Deformation Capacity of RC Members with Brittle Details Under Cyclic Loads

by D.V. Syntzirma and S.J. Pantazopoulou

Synopsis: The sequence of failure in reinforced concrete (RC) prismatic members is used as a tool in estimating dependable deformation capacity. Response mechanisms that may limit the response leading to damage localization are identified (web diagonal cracking, bar buckling, disintegration of compressive struts due to load reversal, and anchorage failure of primary reinforcement). Deformation components are additive only if stable hysteretic response controlled by flexure prevails. In all other cases, the deformation component associated with the controlling mode of failure dominates the overall deformability of the member. Because the sequence of failure depends to a large extent on load history, deformation attained at any particular level of load is also load history dependent. This is why experimental values for deformation capacity reported in international literature are characterized by excessive scatter. The proposed methodology is applied to a number of published column tests. Analytical estimates are evaluated through comparisons with experimental results and by parameter studies conducted in order to examine the sensitivity of the estimated displacement limit at compression bar buckling to important design variables.

Keywords: deformation capacity; displacement-based assessment; nonconforming RC members; seismic design of RC

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INTRODUCTION

A primary objective of displacement–based assessment of existing RC structures is to ensure that dependable deformation capacity Δ_R of each individual component exceeds the anticipated deformation demand Δ_d. The requirement that $\Delta_R > \Delta_d$ refers to an ultimate limit state; reversal of the inequality signifies excessive damage and even collapse of the structural component.

Existing structures, designed up to an array of earlier versions of seismic codes as these evolved through the last half century, are generally classed as "low deformation capacity" systems, i.e. they have limited ability to sustain large inelastic deformation reversals without strength loss. In terms of reinforcement amounts, older structural members typically are lacking in properly detailed transverse reinforcement (confinement and shear reinforcement in beams, columns, joints and lap splices). Reconnaissance studies of damage from past earthquakes show that the above requirement of $\Delta_R > \Delta_d$ may not be always secured by old designs, either because the intrinsically available deformation capacity is negligible due to poor detailing, or because the demand is excessive due to inherent conceptual flaws in the structural system (excessive localized flexibility); the latter is seen frequently even in modern construction.

To estimate the dependable deformation capacity of RC members, it is important to determine simple models that are capable of reproducing the most prevailing parametric sensitivities observed in tests of brittle components. This objective has been pursued in the present paper through analytical modeling and evaluation of published experiments. The analysis is based on a comprehensive concept of sorting through the various alternative failure mechanisms that could control member behavior (often prior to development of full inelastic flexural action), seeking the weakest link of member response for a given cyclic displacement history. This framework is referred to hereon as Capacity-Based Prioritizing, or CBP. Departing from earlier concepts that center or expand on the estimation of drift components using cross–sectional and material properties and aspect ratio, the present approach determines the actual pattern of dependable deformations associated with the prevailing failure mechanism. In identifying the weak link of member behavior the basic premise has been that this mechanism becomes the fuse of the member response, i.e., upon increase of the imposed end displacement, deformation is expected to localize in that fuse, and hence, beyond that stage, all other forms of nonlinear behavior may be irrelevant.

Particular emphasis is placed on alternative failures that would prevail due to poor confinement. Web diagonal tension failure, buckling of compression reinforcement, disintegration of concrete due to reversal of load, and limited anchorage or lap–splice capacity of primary reinforcement are candidate scenarios. The influence that these failure modes may have upon the deformation capacity of a poorly–detailed member depends upon the sequence in which they develop. Thus, the sequence of failure, if seen as a chain of successive events in the response history of the member, *uniquely* defines deformability, although it is not necessarily *unique* in itself, for it depends on the imposed loading history. As a rule, the various strength mechanisms *do not* degrade proportionately with load reversals. This point is reflected repeatedly in the available experimental evidence and it is why, application of the proposed analysis framework affirms conclusively that the process of assessment of deformation capacity is rather complex and cannot easily be treated by unidirectional closed form expressions.

