected on the basis of concrete cylinder strength; that is, of course, a well-known phenomenon.

Fig. 22 shows experimental and analytical moment-curvature diagrams for a doubly reinforced beam (of steel ratio $p=p'=.0079$) under reversed pure bending. The analytical curve was developed using the elastic-perfectly plastic idealization for the steel response shown in Fig. 6a, and the obvious discrepancy was also attributed to the neglect of the Bauschinger Effect.

To explore this further, a basic investigation in the mechanical properties of reinforcing steel was undertaken;[12] machined specimens were subjected to reversed axial loading, and stress-strain curves such as the one shown in Fig. 23 were obtained. The results of many such tests were compiled and an analytical non-linear expression derived which describes the actual steel response with reasonable accuracy.

The non-linear analytical stress-strain relation describing the Bauschinger Effect under reversed loading was incorporated in the beam theory and analytical curves such as the dotted one of Fig. 24 was obtained. It can be seen that the trend of the test results can be predicted by use of the more exact steel stress-strain relation.

The gist of this discussion is summarized in the following:

Reasonable results for under-reinforced beams under reversed loading can be obtained using the bi-linear approximation for the response of the

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[10] Agarwal, G. R., Tulin, L. G., and Gerstle, K. H., "The Response of Doubly Reinforced Concrete Beams to Cyclic Loading," Journal of the American Concrete Institute (submitted for publication).

[11] Sinha, B. P., Gerstle, K. H., and Tulin, L. G., "Response of Singly Reinforced Beams to Cyclic Loading," Journal of the American Concrete Institute, Title No. 6-156, August, 1964, p. 1021.

[12] Singh, Awtan, Gerstle, K. H., and Tulin, L. G., "The Behavior of Reinforcing Steel Under Reversed Loading," Journal of the American Society for Testing Materials, (to be published in Materials and Standards).

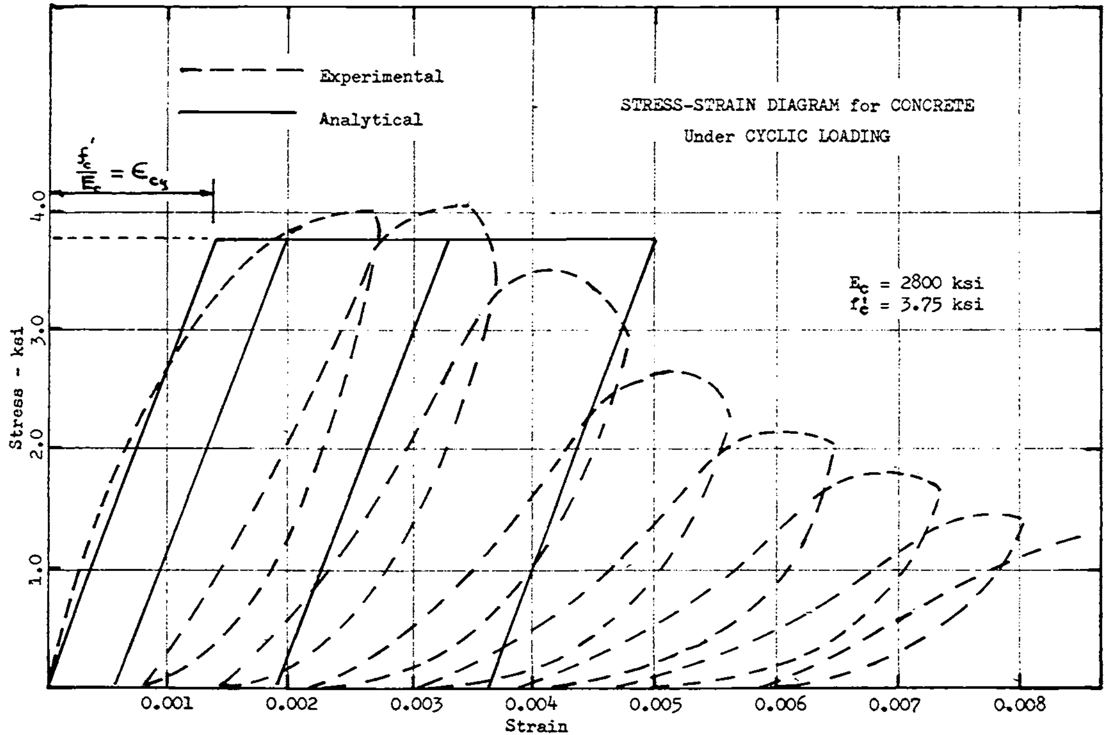
Experimental
Analytical
STRESS-STRAIN DIAGRAM for CONCRETE
Under CYCLIC LOADING
$\frac{f_c'}{E_c} = \epsilon_{cy}$
E_c = 2800 ksi
f_c' = 3.75 ksi
Stress - ksi
1.0
2.0
3.0
4.0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
Strain

