**PAPER NO. 1** An effort is made to bring into focus the knowns and unknowns relevant to the design of reinforced concrete columns. Both theoretical and practical concerns are covered in an effort to distinguish immediate problems which must be solved to make the reinforced column more economical and versatile.

# The Reinforced Concrete Column in Perspective

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BECAUSE OF ITS PROMINENT USE in ancient Egyptian and Greek temples, the column is the victim of understatement. It is most often associated with the image of the post and lintel: a vertical member of regular section subjected to axial compression. Actually the column in the modern reinforced concrete frame may be subjected to reversible axial forces, bending, torsion, and shear. Furthermore, the column may be nonprismatic and have an irregular cross section perforated by openings for conduits running along and across its axis. The relationship of such a column to the Attic shape is far less than that of a Daliesque beauty to a caryatid. Nevertheless, a careful study of most modern codes of practice will reveal that they deal primarily with the regular-shaped "post" with some recognition of bending moments.

Design considerations for vertical and horizontal members of a frame differ (stability of beams, for example, is seldom of consequence), but these differences are fewer and less important than has been assumed. Problems that concern the reinforced concrete column concern other members of the structure. This report attempts to bring to focus the knowns and unknowns in the host of technical information relevant to the design of reinforced concrete columns, and to distinguish the problems

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which need urgent attention. Problems related to the steel reinforcement and to fatigue of concrete are not included.

The general problem is discussed at two levels: the response of the concrete and the response of the reinforced concrete unit. The discussion covers both theory and practice.

# CONCRETE AS IDEA

#### **Failure criterion**

In view of experience with other less heterogeneous materials, the knowledge that we still lack an intelligible general explanation for the strength of concrete under different conditions of loading should not come as a surprise. Despite the overtones of alchemy, efforts toward the development of a criterion or set of consistent criteria to describe the failure of concrete are desirable from the practical as well as the scientific viewpoint.

Concrete has been observed to fail by (1) separation along a plane perpendicular to the maximum tensile stress or (2) sliding along a plane inclined to the axes of principal stress. Most of the failure theories are associated explicitly or implicitly with one of these modes.

Brandtzaeg's approach<sup>1</sup> is noteworthy in that it combines both modes of failure and provides a transition from one to the other. Reinius<sup>2</sup> has

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developed an ingenious model primarily in relation to Mode 1, followed by similar models due to Baker,<sup>3</sup> Roy,<sup>4</sup> and Anson.<sup>5</sup> The Coulomb-Mohr theory of failure is the most popular interpretation of Mode 2. The influence of the intermediate principal stress has been recognized through approaches adopted by Bresler<sup>6</sup> and McHenry.<sup>7</sup> Experiments by Richart,<sup>1</sup> Reinius,<sup>2</sup> Wastlund,<sup>8</sup> Bellamy,<sup>9</sup> Weigler,<sup>10</sup> and Hilsdorf<sup>11</sup> have indicated finite influence of the intermediate principal stress, although the observed influence has varied from negligible<sup>1,11</sup> to significant.<sup>10</sup>

Although some of the experimental observations are explained by all of the available theories and some observations by some of the theories, a theory to explain all bona fide observations of failure is still to come. Furthermore, the available theories are concerned primarily with compressive strength. The tensile strength of the concrete is more important than would be implied by its arbitrary elimination in calculations relating to flexure. Attempts such as that of Kaplan,<sup>12</sup> who made an effort to relate the tensile strength of concrete to Griffith's failure criterion,<sup>13</sup> should be increased.

## Load-deformation characteristics

In structural analysis, it is necessary but not sufficient to know the strength of a material. Its load-deformation characteristics must also be known. Considerable progress has been made in this area in recent years.<sup>14-16</sup> One important point that has received little attention is the conversion from load-deformation to stress-strain. It is well known that, especially at advanced stages of loading, the average unit strain depends on the length and location of the measuring gage. It has also been shown<sup>17</sup> that compressive stresses in an axially loaded short specimen are not uniform. Further study of the stress and strain distributions in short test cylinders or prisms is desirable in order to evaluate the significance of the standard tests.

The modulus of deformation  $E_c$  for concrete has been usually expressed as a function of the compressive strength only. La Rue,<sup>18</sup> Baker,<sup>3</sup> Hansen,<sup>19</sup> and Hirsch<sup>20</sup> have related  $E_c$  to the stiffness of the hardened paste and the aggregate. This approach should be developed further and recognized directly in design recommendations.

The deviation of the stress-strain curve from linearity has been attributed to microcracks.<sup>19,21-23</sup> Hsu<sup>23</sup> has observed that microcracks exist along the aggregate-mortar interfaces before loading and that the number and extent of these cracks increase with increasing load. At stresses equal to 70 to 90 percent of the maximum, cracks develop within the mortar matrix and form a continuous pattern at maximum stress. A better understanding of the relationship between microcracking and the shape of the stress-strain curve may lead to improvements in design, especially where sustained loads are concerned. Future studies should include the complete stress-strain relationship. The portion of the curve beyond peak stress should receive particular attention. The effects of strain gradient, of the presence of reinforcement, and of autogenous healing<sup>24,25</sup> on this portion of the curve need further study.