The proposed methodology is tested on a number of tests published in international literature. Relevant experiments concern *brittle* columns with lap splicing in the critical regions under cyclic load reversals. In the paper, comparisons are made between analytical estimates and experimental results, whereas the sensitivity of the proposed methodology to important design variables is also explored through parameter studies.

RESEARCH SIGNIFICANCE

Quantifying the dependable deformation capacity of RC members, particularly members *non–conforming* to modern standards, is a milestone in the process of assessment of seismic resistance of existing structures. Existing methods of calculating deformation capacity are marked by excessive scatter even when applied to well-detailed members. In this paper a method of capacity-based prioritizing of the alternative modes of failure is used to identify localization of failure and to estimate the associated deformability of reinforced concrete members with substandard detailing representative of former construction practices.

FRAMEWORK FOR ESTIMATING DEFORMATION CAPACITY

Deformation capacity of a RC member as defined for the needs of seismic assessment is the maximum relative translation (or relative drift) the member may sustain without excessive loss of lateral load strength. To evaluate this response parameter, the usual approach is to calculate contributions to drift of the various modes of deformation that occur along the member, namely flexural, shear, and rotation due to reinforcement pullout, which are then superimposed as they are generally considered to act cumulatively [e.g. Lehman[1], Inel[2], Panagiotakos[3], Pujol[4]]:

$$\Delta_y = \Delta_y^{flex} + \Delta_y^{shear} + \Delta_y^{slip} \quad ; \quad \Delta_u = \Delta_y + \Delta_p^{flex} + \Delta_p^{shear} + \Delta_p^{slip} \tag{1}$$

Subscripts y and p in Eq. (1) refer to the yield and plastic (total minus yield) values of the respective terms. In all relevant published works, Eq. (1) is paired with analytical expressions for the various components [e.g. Lehman[1], Inel[2]]. These are derived from the basic components of deformation (curvature, shear angle, steel–strain along the anchorage) either using the mechanics of the so–called *stick–model* or are otherwise obtained empirically [e.g. Panagiotakos[3]]. The *stick–model* is a cantilever having the member geometry and a length equal to the member shear–span L_s, subjected to constant shear that simulates the behavior during lateral sway of a typical frame element segment between midspan and the moment resisting end support. The tip displacement of the stick-model divided by its height is the lateral drift, a response parameter directly comparable with the actual member drift. Previous attempts to correlate the various methods that rely on Eq. (1) for calculating deformation capacity at yield and ultimate with experimental data, are marked by excessive scatter, and great discrepancy between analytical estimates and experimental values of deformation capacity [Inel[2], Panagiotakos[3], Syntzirma[5]]. Scatter is great even in well–detailed elements and deteriorates when members *non–conforming* to modern detailing standards (referred to hereon as *brittle*) are considered.

The systematic inaccuracy of what appears to be a consistent mechanics approach points to a fundamental conceptual flaw. The entire procedure for calculating deformability, either using the stick model or its many variations, is based on the assumption of a robust flexural action. From straightforward moment–rotation calculations it is evident that ideally, flexural drift would account for the largest fraction of the total, i.e. it would represent the basic component. But whereas a moment–curvature analysis could suggest a sufficiently ductile behavior up to a compressive strain level of 0.005 (cover spalling), the actual amount of displacement is often limited by other mechanisms of failure owing to sparsity of stirrups, such as diagonal tension failure of the web, buckling of compression reinforcement, disintegration of the compressive struts due to reversal of load, and limited anchorage or lap–splice capacity of primary reinforcement. Thus, the exact pattern of dependable deformations is not simply a matter of cross–sectional properties. Development of full flexural action may not always be possible in actual circumstances, due to the interaction with shear and the likely influences of all the other mechanisms of response. If such mechanisms prevail prior to development of full inelastic flexural action, then calculated inelastic flexural deformations may even be irrelevant.

