phenomenon of fatigue fracture. It has been observed that fatigue failure is due to progressive internal micro-cracking. Progressive damage under fatigue loading is also indicated by reduction of the slope of the compressive stress-strain curve with an increasing number of cycles. In addition to internal micro-cracking, fatigue loading is also likely to cause changes in the pore structure of hardened cement paste. Similar to the behavior of concrete under sustained loads, the strain of concrete during repeated loading increases substantially beyond the value observed after the first load application[12]. The strain at fatigue failure is much higher than that at static load.

Just as concrete beams would fail in many different ways under static loading, this is also true for fatigue loadings. Identical beams may fail differently, depending on whether they are subjected to static or repeated load. Thus, it is important to examine all modes of failure during the damaging process in fatigue loads, and their resistance relationships need to be modified for describing damage effects.

PROPOSED MODEL FOR DAMAGE ACCUMULATION OF RC BEAMS

An analysis model for predicting the cyclic behavior of concrete beam under shear and flexure load will be discussed in this section. A considerable amount of research has been devoted to the problem of predicting the full deformation response of RC beams in shear. An outcome among these researches is the MCFT proposed by Collins and Mitchell. This theory can be used not only to describe the ultimate state for strength design, but also to explain the service load behavior.[13 14] In this study, it is important to describe the gradual damage accumulation of RC beams under fatigue loading. Thus, the MCFT is selected for the analysis of cyclic response of RC beams under shear and flexure.

To describe the damage process of RC beams under repeated loads, the MCFT developed originally for static loads has been modified in this study. The modification is mainly based on the simple scalar damage concept. As strain increases with loading cycles influences the stress-strain relationships of diagonal compression struts, the cyclic behavior of RC beam under repetitive loads can be expressed in the proposed model.

Concrete Stress-Strain Relationships

The concrete stress-strain relationships in compression strut of a beam are based on the average stress and strain concepts. Relationships linking average stresses to average strains are required for both the reinforcement and the concrete. It follows that the average stress-average strain relationships for the reinforcement and the concrete may not be completely independent of each other. However, for the sake of simplicity, it will be assumed that these relationships are independent.

According to the Vecchio's proposition, the expression for the parabolic stress-strain relationship of principal compression may be written as Eq. (1).

$$f_d = f_c' \left[2\left(\frac{\varepsilon_d}{\varepsilon_o}\right) - \beta\left(\frac{\varepsilon_d}{\varepsilon_o}\right)^2 \right] \tag{1}$$

where f_c' is the cylinder compressive strength, ε_o is the strain at peak stress f_c', and peak stress f_c', and β is a function of the strain ratio $\varepsilon_{dt}/\varepsilon_d$. ε_d is the strain of diagonal strut, and ε_{dt} is the strain in transverse direction of that.

The specific form of β in Eq. (1) was developed by plotting the values of f_c'/f_p' against $\varepsilon_{dt}/\varepsilon_d$ and determining the equation of the line of best fit by Vecchio and Collins. Thus, it can be seen that an approximation for β is :

$$\beta = 0.85 + 0.27\,\varepsilon_{dt}/\varepsilon_d \tag{2}$$

It is assumed that the average tensile stresses may exist even after cracking due to the tension stiffening effects in the cracking zone.

Cracking Stress

The cracking stress was taken equal to the principal tensile stress in the concrete corresponding to the load where the crack is first detected.

$$f_{cr} = 0.33\sqrt{f_c'}, (MPa) \tag{3}$$

This cracking stress is needed to define the stress-strain relationships in tensile direction. In case of stresses in the concrete below this cracking stress, a linear relationship between stresses and strains is used. Moreover the relationship based on the tension stiffening concepts was used in stresses in the concrete beyond this cracking stress.

Steel Stress-Strain Relationships

As discussed in the previous section, relationships linking average stresses to average strains are also applied to the reinforcing steel. Because the actual strain distributions along the bar are unknown, it will be rational that the average stress-strain relationships are need in steel.

COMPATIBILITY CONDITION

The strains in reinforcements and concrete are all defined by the Mohr's circle of strain shown in Fig. 1. The compatibility condition was constructed by the relationships in this figure as in Eq. (4) ~ Eq. (6).

$$\varepsilon_l = \varepsilon_d \cos^2\alpha + \varepsilon_{dt} \sin^2\alpha \tag{4}$$

$$\varepsilon_t = \varepsilon_d \sin^2\alpha + \varepsilon_{dt} \cos^2\alpha \tag{5}$$

$$\frac{\gamma_{lt}}{2} = (-\varepsilon_d + \varepsilon_{dt}) \sin\alpha \cos\alpha \tag{6}$$

where ε_l is the longitudinal strain, ε_{dt} is the vertical strain, and γ_{lt} is the shear strain in stress field.

