at this face. This higher creep rate on the upper face was in fact observed. The initial eccentricity credited to each column was the distance from the line joining the knife-edges to the column center line at midheight at the time of commencement of the test.

Testing

The column specimens were tested in a universal testing machine of 400,000 lb capacity. Each specimen was aligned in the machine with the aid of a theodolite which was subsequently used to take readings of lateral deflection. Loading was in stages, to give about ten sets of readings. The load increments were smaller as the ultimate load was approached.

Deflections at the sixth-points were observed by theodolite. The mid-height readings were taken first and end readings last to minimize variations due to creep, which were noticeable at higher loads. The theodolite read to 1 sec of arc and was situated 100 in. from the test piece. Allowing for a pointing error of 5 sec of arc, the error in deflection readings is

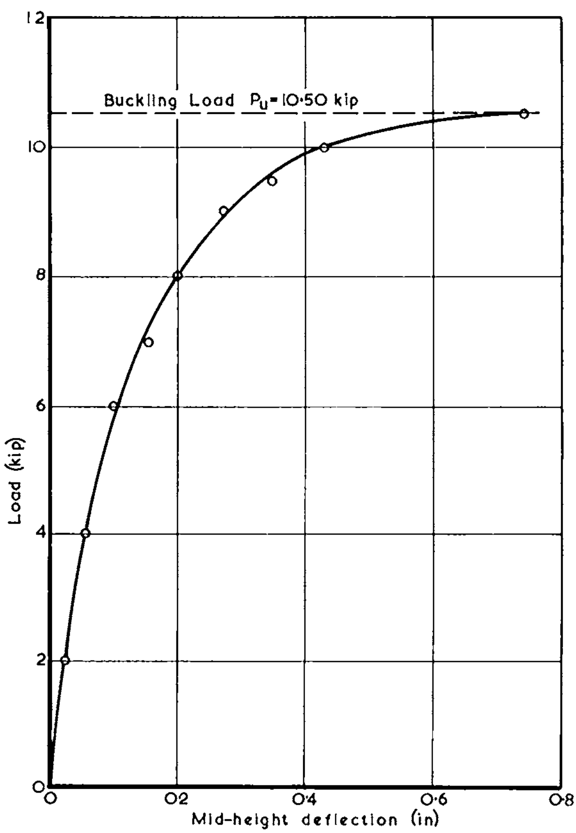

Fig. 8-3 Experimental load-deflection curve for column in Series D with $e = 0.26$ in.

±0.0005 in. Strain distribution across the section was measured at mid-length by means of a Huggenberger Tensotast of 10 cm gage length. A dial gage enabled the total shortening of the column to be measured.

A complete set of readings was taken at each load increment and the duration of the test was about 20 min.

TEST RESULTS

As mentioned earlier, most specimens developed a small initial curvature before testing. The actual initial eccentricity (at midheight) was therefore slightly different from the intended value as provided by the steel loading heads. The eccentricity referred to in these results is this actual eccentricity.

TABLE 8-1 FAILURE LOADS OF SPECIMENS

Series	Eccentricity e, in.	Concrete strength f'_c, psi	Prestress, f_{cp}, psi	Failure load P_u, lb
A	(0) 0.08 0.09 0.26 0.50 1.50 3.93	4170	100	(16,500) 12,600 11,970 7,520 4,960 2,700 1,780
B	(0) 0.02 0.22 0.33 0.61 1.63 3.92	4870	1110	(15,100) 14,460 9,660 8,500 5,800 3,120 1,800
C	(0) 0.10 0.22 0.30 0.57 1.75 3.94	4670	1520	(13,900) 12,000 10,500 9,000 7,040 3,400 1,780
D	(0) 0.05 0.26 0.57 0.84 1.64 3.99	4520	2160	(13,800) 13,000 10,500 7,500 5,920 3,940 2,000
E	−0.03* (0) 0.11 0.30 0.70 1.77 4.36	4520	2660	15,160 (14,500) 12,980 10,000 7,200 4,130 2,100

*This negative value resulted because the initial curvature of the column was greater than, and in the opposite direction to, the eccentricity determined by the loading heads.

The failure loads of the columns are given in Table 8-1. Because of the accidental eccentricity, no specimen was tested with truly axial loading. However, the experimental values plot as smooth curves of somewhat hyperbolic shape. Consequently, values corresponding to zero eccentricity were obtained by extrapolation. A plot of eccentricity against reciprocal of failure load was used for this purpose. These extrapolated values are shown in parentheses in Table 8-1.

Fig. 8-3, 8-4, and 8-5 show typical deflection and strain behavior. They refer to the same specimen for which the above strain data were given. Fig. 8-3 shows the variation of midheight deflection with load. Fig. 8-4 gives the same information in a different form. At the point of instability the tangent to the M-y curve has a gradient equal to the buckling load and this tangent cuts the y axis at a value equal to the initial eccentricity.