The sequence in which the various response (and failure) mechanisms will occur defines a unique pattern of deformation output for the member that clearly depends on load–history since the strength values of the various modes of behavior degrade disproportionately with the extent of cracking and the number of load cycles. A critical step in the direction of determining the deformation response is to identify *the weak link of behavior*, where localization is expected to occur. To settle this issue the dependable strengths of the alternative mechanisms of failure are prioritized as described by the following qualitative statement (CBP method):

$$V_{u,\lim} = \min\{V_{flex}, V_{shear}, V_{lap}, V_{buckl}\} \tag{2}$$

The static relationship between the various strength indices in Eq. (2) refers to the basic flexural stick–model described. These strength terms need be estimated by considering the cyclic load effects and degradation due to cracking. $V_{u,lim}$ is the shear force sustained by the member at failure, V_{flex} is the shear force required to develop the flexural capacity at the support (i.e. $V_{flex}=M_u/L_s$), V_{shear} is the web shear strength, V_{lap} is the shear force sustained when the capacity of the anchorage or lap splice of reinforcement is attained (i.e., $V_{lap}=M_{lap}/L_s$, with M_{lap} the moment sustained at the support at that instant). V_{buckl} is the shear force sustained when compression reinforcement buckles at the critical section; this magnitude is associated with the displacement limit at which bars are expected to buckle, which is a load-history dependent quantity. Therefore, although it is used in comparing strengths, should V_{buckl} control, it would necessarily be associated with realization of the associated displacement limit in the response curve.

Practical Implementation of the Proposed Method for Old–Type RC Members

Considering that localized deformations of any kind other than flexural may prove fatal for member integrity, it is natural to associate the ultimate drift with the onset of localization. To recognize that the contributions of the various modes of deformation to the total drift at yield or ultimate depends on the mode of localization, the familiar Eq. (1) is *modified conceptually* as shown in Eq. (3) by the introduction of weight factors, w_y and w_u, that operate on the individual contributions. *The weight factors represent the relative strength (for strength controlled mechanisms) or the relative deformation capacity (for deformation-controlled mechanisms) of the weak link controlling localization as compared with the mechanism under consideration*:

$$\Delta_y = w_y^{fl}\cdot\Delta_y^{fl} + w_y^{sh}\cdot\Delta_y^{sh} + w_y^{sl}\cdot\Delta_y^{sl} \quad ; \quad \Delta_u = \Delta_y + w_u^{fl}\cdot\Delta_p^{fl} + w_u^{sh}\cdot\Delta_p^{sh} + w_u^{sl}\cdot\Delta_p^{sl} \quad (3)$$

Apart from shear action which is considered a strength-controlled mode of behavior, all other modes are deformation-controlled. Individual components are obtained from basic mechanics [Pantazopoulou[6]]:

$$\Delta_y^{fl} = \frac{1}{3}\cdot\phi_y\cdot L_s^2\ ; \quad \Delta_p^{fl} = (\phi_u-\phi_y)\cdot\ell_p\cdot(L_s-0.5\ell_p); \quad \Delta_y^{sh} = \frac{V_c}{0.4\cdot E_c\cdot 0.8A_g}\cdot L_s;$$

$$\Delta_p^{sh} = \varepsilon_{st}\cdot L_s \qquad \Delta_y^{sl} = \frac{\phi_y\cdot D_b}{8}\cdot\frac{f_y}{f_{b,y}}\cdot L_s\ ; \quad \Delta_p^{sl} = (\phi_u-\phi_y)\cdot\frac{D_b}{4}\cdot\frac{\beta\cdot f_u}{f_{b,u}}\cdot L_s \qquad (4)$$