EQUILIBRIUM CONDITION

The equilibrium conditions are concerned with the stresses in the reinforcements, the stresses in the concrete, and the relationship between these stresses and strains in the beam layer. The stress conditions in the concrete are described by the Mohr's circle of stress shown in Fig. 2. The geometry of the stress circle establishes three independent equilibrium requirements for the concrete stress field.

$$f_l = f_d \cos^2\alpha + f_{dt} \sin^2\alpha + \rho_l f_{sl} \tag{7}$$

$$f_t = f_d \sin^2\alpha + f_{dt} \cos^2\alpha + \rho_t f_{st} \tag{8}$$

$$v_c = (-f_d + f_{dt}) \sin\alpha \cos\alpha \tag{9}$$

where f_l is the stress of longitudinal direction, f_t is the stress of vertical direction, f_{dt} is the transverse stress of diagonal strut, v_c is the shear stress, and $\rho_l f_{sl}$, $\rho_t f_{st}$ are the longitudinal and vertical stresses of steel respectively.

CYCLIC DAMAGE PROCESS

In this study, an analytical model is formulated to describe the phenomenal state of beams under fatigue loads, that is, the increase in deflection, strain and crack widths of reinforced concrete beams. This could be conducted by using the fatigue properties of the constituent materials. Almost all investigations for the fatigue properties of RC beams have been conducted mainly to obtain the fatigue life of RC beams. Only a few researchers have been interested in the serviceability aspect of the RC beams subjected to fatigue loading[15]. Furthermore, research on the cyclic behavior of concrete beams under shear loads has seldom been performed.

As described previously, fatigue loads would cause the increase of strains and deflections in RC beams even under the same load levels. This increase in deflection and strain of a reinforced concrete beam subjected to repeated loading can be attributed primarily to:

(i) the cyclic creep of concrete in the compression zone,
(ii) the reduction in stiffness contribution of the tension zone concrete due to the fatigue-tensile cracking and the progressive deterioration of the bond between steel and concrete, and
(iii) the cyclic strain softening of reinforcing steel.

Increasing deformations of concrete subjected to cyclic compression have been studied and reported by Whaley and Neville[16]. They have shown that the cyclic variation of stresses from f_{max} to f_{min} leads to an increasing strain in the uniaxial compression specimens. This means that the increasing strain of the compression zone of concrete beams subjected to cyclic loads should be a significant factor in increasing deflections and crack widths. This phenomenon could also be used to describe the cyclic effects on the RC beams as the expression of changes in strain-stress relationships.

Based on the their test data, Whaley and Neville have shown that the cyclic increasing strain can be expressed as the sum of two strain components. The mean strain component is the creep strain produced by the static mean stress $f_m = (f_{max} + f_{min})/2$. The additional cyclic component was found to depend on both the mean stress f_m and the stress range $f_r = (f_{max} - f_{min})$. They proposed the following predictive equation for the total cyclic increasing strain.

$$\varepsilon_c = 129 f_m t^{1/2} + 17.8 f_m f_r N^{1/3} \qquad (10)$$

where, ε_c is the cyclic increasing strain in micro mm/mm,

f_r is the stress-range expressed as a fraction of the compressive strength,

f_m is the mean stress expressed as a fraction of the compressive strength,

t is the time from start of loading in hours,

N is the number of cycles of repeated loads.

Knowing the maximum and minimum stress, the number of cycles and the frequency of loading, the increasing strain can be predicted using Eq. (10). As noted by Whaley and Neville the fit may be considered acceptable for values of $f_m < 0.45$. This limitation may, in most cases, not be restrictive because the maximum working stresses normally do not exceed this value.

The reduction in stiffness can be expressed as the cycle-dependent secant modulus due to increasing strain. Eq. (11) can be used to develop a cycle-dependent secant modulus for concrete in compression. For a given number of cycles, the total strain (instantaneous + cyclic) corresponding to f_{max} can be obtained using Eq. (11) and knowing the initial secant modulus. (see Fig. 3). The relation can be expressed as:

$$E_s = \frac{f_{max}}{\dfrac{f_{max}}{E_o} + \varepsilon_c} \tag{11}$$

where, E_o is the initial secant modulus and,

E_s is the cyclic modulus after N number of cyclic loads.

In this study, the MCFT which has been previously used in the static case, is adopted for predicting the cyclic behavior of concrete beams under shear and flexure. Thus, a new relationship is needed to explain the cycle dependent principal compressive stress-strain relation.