There has been considerable study of time-dependent effects on the stress-strain curve.<sup>15,19,26,27</sup> Creep of the concrete is an important issue in design. Although the introduction of the concept of creep with and without moisture exchange<sup>19</sup> is promising, there still does not exist a rational method for a quantitative prediction of creep.

# CONCRETE AS A BUILDING MATERIAL

#### Strength

The actual strength of the concrete in the structure is a subject that appears to have eluded open discussion of any extent. The basic problem transcends control cylinder statistics. Even if the control cylinder strengths have no dispersion, the compressive strength of the concrete in the column may differ from that of the cylinder depending on casting and curing conditions. An obvious sample is the use of 85 percent of the cylinder strength for column strength. This reduction has been ascribed to an increase in the water-cement ratio in the upper portion of vertically cast columns by many investigators.<sup>28-31,49</sup> On the other hand, Petersons' test results<sup>32</sup> have indicated an increase in strength near the bottom of the vertically cast column rather than a decrease in strength near the top. Superimposed on this are the problems related to the effects of forming, curing, reinforcement, vibration, and "normal" dispersion.

Similar problems exist for tensile strength, compounded by the palpable effect of restrained shrinkage and the questions regarding the significance of the standard tests: the direct tension test, the modulus of rupture test, and the split-cylinder test.

The relationship between the actual strength of the concrete in the structure and the indication of the control specimen deserves immediate and concentrated attention. New developments in nondestructive testing methods may accelerate these studies.

#### Lateral restraint

For design, it has been found satisfactory to express the axial strength of confined concrete as the unconfined strength plus a constant times the uniform lateral pressure. This constant, used implicitly for the design of spirally reinforced columns in the ACI Building Code,<sup>33</sup> has been derived from experiments on a particular type of concrete.<sup>28</sup> Its validity for different types of concrete deserves critical study.<sup>34</sup>

In the case of circular transverse reinforcement, a rational method is still lacking to determine the transverse reinforcement stress corresponding to maximum capacity of column.<sup>35</sup> Nor are reliable methods available for predicting the complete load-deformation curve of the confined concrete. There is even less information on the problem if the confinement is pro-

vided by noncircular transverse reinforcement.<sup>36</sup> Further information is also needed on the effect of confinement by adjoining concrete elements.<sup>37</sup>

# THE REINFORCED CONCRETE COLUMN AS IDEA

### Axial load and bending

The reinforced concrete column section may be subected to combinations of axial load, bending, shear, and torsion. Because of the classical definition of the column, most analytical work has been on combined axial load and bending.

If the critical properties of the materials involved are known or are assumed, the analysis of the response of the column section to axial loading and bending becomes simply a matter of the manipulation of the principles of statics and geometry. Nevertheless, because of the reluctance of the profession to move away from the realm of working stress design and to recognize bending in columns, the first comprehensive study of the problem was published in 1951.<sup>38</sup> The effects of length on an inelastic column were considered analytically by Broms,<sup>39-41</sup> Pfrang,<sup>42-44</sup> and Mauch.<sup>45</sup> A significant recent development is the initiation of tests on long columns by Breen<sup>46</sup> in an effort to reconcile results of the analysis and the response of the column in a structural frame.

Recent progress in this area is rapid and promising, but it may be out of proportion to what is known about the loading conditions. Over and above the problem of predicting the possible loads on a structure, there is the problem of establishing what contraints are provided by other elements of the building, structural or nonstructural. Knowledge of what a column is asked to do should not be too far behind knowledge on what the column can do.

## Shear and torsion

Shear and torsion may be dominant effects in columns, especially where lateral loads are involved. Nevertheless, they are often ignored. For example, although the report of ACI-ASCE Committee 426, Shear and Diagonal Tension, considers the influence of axial load on shear strength, applying the design recommendations given to a nonrectangular section requires more than routine interpretation.<sup>50</sup>

When further work is done on shear and torsion, the fact that these phenomena are not limited to beams should be realized by the investigators.

# THE REINFORCED CONCRETE COLUMN AS A BUILDING ELEMENT

There has been a rapid evolution in the types of reinforced concrete buildings. Consequently, there is urgent need for a critical reappraisal of certain practical considerations regarding construction and design. These are discussed in the following paragraphs.

#### Design limits

Limiting dimensions for columns—An 8 ft long reinforced concrete column with a 6 in. square section using 5000-psi concrete and four #4 intermediate grade steel reinforcing bars should develop an axial load capacity of 185,000 lb, a quantity which cannot be dismissed as insignificant. Nevertheless, this column cannot be used as part of a structure governed by the current ACI Building Code.<sup>33</sup> The development of precasting techniques and the attendant necessity of using minimum weight concrete structures demand a reappraisal of the limiting cross-sectional dimensions of reinforced concrete columns.