Parameters ϕ_y and ϕ_u are the yield and theoretical ultimate curvature of the member; V_c is the concrete contribution to the nominal shear strength; E_c is the concrete modulus of elasticity; A_g is the gross cross sectional area and D_b is the diameter of longitudinal tension reinforcement. Variable f_u in Eq. (4) may be *less* than the actual tension capacity of the reinforcement, as the true stress developed in the primary bars may be limited by premature failure of the member. Ratio $\beta=(f_u-f_y)/f_u$ in Eq. (4) is the normalized strength increase from the point of yield to the maximum stress attained by the tension steel during the response. Thus, β is a variable, taking on different values throughout the response as the stress in the tension steel increases. l_p is the associated length of plastic hinge, $f_{b,y}$ is the bond stress at yielding of tension reinforcement, and $f_{b,u}$ is the bond stress corresponding to the ultimate state developed along the depth of yield penetration (here $f_{b,u}$ is taken without loss of generality as a fraction of $f_{b,y}$, i.e., $f_{b,u}=\lambda \cdot f_{b,y}$, where λ ranges between 0.5 and 0.8). Variable ε_{st} is the strain of stirrup steel (at stirrup yielding, it is calculated as $V_w/n_{st}A_{st}E_s$, where V_w is the stirrup contribution to shear strength, n_{st} is the number of stirrup layers intersecting a crack plane at 45^o with the member axis, A_{st} is the area of stirrup legs in a single stirrup layer, and E_s is the modulus of elasticity of stirrup reinforcement). Clearly, the maximum value for ε_{st} for which the calculation of Δ_p^{sh} is meaningful is at stirrup yielding, for beyond that point the member is considered to have failed in shear, [Pantazopoulou[6]].

From Eq. (4) it follows that the nominal flexural deformation capacity, $\Delta^{fl} = \Delta_y^{fl}+\Delta_p^{fl}$, may not be realized unless flexure prevails from Eq. (2) as the weakest mode. Clearly there are several alternative combinations, such as premature bar buckling, or inadequate development capacity along the anchorage of a brittle member; this explains the dramatic scatter between experimental values and analytic expressions that are blind to localization. A critical parameter in Eq. (4) is the length of plastic hinge next to the moment–resisting support. The general form is,

$$\ell_p = \beta L_s + \frac{D_b}{4} \cdot \frac{\beta}{(1-\beta)} \cdot \frac{f_y}{f_{b,u}} \qquad (5)$$

In most of the available variations of the stick model known in the literature, the second term in Eq. (5), which represents slip/pullout effects on the length of inelastic activity is taken as a constant associated with the ultimate tension capacity of reinforcement [e.g. Priestley[7]]. In reality, the plastic hinge length is a variable that depends on the maximum steel tension stress attained at the critical section (i.e. on the current value of β) and the bond stress conditions along the anchorage, particularly if premature failure terminates the response of the member prior to development of full inelastic flexural capacity.

Strength Estimates for the Various Modes of Failure

The accuracy of the proposed CBP procedure for displacement capacity estimations requires a reliable assessment of the strength terms entering Eq. (2). When correlating with test results these may be calculated from standard theory, using pertinent simplifying assumptions for expediency. One such approach is outlined in the following.

Flexural Strength — The ideal flexural strength is meaningful only if it may be safely assumed that it is supported by all other mechanisms of behavior (i.e., if $V_{flex} < \{V_{shear}, V_{lap}, V_{buckl}\}$). The bending moment sustained when the flexural reinforcement reaches yield for the first time is M_y^m, and in a monotonic loading history the ultimate flexural resistance is M_u^m. Both M_y^m and M_u^m may be calculated from standard theory.

The reduced flexural yield strength that may be sustained after cyclic reversal of load, need consider that the compression steel has yielded in tension in the preceding cycle [Pantazopoulou[6]]. At this point, cracks will remain open in the compression zone, and hence all the compressive force will be resisted by compression reinforcement. Only if this reinforcement yields in compression may the cracks close so that the concrete may contribute to compression resistance. For equal areas of top and bottom reinforcement, the cyclic yield moment, M_y^c is $M_y^c=A_{s1}f_y(d\text{-}d_2)$. For the same case, the ultimate cyclic moment theoretically is $A_{s1}f_u(d\text{-}d_2)$, but in the absence of confinement the maximum compressive strain, ε_{cu}, is not expected to exceed 0.005 (the cover spalling strain) and it is therefore best to take $M_u^c=M_y^c$. For unequal areas of compression and tension reinforcement, $M_y^c=A_{s2}f_y(d\text{-}d_2)$, and $M_u^c=A_{s2}f_u(d\text{-}d_2)+(A_{s1}\text{-}A_{s2})f_u\cdot jd$ (the last term to be included if confinement is available), where A_{s1} and A_{s2} is area of the tension and compression reinforcement respectively; d and d_2 is the distance from the extreme compression fiber to the centroid of the tension and compression reinforcement and jd is the distance of the tension steel to the centroid of the compression zone on the cross section. The corresponding values of V_{flex} are, $V_{flex,y}{}^c= M_y^c/L_s$, $V_{flex,u}{}^c = M_u^c/L_s$.