As discussed in the previous chapter, the principal compressive stress-strain relationship was described in the Eq. (1). This equation can be modified to explain the reduction of stiffness due to the increasing strain under repeated load cycles. This modification is conducted simply by introducing the scale factor α that represents the scalar damage in the Eq. (12). This scale factor modifies the parabolic principal compressive stress-strain relationship by reducing the normalized ordinate as in Eq. (13).

$$f_d = \alpha f_c' \left[2\left(\frac{\varepsilon_d}{\varepsilon_o}\right) - \beta\left(\frac{\varepsilon_d}{\varepsilon_o}\right)^2 \right] \tag{12}$$

$$\alpha = \frac{2\left(\frac{\varepsilon_d}{\varepsilon_o}\right) - \beta\left(\frac{\varepsilon_d}{\varepsilon_o}\right)^2}{2\left(\frac{\varepsilon_d + \varepsilon_c}{\varepsilon_o}\right) - \beta\left(\frac{\varepsilon_d + \varepsilon_c}{\varepsilon_o}\right)^2} \tag{13}$$

where f_c' is the cylinder crushing strength, ε_o is the cylinder strain at peak stress f_c', β is a function of the strain ratio $\varepsilon_{do}/\varepsilon_d$, and α is a function of cyclic increasing strain as shown in Fig. 4.

In Eq. (12), α should be calculated at each load repetition because of stress redistribution, but for the simplicity of calculation, it was extracted in constant intervals.

ANALYSIS SCHEME

Basically all schemes in the analysis were similar to those in MCFT except the damage process due to cyclic effects. A RC beam can be discretized into a series of concrete strips, with superimposed longitudinal steel components. Each concrete strip is defined by its width(b), depth(d), amount of transverse reinforcing steel(ρ_t), and position relative to the top of the beam(y_c). Similarly, the longitudinal steel components are defined by their cross sectional area(A_s), yield strength(f_{yl}), and position relative to the top of the beam(y_s). Properties common to the entire beam cross-section include the concrete cylinder strength(f_c'), concrete strain at peak stress(ε_o), yield strength of the transverse steel(f_{yt}), and Young's modulus for steel(E_s). Each concrete strip and longitudinal steel component is decoupled and analyzed individually. The stress level of each concrete strip is determined by the above iterative analytic procedure. The increase in diagonal principal compression in the struts due to cyclic strain is calculated by the assumed cyclic strain increase function. New stress-strain relationships of diagonal compressive concrete strut are determined, and the re-analysis of the beam section is performed.

ANALYSIS RESULTS

The analysis of the response of RC beams under repeated shear-flexure loading will be estimated, and compared with the experimental results. In this section, the variations of deflection at the maximum load condition will be plotted, corresponding to the increasing number of loading cycles. At each loading cycles, the same maximum and minimum loads are applied, but the damage caused by the repeated load made lower the stiffness of the diagonal strut and higher the stresses in transverse stirrup steel, which carry the principal compressive stress and transverse tension stress, respectively.

The deflections and strains of the RC beams are gradually increased with the repetition numbers, although the same magnitude of loading is applied cyclically. The reason of these deflection and strain increase can be explained from the stiffness reduction, cyclic creep, bond slipping, and increasing number of cracks. As shown in the experimental results, the diagonal tension cracks were developed abruptly from the end of the flexural tension crack to the longitudinal compression steel point at a certain load although the transverse stirrup steel had been installed in the beam.

RC beams under repeated shear-flexure loadings are analyzed by the proposed method. Those are loaded with simply supported four-point loading condition, and their responses such as maximum deflections, steel strains, concrete strains are monitored at each loading stage as shown in Fig. 5.

The analytical results of maximum deflections and curvatures at the maximum load of stage at each cycle show the degradation effect due to the repeated shear loads. As shown in Fig. 6, and Fig. 7, the deflections at 10^6-th cycle increase up to 14% larger than those at the first cycle in the same condition. The curvatures also increase up to 15% at 10^6-th cycle, compared with these at the first cycle.

The analytical results are compared with the experimental results to verify the validity of the proposed method. Those comparisons are shown in Fig. 8 ~ 9. The present method was also compare with Lovegrove's results[17] as in Fig. 10. The analytical results show same trend as shown in the experimental results and these trends correlate well to the experimental results of a large number of cycles.

By the proposed analytical method, it is possible to determine the tensile, compressive and stirrup steel strain variations under the repeated loads. The strains also increase when the number of loading cycles increases. Bearing in mind the random nature of crack development in the concrete section of test members, some variations of steel strains that are much influenced by the crack path in the test, are unavoidable. Thus, the steel strains are determined by the value of the average concepts that was assumed in the proposed analytical procedures, and this value may be used in the design procedure.