The graph of strains at the midheight section (Fig. 8-5) is also of interest as indicating that, for this small eccentricity, tension does not develop on the "tension" face until close to the failure load. Clearly cracking would not be expected at any stage for this column.

Although deflection measurements were taken at a number of points along the columns, it was not possible, with the method used, to obtain

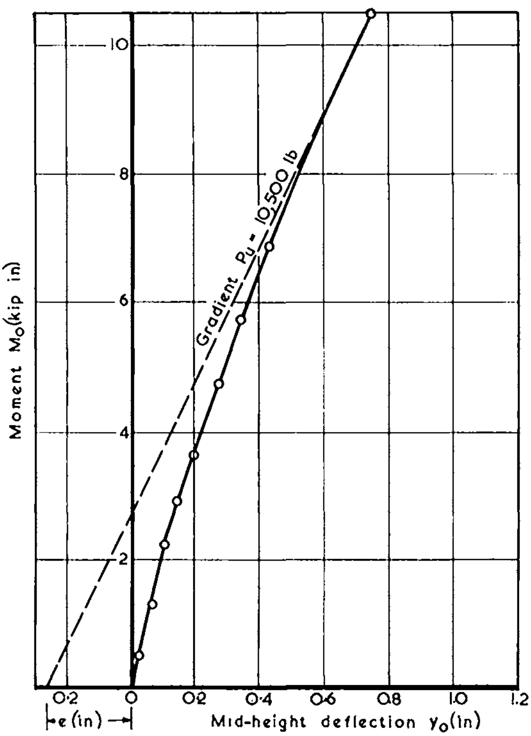

Fig. 8-4 Experimental moment-deflection curve for column in Series D with e = 0.26 in.

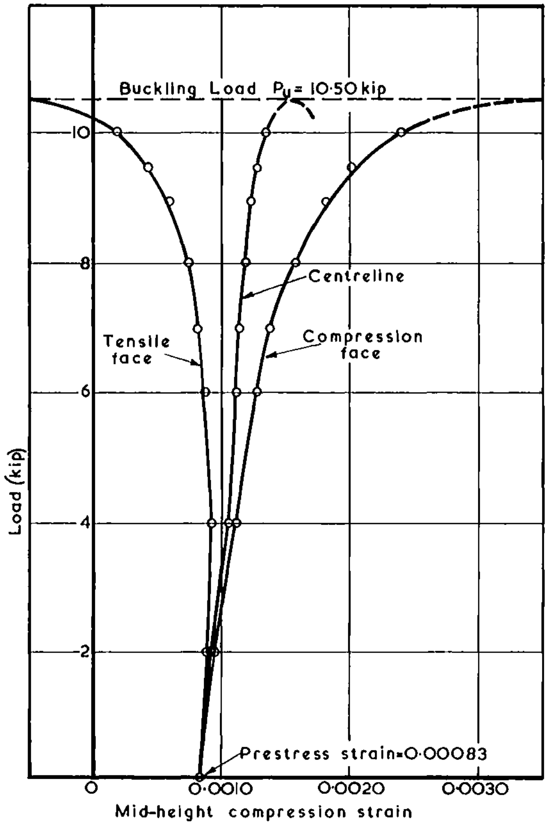

Fig. 8-5 Experimental load-strain variations for column in Series D with e = 0.26 in.

reliable sets of readings at loads approaching the ultimate. Plots of the deflected shapes were compared with parabolic, circular, hyperbolic, and cosine curves. At low and moderate loads the cosine curve was always found to provide the closest fit. At advanced loads, the time interval required for the readings, although short, prevented the readings from forming a comparable set in view of the rapid creep. Strain readings at the center section usually indicated a curvature greater than that consistent with a cosine curve for the given deflection. It would thus seem probable that, close to failure, a departure from the cosine shape took place.

DISCUSSION

It is possible to plot a graph of P_u against eccentricity for each series of tests. The test results lie on well defined curves (which are not shown for reasons of space), and naturally indicate a reduction of P_u for increasing eccentricity. Of more interest in the present investigation is the effect of prestress on P_u for a given eccentricity of loading. Since the tests in the