In the correlation study presented in the following sections, flexural strength is calculated conventionally under the assumption that flexure is the predominant mechanism of failure, so as to point out the dramatic influence on deformation capacity owing to premature occurrence of other failure mechanisms.

Shear Strength — Based on recent tests it has become evident that shear strength of reinforced concrete degrades faster with cyclic load, for higher ratios of shear demand to shear supply. Wood and Sittipunt[8] had proposed a limit of 60% as a cutoff point in identifying shear failures from other types of failures when processing the experimental literature on walls. Thus, a shear failure is likely to occur when $0.6V_{shear}< V_{flex}$, even if the nominal check prescribed by the code holds, namely that $V_{shear}<V_{flex}$.

Various models have been proposed to establish shear strength of reinforced concrete as a function of deformation [e.g. Priestley[9], Moehle[10]]. A common working hypothesis is that the shear strength of cracked reinforced concrete comprises a primary contribution, V_w of the web reinforcement (the tension ties of the Ritter–Moërsch truss analogy) and secondary contributions, V_c. Here, these terms are operated on by a strength reduction coefficient, k, which is a function of imposed displacement ductility, μ_Δ [Lynn[11]]:

$$V_{shear} = k\cdot\left(V_w + V_c\right);\quad 0.7\leq\ k=(1.15\text{-}0.075\mu_\Delta)\ \leq\ 1.0 \tag{6a}$$

$$V_w = A_{st}\sum_{n_{st}} f_{st,i} \tag{6b}$$

The expression for k was selected from among the various published alternatives due to its simplicity but also because it was found to produce conservative estimates when tested against a large database of experiments on prismatic RC members under reversed cyclic load [Syntzirma[5]]. Note that the reduced shear strength value is 70% of the initial value at a displacement ductility of 6. This is consistent with theoretical investigations based on the diagonal compression field theory, which have shown that at a ductility of shear distortions in the order of 2 (yielding of shear reinforcement occurring in a range of shear distortion of 0.004–0.006), the total shear resistance is reduced to 75% of the initial value [Tastani[12]]. (Ductility of shear distortions concerns the ratio of peak to yield shear strain in the plastic hinge region, γ_u/γ_y; therefore the total displacement ductility is much higher than the reference value of 2). Through k it is recognized that the truss component V_w is also affected by damage accumulation due to the tension–softening of the compressive struts that support the truss function [Lynn[11], Vecchio[13], Martin–Perez[14]]. V_w is nonzero only if ties are spaced close enough to secure that any diagonal crack (taken for simplicity at an angle of 45° with respect to the longitudinal axis of the member) is crossed by at least one stirrup layer. In Eq. (6b), n_{st} is the number of stirrups crossing a diagonal web crack, obtained as the integer part of the d/s ratio, where s is the spacing of stirrups. A_{st} is the area of all stirrup legs in a single stirrup layer, crossing the crack plane. $f_{st,,i}$ is the maximum stirrup tensile stress developed over the anchorage length of the i–th stirrup. Open stirrups, or inadequately anchored stirrups may not be able to develop their full yield strength, $f_{y,st}$, if the critical section where they cross the crack is very near to their anchorage. Thus, $f_{st,i}=f_{y,st}\cdot L_{b,i}/0.7L_b$, where $L_{b,i}$ is the available anchorage length of the i–th tie measured from the point where it is intercepted by the crack to the end hook, and L_b is the standard straight development length for the bar diameter of the ties considered. For open stirrups and usual section heights, the sum in Eq. (6b) corresponds to $0.5n_{st}f_{y,st}$, i.e. only half the stirrups are effective.