Fig. 11 ~ 13 show the variation of maximum compressive concrete strains, maximum diagonal principal compressive strains, stirrup steel strains and longitudinal tensile steel strains with the number of repeated loads respectively. The diagonal principal compressive strains and stirrup strains are greatly increased up to about 50% in the 10^6-th cycle, compared with those at the first cycle. Especially, the stirrup strains at higher load ranges show considerable increase of about 60% in the 10^6-th cycle, compared with those at the first cycle. Fig. 19 and Fig. 20 also show the comparison of proposed analytical results with the test results. In these figures, the comparison is made with the present experimental results of longitudinal tensile, compressive and stirrup steel strains. Fig. 20 implies that experimental results show a little bit more degradation than the analytic results in higher cycles. These might have been caused by other mechanisms which are not considered in the present analysis such as bond slips and abrupt crack propagation just before failure.

CONCLUSIONS

In this study, a quantitative analysis technique for the damage process of reinforced concrete beams under shear fatigue loading is proposed, which can express the progressively increasing strain and stiffness degradation. The analysis technique is mainly based on modifying the MCFT and scalar damage concept, which describes the strain and stress configuration in the shear zone by considering the 2-dimensional effect, and degradation of principal compressive strut by cyclic strain increase, secant modulus decrease, and modifying the parabolic stress strain relationship. Through this study, the following conclusions can be drawn.

1. An analytical method which can describe the cyclic damage of reinforced concrete beam under repeated loading is proposed. The proposed analysis technique is based on the modified compression field theory and considers the damage of the principal compression strut according to strain increase, and introduced an adjusted parabolic principal stress-strain relationship.
2. The analytical results indicate that even though the required stirrup steels are installed, the deflections, curvatures and stirrup strains increase up to 14%, 15%, 60% after 10^6 load repetitions respectively, compared with those at first cycle. These results imply that the diagonal cracks in the beam may induce serious serviceability problems under the repeated loads.
3. It is important to evaluate the damage accumulation of beam under repeated loads for not only safety, but also serviceability aspect. The proposed analytical technique can be used to predict the response of reinforced concrete beams subjected to repeated loads and to evaluate the serviceability of those structures under fatigue loads.

REFERENCES

1. Hsu, T. C., "Is the staggering concept of shear design is safe?," Journal of ACI, Proceedings Vol.79, No.6, Nov.-Dec., 1982, pp.435-443.

2. Collins, M. P., and Mitchell, D., "Shear and Torsion Design of Prestressed and non-prestressed Concrete Beams," Journal of the Prestressed Concrete Institute, Vol.25, No.5, Sept.-Oct., pp.32-100.

3. Ueda, T, "Behaviour in Shear of Reinforced Concrete Beams fior Static and Repeated Loads," Dr. Thesis, University of Tokyo, April 1979.

4. Rahal, K. N., and Collins M. P., "Analysis of Sections Subjected to Combined Shear and Torsion - A Theoretical Model," ACI Structural Journal, V. 92, No. 4, ACI, July-August, 1995, pp.459-469.

5. Ritter, W., Die Bauweise Hennebique (Construction Techniques of Hennebique), Schweizerische Bauzeitung, Z rich, Feb. 1899.

6. Morsch, E., Concrete-Steel Construction, McGraw-Hill Book Company, New York, 1909, pp. 368. (English translation by E.P. Goodrich of 3rd ed. of Der Eisenbetonbau, 1st ed., 1902.)

7. Kupfer, H., "Erweiterung der M rsch'schen Fachwerkanalogie mit Hilfe des Princips vom Minimum der Form nderungsarbeit," CEB, Bull, Inf., No.40, June 1964, pp.44-57.

8. Thurliman, B., "Plastic Analysis of Reinforced Concrete Beams," IABSE Colloquium, Copenhagen, 1979. Plasticity in Reinforced Concrete, Introductory Report, Zurich, International Association for Bridge and Structural Engineering, Reports of the Working Commissions, Vol.28, Oct. 1978, pp.71-90.

9. Vecchio, Frank J., "Reinforced Concrete Membrane Element Formulations," Journal of Structural Engineering, ASCE, Vol. 116, No.3, March 1990, pp.730-750.

10. Hsu, Thomas T. C., "Softened Truss Model Theory for Shear and Torsion," ACI Structural Journal, V.85, No.6, Nov.-Dec. 1988, pp.624-634.