FIG. 20

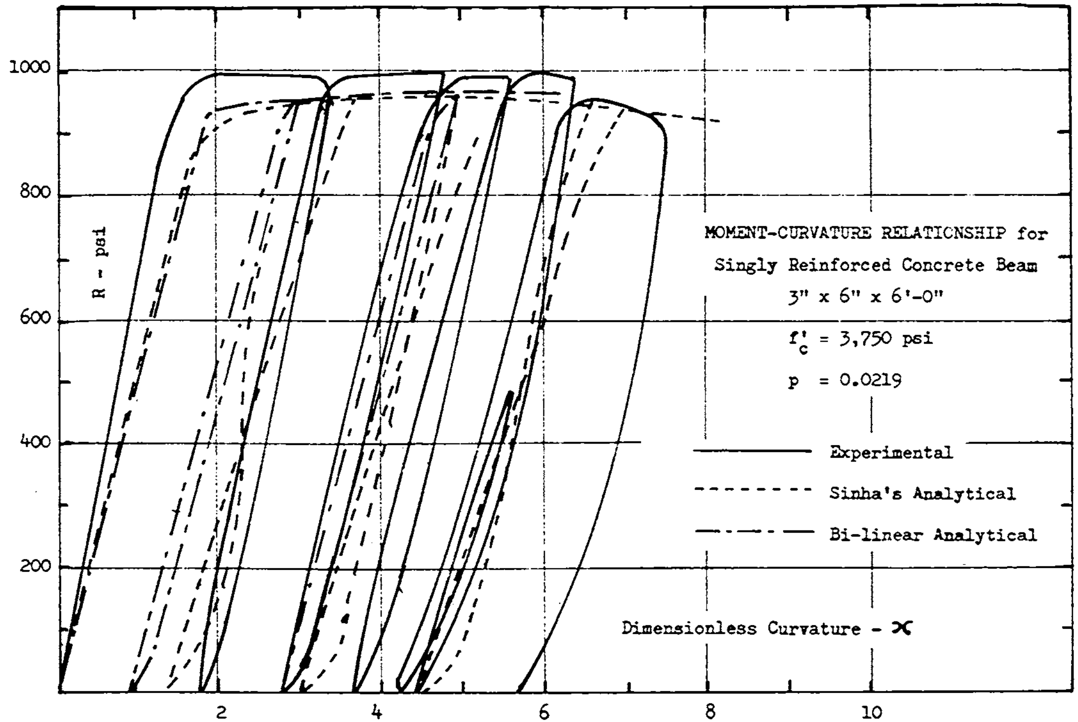
MOMENT-CURVATURE RELATIONSHIP for
Singly Reinforced Concrete Beam
3" x 6" x 6'-0"
f'c = 3,750 psi
p = 0.0219
Experimental
Sinha's Analytical
Bi-linear Analytical
Dimensionless Curvature - ϰ
R - psi
1000
800
600
400
200
2
4
6
8
10

FIG. 21

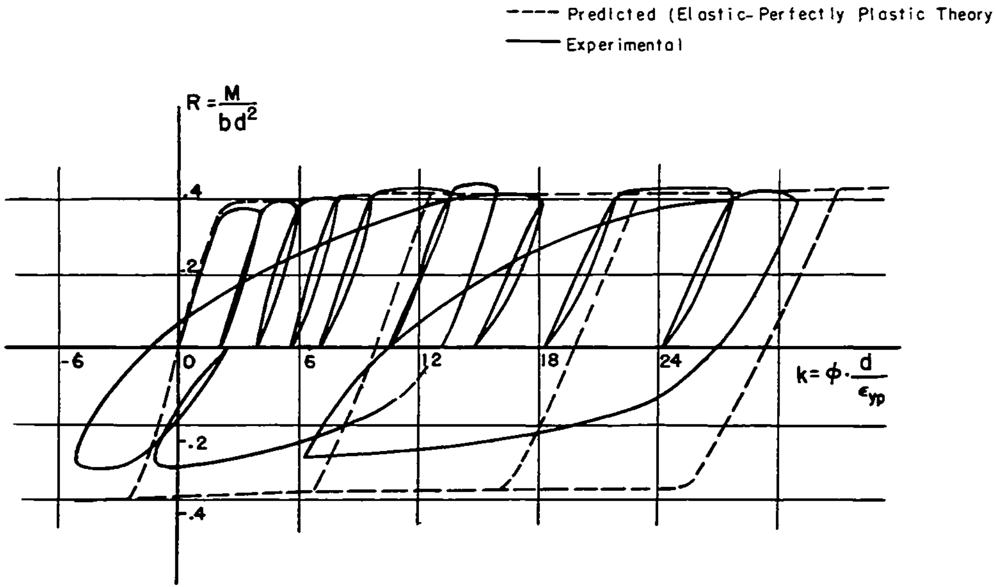
Predicted (Elastic-Perfectly Plastic Theory)
Experimental
$R = \frac{M}{bd^2}$
.4
.2
-.2
-.4
-6
0
6
12
18
24
$k = \phi \cdot \frac{d}{\epsilon_{yp}}$