The concrete contribution term is attributed to other mechanisms of resistance that get mobilized through diagonal tension of the concrete web, i.e., the dowel action of longitudinal reinforcement spanning across cracks, the frictional interlock between cracked interfaces and reinforcement to concrete bond (tension–stiffening) along the bar between successive cracks. A point of difference between the various models is whether the contribution of axial compression to shear resistance (which is believed to delay opening and affect inclination of cracks) be accounted for under V_c or whether it be added separately to highlight its significance as a distinct mechanism of resistance (referred to as arch action). It can be shown that shear distortion imposes a tensile strain on all reinforcements, the magnitude of which may be estimated from equilibrium and compatibility requirements. If the axial stress level in the member is low or negligible, the net stress in all longitudinal reinforcement may be tensile, particularly for low aspect ratios (high shear demands) [Tastani[12]]. Based on the 45° truss model, the tensile strain increment in the longitudinal reinforcement resulting from shear action alone is, $\varepsilon_s=0.5V_s/E_sA_s$ where V_s is the shear force that is actually resisted by the stirrups (Eq. 6b). Similarly, strain owing to shear develops in transverse reinforcement: ε_{st} is given as $V_s/E_s\cdot n_{st}\cdot A_{st}$ (defined in Eq. 4). Considering these strain values as additive to the flexural strains, it is concluded that unless the compression strains caused by flexure or axial load in the compression zone are very high, the compression reinforcement may carry a net tensile strain upon load reversal (i.e., a full reversal of load will not produce symmetric strains in tension and compression). This means that compression reinforcement is principally susceptible to sideways buckling as described in the following section. A simple assumption is that upon load reversal, if the normalized axial load is less than $(\rho_{s1}-\rho_{s2})f_y/f_c'$ it should be assumed that the cracks remain open and therefore the V_c term in Eq. (6a) should be taken zero (this is similar to the limit $N/A_gf_c' < 15\%$ prescribed by Codes for critical zones of beams). Hence:

$$\text{if } \frac{N}{A_g \cdot f_c^{'}} \geq (\rho_{s1} - \rho_{s2})\frac{f_y}{f_c^{'}} \quad \Rightarrow V_c = 0.5\sqrt{f_c^{'}} \cdot \left(\frac{d}{L_s}\sqrt{1 + \frac{N}{0.5\sqrt{f_c^{'}} \cdot A_g}} \right) \cdot A_g \ (MPa) \quad (7a)$$

$$\text{otherwise,} \quad V_c = 0 \qquad (7b)$$

Thus, the V_c term in Eq. (7) is defined as the product of the principal tensile stress at diagonal cracking by the gross section area. N is the axial load and ρ_{s1}, ρ_{s2} are the tension and compression reinforcement ratios respectively. The tensile strength of concrete is taken as $0.5\sqrt{f_c^{'}}$ (MPa) ($6\sqrt{f_c^{'}}$ in psi).

Anchorage and Lap–Splice Strength — Premature failure of a lap–splice or anchorage effectively limits the force developed in the reinforcing bar to a value lower than its axial strength. In brittle construction, common bond–related problems are owing to (a) the practice of splicing the main column reinforcement just above the base of each floor (i.e., within a critical section), usually providing small lap lengths without the necessary confinement through stirrups, (b) the use of smooth reinforcement, where anchorage capacity depends on frictional mechanisms mobilized along the anchored length.

The force F_s, that a lap–splice or anchorage zone of length L_b may develop, is equal to the total frictional force that develops on the bar lateral surface within the length L_b. The frictional force is proportional to the normal clamping force, through the frictional constant μ_f; for concrete, μ_f is usually taken between 1 and 1.5 [Priestley[7]]. In the absence of stirrups in the lap region, the clamping pressure is only provided through the tensile resistance of the concrete cover, $f_t^{'}$, developing over a crack path of length p as defined by Priestley[7]. Therefore, the force that an unconfined lap splice may develop equals to, $F_s = p \cdot f_t^{'} \cdot L_b$, and the corresponding bar stress $f_s = p \cdot f_t^{'} \cdot L_b / A_b$, where A_b is the cross sectional area of one lapped bar. Even if $f_s \geq f_y$, the actual strength that may be supported by the lap length without stirrups will disintegrate upon load reversal. The lap region being also a critical column segment, will be located alternatingly in the compression and tension zone of the column cross section as the direction of the seismic force reverses. For axial compressive strains in excess of 0.0015–0.0020, tensile cracks parallel to the direction of the compressive force are expected to occur (with axial compressive strains in that range, the corresponding transverse strains are approximately half that value, i.e. Poisson's ratio near peak stress ≈0.5). Upon reversal of the load, the lap region will try to develop tension force in the bar, but with the cover cracked, the clamping force will be diminished.