11. Schlaich, J., Schafer, K., and Jennewein, M., "Toward a Consistent Design of Reinforced and Prestressed Concrete Structures," PCI Journal, Vol.32, No.3, pp.74-151.

12. Raju, N. K., "Small Concrete Specimens Under Repeated Compressive Loads by Pulse Velocity Technique," Journal of Materials, V.5, No.2, June 1970, pp.262-272.

13. Khaldoun, N. R., and Collins, M. P., "Analysis of Sections Subjected to Combined Shear and Torsion - A Theoretical Model," ACI Structural Journal, Vol.92, No.4, July-August, 1995, pp.459-469.

14. Chung, W. K., and Ahmad, S. H., "Model for Shear Critical High-Strength Concrete Beams," ACI Structural Journal, Vol. 91, No. 1, Jan.-Feb., 1994, pp.31-41.

15. Shah S. P., and Chandra S., "Fracture of Concrete Subjected to Cyclic and Sustained Loading," ACI Journal, Vol. 67, No. 49, Oct. 1970, pp.816-825.

16. Whaley, C. P., and Neville, A. M., "Non-Elastic Deformation of Concrete Under Cyclic Compression," Magazine of Concrete Research, Vol.25, No.84, Sep. 1973, pp.145-154.

17. Lovegrove, J. M. and Salah, E. D., "Deflection and Cracking of Reinforced Concrete under Repeated Loading and Fatigue," ACI SP75-6 Edited by Shah, 1982, pp.133-152.

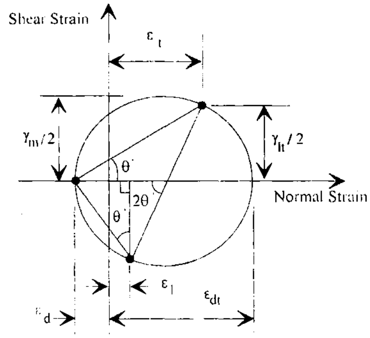

Fig. 1 Mohr's circle of strain in concrete

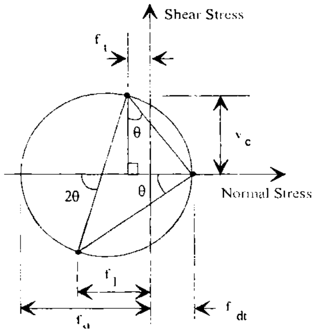

Fig. 2 Mohr's circle of stress in concrete

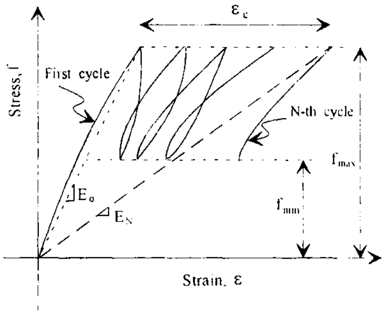

Fig. 3 Cycle dependent secant modulus of elasticity

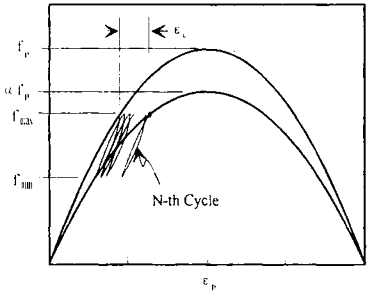

Fig. 4 Principal compressive stress-strain relationships considering the cyclic effect

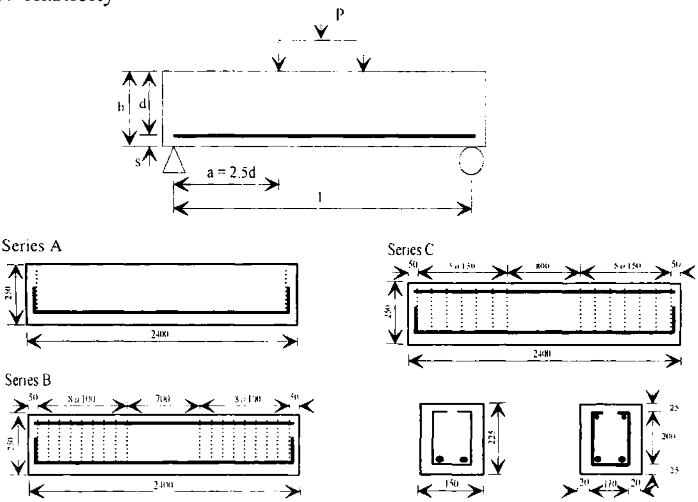

Fig. 5 RC members used in this analysis and experiment