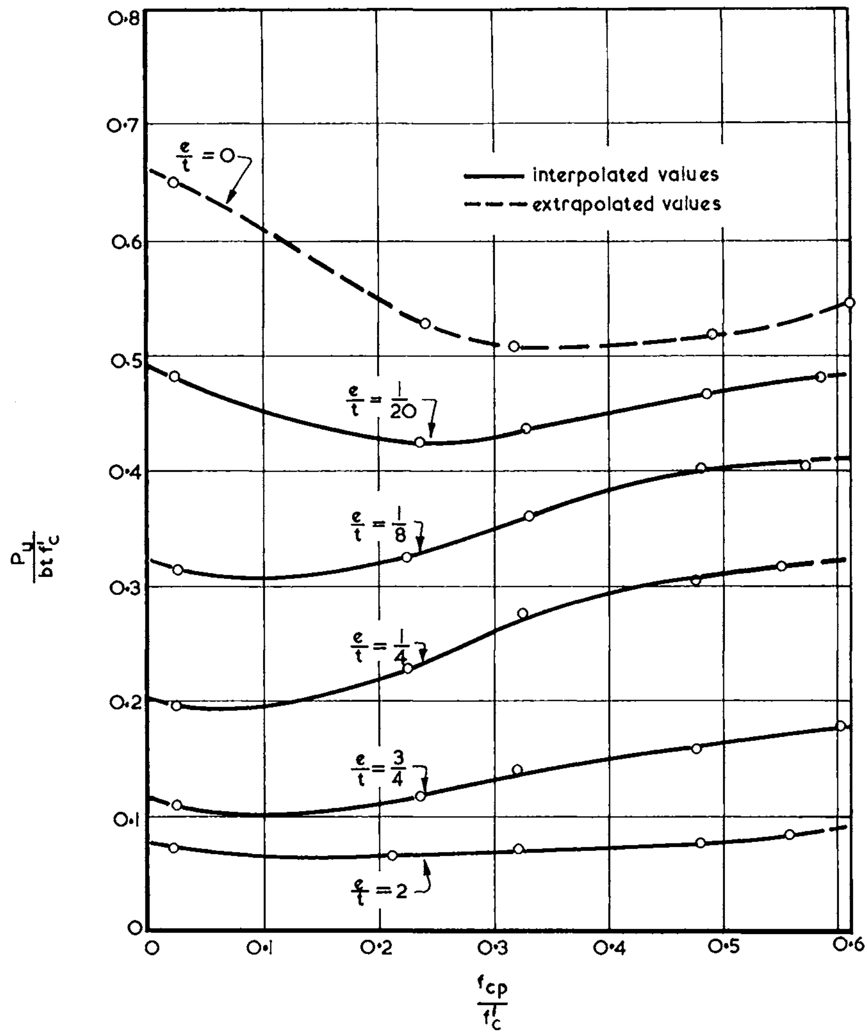

Fig. 8-6 Experimental variation of buckling load with prestress for all column series

different series did not correspond to precisely the same eccentricities, the results for a chosen eccentricity were obtained by interpolation. This is considered to be amply justified in view of the smoothness of the P_u-e curves referred to above.

In this form the results are shown in Fig. 8-6, where each curve corresponds to a particular load eccentricity and shows the variation of failure load as prestress increases. In the range of low prestress, these curves behave much as might be expected. Thus for centrally loaded specimens, the presence of prestress decreases the capacity of the column. The applied load produces mainly axial compression in the member, and consequently any preexisting compression is detrimental. For comparatively large eccentricities, the applied load produces mainly bending in the member. Since prestress causes little variation in the ultimate flexural strength, it might be expected that prestress would have little effect upon the capacity of columns with large eccentricity of loading, and this appears to be substantially the case.

When the prestress is appreciable, however, its effect differs from that which might be expected as a result of the cursory reasoning just outlined. A theoretical analysis of these columns is dealt with in Paper No. 7 in this volume. However, it appears that, irrespective of the precise theory adopted, the prestress is detrimental to the ultimate capacity, provided it is assumed that the stress-strain characteristics of the concrete are independent of prestress. One is thus forced to conclude that the beneficial effects indicated in Fig. 8-6 must be attributed in large measure to variations, induced by prestressing, in the stress-strain relationship of the concrete. Such changes have been noted by several investigators[16,17] and the present tests appear to afford further evidence of this effect.

The measurements of lateral deflection show that, over most of the load range, the deflected shape of the column closely approximates a cosine curve. Such a curve results if the curvature of the elements is proportional to the bending moment. For small eccentricities, although the percentage variation of bending moment along the column is considerable, the absolute values are everywhere small. Because of this, and also because of the axial compression, the member is uncracked except at loads near the ultimate. For large eccentricities, the absolute variation of moment between the ends of the member and the center is greater. One would expect therefore that at some stage of loading some sections would be cracked while others remained uncracked. For a member in pure flexure there is a fairly rapid decrease of flexural stiffness at the onset of cracking. However, in the presence of axial compression, this difference of stiffness takes place more gradually. For this reason the departure from a cosine curve at a given load is probably masked by inaccuracies of measurement.

CONCLUSIONS

The following conclusions may be drawn from the foregoing discussion:

1. For an axially loaded column with an l/t ratio of about 33, the presence of a small prestress (up to 0.30 f_c') will decrease the load-carrying capacity.