Therefore, even if the theoretical flexural strength is reached once, upon the first cycle of loading to a full reversal, response is expected to degrade rapidly. Using the familiar k_{tr} concept of ACI 318[15] the development capacity of each lapped longitudinal bar is obtained from the maximum clamping force that stirrups may provide, determined as

$\mu \cdot n_{st} \cdot A_{tr} f_{yst}/n_b$ where n_b is the total number of bars restrained by a total of n_{st} stirrups in the direction considered along the lap length L_b and A_{tr} is the area of stirrup legs along one direction of restraint (for stirrups to be accounted for in this calculation, they should interrupt the crack path p mentioned in the preceding). A coefficient of friction equal to *1.4* was suggested when transverse reinforcement is used as a restrainer. Thus, the axial force supported by each lapped longitudinal bar is,

$$F_s = 1.4 \cdot A_{tr} \cdot f_{y,st} \cdot n_{st} / n_b + p \cdot f_t' \cdot L_b \qquad (8)$$

The second term is to be ignored if the maximum axial compression strain exceeds 0.002.

<u>Dependable Ductility at Buckling of Compression Steel</u>

Upon reversed cyclic loading the strength of compression reinforcement is affected primarily by the load history and the arrangement of stirrups thereby influencing the value of V_{buckl} in Eq. (2). Here $V_{buckl}=M_{buckl}/L_s$ where M_{buckl} the flexural moment of the cross section when compression steel reaches imminent buckling conditions.

In the absence of well anchored, stiff and closely spaced ties, elastic symmetric buckling may occur before the bar may reach yielding. An *S400 (GR60)* compressed bar yields in compression before it buckles if stiff ties are spaced at $s/D_b=17.5$; the corresponding ratio for *S220 (GR33)* is *23.7*. If ties are very flexible, it is conservative to assume that the length of the buckled bar may extend over the entire length of the plastic hinge, which may encompass several tie spacings. The corresponding elastic axial compressive strain sustained by the bar is $\varepsilon_{s2}=\pi^2 D_b^2/16 l_p^2 \le \varepsilon_y$. If elastic buckling may be safely precluded, the strain ductility of compression reinforcement is calculated considering the following:

<u>(1) Effect of s/D_b ratio on dependable strain —</u> In the plastic hinge region where cracks remain open under cyclic shear reversal, sideways buckling is the expected mode of failure of compression reinforcement. It can be shown that the relationship controlling the critical spacing of ties and the reinforcement stress that may be sustained prior to buckling is given by [Syntzirma[16]],

$$s/D_b = \psi\sqrt{E_s/f_s} \;\; ; \quad E_s = 200\,GPa, \text{ if } f_s \le f_y; \; E_s = E_r, \quad \text{if } f_s > f_y \qquad (9)$$

where ψ is a coefficient taking on the values of 1.5 and 0.785 in the cases of symmetric and sideways buckling, respectively. In the remainder of this study, the value of ψ=0.785 will be used to account for the effect of strain reversal and permanent distortion in the plastic hinge region on the shape of bar buckling. E_r is the double modulus stiffness which is a weighted average between the tangent stiffness of the bar, E_h, and the initial elastic modulus E, to account for the elastic unloading of the tension side of the buckling bar as it bends. For easy reference the ratio of E_r/E, is plotted against the ratio E_h/E in Fig. 1a [Papia[17]]. Given the tie spacing in a member with poor detailing, the dependable axial compressive strain at which reinforcement is likely to buckle, ε_{buckl}, is calculated