FIG. 22

concrete, but the steel behavior must include a description of the Bauschinger Effect.

HIROYUKI AOYAMA.—The writer appreciates the valuable contribution made by Gerstle and Tulin. He certainly agrees with their opinion that the Bauschinger effect of steel should be incorporated in the analysis. However, this alone cannot make a satisfactory improvement. As was stated in the paper, the discrepancy shown in Figs. 14 to 16 was the results of relatively simple assumptions and limited scope of the work. The possible improvement was suggested in the paper.

The main difficulties for considering the Bauschinger effect are: first, that the general stress-strain relationship for steel is not yet formulated,

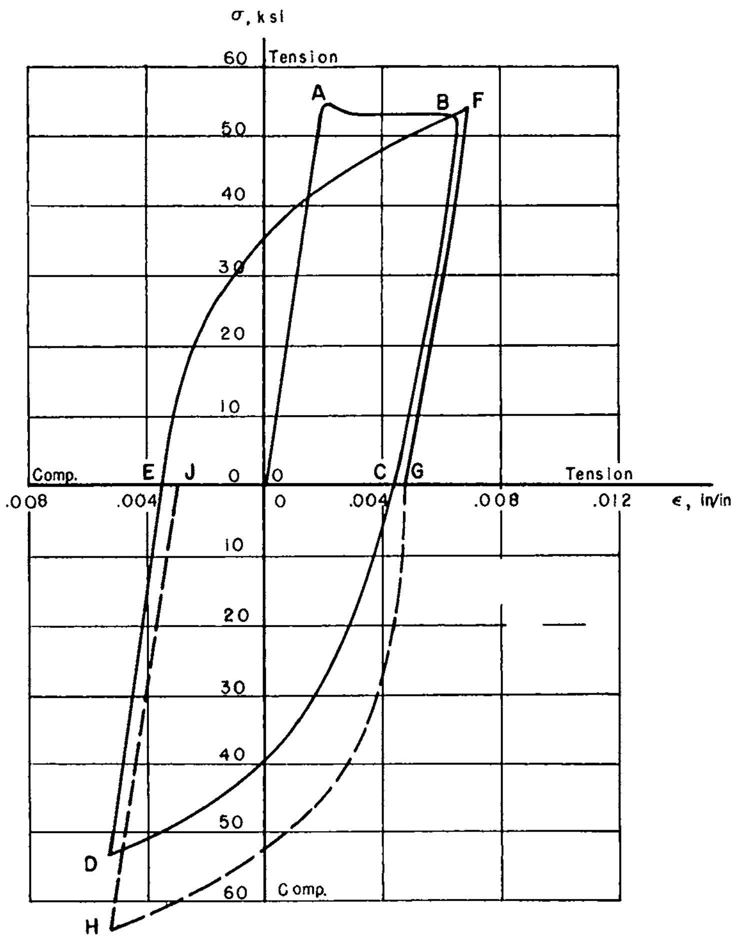

FIG. 23

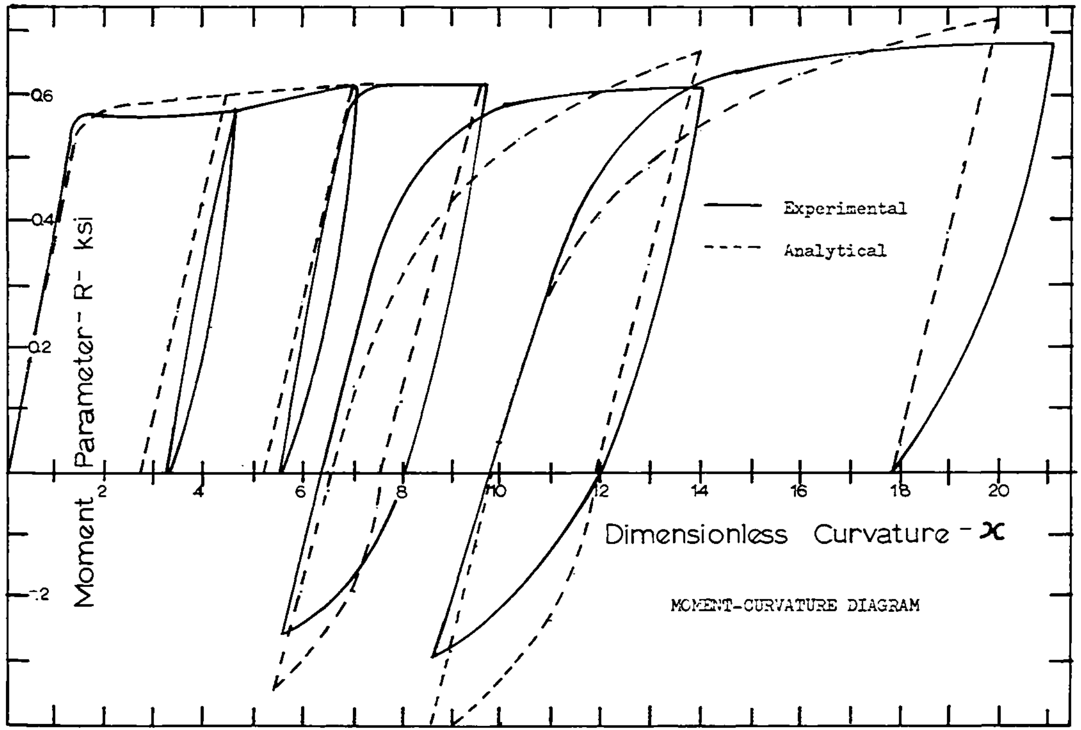
0.6
0.4
0.2
-.2
Moment Parameter - R - ksi
2
4
6
8
10
12
14
16
18
20
Dimensionless Curvature - x
Experimental
Analytical
MOMENT-CURVATURE DIAGRAM

FIG. 24

although there are some experimental data available;[13,14] and second, that it may be necessary to consider the strain concentration of steel at a cracked section, because the Bauschinger effect, being a function of the plastic residual strain, will be dictated by the peak plastic strain at the cracked section rather than the average strain of the reinforcement.

The work is being extended at the University of Tokyo to include not only the Bauschinger effect but also the general stress-strain relation for concrete, and to consider several consecutive loading cycles.

The writer would like to point out that the analysis should be extended further to include the moment vs. rotation and ultimately the load vs. deflection relation of a given structure. Combining this with records of earthquake motions it would be possible to obtain an accurate estimate of the response of a reinforced concrete structure to earthquakes.

In this connection, V. Bertero of the University of California made a very interesting statement in the free discussion. He stated that the results obtained in his test were similar to those presented by the writer and Gerstle and Tulin. From analysis he found that the drop in stiffness due to reversal of loading was considerably larger than that predicted even in the case where the Bauschinger effect was included. In