2. For large eccentricities, the presence of prestress affects the capacity of a column only slightly.

3. Contrary to anticipated theoretical predictions, the capacity of a column is increased by the presence of a large prestress (up to 0.6 f_c'). This phenomenon has been explained by changes, due to prestress, in the stress-strain relationship for the concrete.

4. Measurements indicate that the deflected shape is closely approximated by a cosine curve for most of the load range. Near failure there is some departure due to rapid changes in flexural stiffness.

5. For the columns tested in this investigation, with prestress up to $0.6 f'_c$, there was no evidence of an optimum prestress level at which, for a given eccentricity of load, the failure load was a maximum. In fact, within the prestress range used, only minimums were encountered.

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PAPER NO. 9 Longitudinally and circumferentially prestressed cylinders and columns of varying slenderness were tested to destruction to determine what influence the amount and kind of prestress had on ultimate load and stability characteristics. Previous tests of triaxially prestressed elements were reviewed, and their findings compared with results of the present study.

Effect of Active Triaxial Stress on the Strength of Concrete Elements

By E. Ben-Zvi, G. Muller, and I. Rosenthal

THE HIGH LOAD CAPACITY OF MATERIALS under triaxial compression has been known since the beginning of the century. Tests and basic research on failure conditions under triaxial compression were undertaken by Von Karman,[1] Richart, Brandtzaeg, and Brown,[2] Balmer,[3] and Bridgman,[4] but much less work was done on the strength of structural elements under the same conditions, e.g. on finding ways and means for practical exploitation of such a favorable state of stress for the erection of engineering structures. As far as known to the authors, the earliest practical contribution toward use of triaxial prestress was made through spiral prestress, as proposed by Maney[5] in 1944. Further investigations of this concept were carried out by Johnson[6] in 1943 and by Feeser and Chinn[7] in 1962. Similar investigations on structural-size specimens were published by Gitman[8] in 1958, by Gambarov[9,10] in 1961–62, and by Mikhailov[35] in 1963.

Triaxially prestressed concrete is being advocated by structural engineers as the building material of the future.[11] Up to now application of triaxial prestress has been made only in a limited number of cases such as the Marne bridge construction by Freyssinet.[12] It has also been reported from the U.S.S.R.[13] that a huge project for a television tower 2000 m high (1.25 mile) using triaxially prestressed precast elements is under construction.

NOTATION

A_c = cross sectional area of concrete, excluding any finishing material and reinforcing steel
A_g = gross area of concrete section, including area of steel
A_k = cross sectional area of concrete in the core, excluding the area of longitudinal reinforcement
A_{st} = cross sectional area of the longitudinal steel
A_h = equivalent area of helical reinforcement (volume of helix per unit length of concrete)
d = diameter of reinforcing or prestressing wire
D = net diameter of concrete column (excluding external shell)
E = Young's modulus of elasticity
E_{col} = Young's modulus of the concrete core
E_{sh} = Young's modulus of an external shell
k = empirical coefficient; increase in ultimate load due to lateral pressure
K = stability factor
N = total failure load of column
N_c = load carried by concrete
N_o = permissible load on a short column
N_{st} = load carried by longitudinal reinforcement, including spiral ties and prestressing wires
N_{sp} = load carried by spiral prestressing wire through lateral pressure
p = ratio of volume of reinforcement to A_c. Same symbol with prime and double prime refers to spiral ties and longitudinal prestressing wires, respectively.
P = tensile force in spiral prestressing wire
R = radius of wrapped specimen
s = pitch of spiral prestressing wire
β = empirical coefficient dependent on slenderness ratio, and affecting the coefficient k
ϵ = unit strain
ν = Poisson's ratio
σ_e = effective stress on the prestressed member
σ_{prism} = prism strength of concrete
σ_y = yield stress of steel. The same symbol with prime and double prime marks refers to spiral ties and longitudinal prestressing wires, respectively.
σ_{ux} = total ultimate strength of spirally prestressed concrete cylinder, equal to $(\sigma^c_{ux} + \sigma^s_{ux})$
σ_{ax} = total allowable stress for the prestressed member in direct compression, equal to $\sigma^c_{ax} + \sigma^s_{ax}$
σ^c_{ax} = permissible compression stress of concrete alone
σ^c_{ux} = compressive strength of concrete alone
σ^s_{ux} = compressive strength increase due to reinforcement and longitudinal prestress
σ^s_{ax} = premissible compression stress for column bars
σ^s_u = ultimate stress of steel
σ_{lat} = lateral pressure on specimen due to restraint of spiral prestress wires
σ_{oct} = octahedral ultimate normal stress of concrete
τ^c_u = ultimate shear strength of concrete
τ_{oct} = octahedral ultimate shear stress of concrete