Limits on longitudinal reinforcement—The minimum limit of reinforcement in a column should be critically studied in view of the new knowledge developed on creep and shrinkage of the concrete and safety considerations of the column. The elimination of the maximum limit of reinforcement, with explicit warnings about its consequences, should be considered. In this connection, it should also be mentioned that the definitions of "composite column," "pipe column," and "combination column" may have no place in a modern building code. The design considerations for such columns could be approached at a more fundamental level.

ACI Committee 347, Formwork for Concrete, recommends<sup>47</sup> a tolerance of minus 0.25 in. and plus 0.5 in. for the cross-sectional dimensions of the column, and a tolerance of 0.25 inches in 10 ft of height, 0.375 in. for a height of 20 ft, and 0.75 in. for 40 ft or more. The impact of this provision on the safety of the column as designed by the ACI Building Code should be carefully studied. The same observation can be made with respect to the tolerances in placing the steel reinforcement.

Conduits and pipes embedded in concrete—In high-rise buildings it is architecturally convenient to place large-diameter conduits and piping within the column. The effect on column resistance of the eccentric positioning of the conduits or piping, taking into consideration the tolerance in their vertical placement and the effect of the horizontal piping connections, deserves careful study. The current limit on the column area which can be displaced by conduits without penalty should be related to the pertinent variables.

*Reinforcement splices*—The use of high strength concretes and more slender columns has contributed to reductions in the size of column sections. It is costly and virtually impossible to make conventional reinforcement splices in columns with small sections, especially if largediameter high strength bars are used. This development, plus the fact that as more research is done on bond the required lap lengths get longer, creates a need for new types of splices such as welding or mechanical couplers.

Transverse reinforcement and concrete cover—The 1963 ACI Building Code has liberalized the requirements for lateral reinforcement of tied

columns. However, there appears to be room for further simplications in these requirements, especially in the case of columns using high strength concrete and large amounts of reinforcement.

Architectural requirements and the desire to save space make it necessary to reduce the section of reinforced concrete columns to the point that in certain cases a concrete cover of 1.5 in. provides a hindrance in design. In the case of a column with an 8-in. side, this requirement leaves only 62.5 percent of the width of the column as the space available for placing reinforcement. Further work on the fire resistance of columns should provide an explicit relationship between concrete cover and time of resistance.

*Bar spacing*—The tendency to use high strength concrete in columns with high slenderness ratios, together with the different methods of depositing and vibrating concrete require an investigation of the limitations of minimum spacing for the bars of the columns. The investigation of the behavior of bundled reinforcement should be continued. Although the use of bundled reinforcement has been permitted by the ACI Building Code, investigations reported so far have been limited.<sup>48</sup>

### Construction

*Tolerances*—Eccentricity of loading is an important factor in columns with high slenderness ratios. The tolerances permitted in column construction affect the eccentricity of the load as well as the economy of the structure. The minimum eccentricity required by the building code should be related to the tolerance specified. A reinvestigation of permissible tolerances is in order.

*Exposed columns*—The use of exposed reinforced concrete columns in modern structures has raised new problems about the effects of dimensional stability of concrete at varying temperatures. The behavior of existing buildings with exposed columns should be carefully scrutinized and reconciled with theoretical considerations with the help of further tests under controlled conditions.

## Shear walls

Concrete shear walls have been one of the successful solutions used by designers in high-rise buildings. The reinforced concrete shear wall resists vertical as well as lateral loads and is in effect a column.

The inherent properties of shear walls create problems in analysis and interpretation in the following areas:

1. Great differences between the dimensions of their sections

2. Stress concentrations at meeting points of orthogonal walls where one wall supports the other

3. Stress concentrations occurring at intersections with floor systems

4. Transmission of loads from walls to reinforced concrete columns

5. The effect of openings

6. Slenderness of the wall (If the requirements of Sec. 2202 of ACI 318-63 are applied, a reinforced concrete wall 402 ft high would require a

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minimum thickness of 22 in. However, buildings of that height have been constructed with walls only 11 in. thick.)

7. Limits on the amount of vertical and horizontal reinforcement.

# CONCLUDING REMARKS

Any survey, either from the purely scientific or the practical viewpoint, of what is known about the reinforced concrete column would inevitably conclude that more needs to be known. To elevate that conclusion from a shibboleth to a serious call for action, recommendations must be given some order of priority. That requires a specific criterion and bitter pruning.

The writers have adopted the mundane criterion that research should makes a product cheaper or more versatile. Naturally, any and all endeavors in fundamental research may lead to a breakthrough and more than satisfy the criterion. However, the following topics have been selected for study from among those suggested on the assumption that further work along these lines promises reasonably quick and certain returns:

1. The relationship between the actual properties of the concrete in the structure and those in the control specimen

- 2. Determination of the actual forces acting on a column in a building
- 3. Reinforcement splices
- 4. Shear strength of columns with nonrectangular cross sections
- 5. Load-deformation characteristics of multistory shear walls
- 6. Limiting values on column size and amount of reinforcement

7. Construction tolerances.

Progress in the first two items would reduce to a minimum that part of the design process subject to judgment. The last two items are areas requiring immediate interaction among ACI committees concerned with structural analysis, materials, and construction.

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