the response of concrete framed structures subjected to reversal of loading, he stated, a sudden drop in stiffness might occur due principally to the presence of shear force. Under load reversals we can have cracking at critical regions through the entire section, and shear at such a section may lead to longitudinal splitting of concrete along the reinforcement.

The writer's work was primarily concerned with the flexure, and his test specimens had no shear in the measuring region. Therefore the effect of shear was not studied, nor inferred, in the work. When the analysis is extended to load vs. deflection relation of a framed structure, however, this can be a very important factor to be considered. Even in case of beams subjected to concentrate load at midspan, Burns, Yamashiro and others had much difficulty in interpreting the behavior. In case of framed structure subjected to reversal of horizontal loads, the writer would like to introduce a still more serious problem, the shear in the beam-column connection, usually referred to as "panel zone." According to the anti-symmetric moment distribution, the shear several times greater than the shear in the framing members will be induced in the beam-column connection. Some pilot tests to explore the effect of this shear indicated[15] that the failure of the frame can result by the diagonal failure in the panel zone.

As far as the stiffness is concerned, the writer believes that the effect of shear is not very important from practical point of view, provided that no major diagonal crack occurs.

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[13] Dubuc, J., "Plastic Fatigue Under Cyclic Stress and Cyclic Strain with a Study of the Bauschinger Effect," thesis presented to the University of Montreal, at Montreal, Quebec, Canada, in 1961, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

[14] Udoguchi, T., and Okamura, H., "On the Mechanism of the Bauschinger Strain Part 1 & 2)," 5th and 6th Japan Congress on Testing Materials, 1962 and 1963.

[15] Umemura, H., and Ikeda, A., "Study on Improvement of Structural Ability of Reinforced Concrete Construction—Tests of the Beam-Column Connections under Repeated Loading (Part 1 & 2)," (in Japanese), Transactions of the Architectural Institute of Japan, No. 89, 1963; No. 103, 1964.