Mathematical Modeling of Infilled Frames

By Richard E. Klingner

Synopsis: Two approaches are discussed for mathematical modeling of the elastic and inelastic response of infilled frames. The first approach is based on idealizations of local behavior, while the second is based on observations of overall behavior. Both approaches are found to give good representations of nonlinear response. The second approach, based on the equivalent strut concept, is believed to be efficient for use in analyzing the response of complex, infilled frame structures.

Keywords: computer programs; cracking (fracturing); dynamic loads; earthquake resistant structures; elastic analysis; frames; infilled frames; lateral pressure; mathematical models; reinforced concrete; structural analysis.

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BACKGROUND

There is increasing recognition among structural engineers that elements which have traditionally been considered nonstructural can significantly affect the behavior of a structure subjected to dynamic loads such as wind and earthquake. This recognition is due, among other things, to the development of improved analytical techniques for modeling the mechanical behavior of nonstructural elements in the inelastic as well as elastic ranges. These techniques may be classified into two broad groups: (1) local or microscopic approaches, in which idealized local behavior (cracking, yielding, bond-slip, etc.) is used to predict force-deflection relationships at the subassemblage level, and (2) macroscopic approaches, in which idealizations are developed predicting observed subassemblage behavior at an overall level only. This paper will discuss examples of each approach which are relevant to the modeling of infilled frames. These approaches are general. However, the specific examples discussed herein apply to reinforced masonry infills bounded by reinforced concrete frames.

Many investigators have studied the effects of infill panels on the response of frame structures to lateral loads. Experimental studies (5,7,8,10,15) have shown that in the elastic range infill panels stiffen and strengthen the frame. In the inelastic range, infill panels can contribute significantly to the frame's energy dissipation capacity through the development of friction across distributed panel cracks (4,8).

Under very low levels of lateral load, the infill panel does not crack, nor does it separate from the bounding frame. Lateral force-deflection behavior is linear and elastic. Small lateral loads cause the formation of cracks along the boundary between the frame and the panel. Although these boundary cracks do not significantly reduce the lateral stiffness of the infilled frame (8), their presence can be included in conventional finite element analyses using bond-link elements (9,13).

Increased lateral loads, however, damage the panels themselves. Figures 1-3, taken from Ref. 8, show characteristic steps in the deterioration of an infilled frame subjected to increasing levels of reversed lateral load. Figures 1 and 2 show the formation of extensive cracks parallel to the compression diagonal, followed by crushing and spalling of the infill along the compression diagonal. As shown in Fig. 3, deterioration of the infill panel at one story level causes a reduction in the infilled frame's lateral story stiffness at that level. Further cycles of reversed loading cause severe deterioration in that panel, and lead to the formation of a sidesway mechanism in the frame (5,8).

It is evident that infilled frame structures exhibit many kinds of nonlinear inelastic behavior following the onset of extensive panel cracking. The panel cracks cause a reduction in the panel's shear stiffness. Increasing stress levels lead to nonlinear stress-strain behavior in local regions of the panel. After portions of the panel spall or crush, panel cracks increase greatly in width, and significant slip displacements occur along these cracks. Reversed cycling to large lateral displacements results in severe deterioration along the cracks, and causes the formation of hinge regions in the bounding frame. Lateral load resistance of the infilled frame at this stage depends on the stiffness of the frame's beam-column connections, the available rotation capacity of the hinge regions, and the rate at which the panels deteriorate.

After the formation of extensive panel cracks, infilled frame behavior cannot be modeled accurately using linear elastic idealizations. The resistance of the cracked shear panel must be idealized using either local or macroscopic approaches. An example of each type of approach will be discussed, and the results will be compared.

ANALYSES USING LOCAL MODELS

Infill panels and other types of shear elements have been analyzed by several researchers using local models (2,3,6). An example of this type of analysis, applied to reinforced concrete shear panels, is discussed by Darwin and Pecknold in Ref. 3. Plain concrete was modeled as a nonlinear orthotropic material. The effects of softening under compressive stresses were included, as well as the formation of cracks under tensile stresses. Crack closing under load reversal was accounted for. Figure 4, taken from Ref. 3, shows that Darwin and Pecknold's proposed model agrees satisfactorily with the experimental load-deflection curve of a cyclically loaded shear panel (2).

Local models for infilled frames can consider other phenomena: the nonlinear moment-curvature behavior of the frame members can be modeled, as discussed by Aktan, Pecknold, and Sozen (1); local bond slip in the panels and frame members can be modeled using idealizations such as that of Morita (12); boundary cracks between the frame and panel can be idealized using techniques similar to those of Refs. 3 and 6. As noted previously, local models can give good reproductions of overall load-deflection characteristics. They can also be useful for studying local behavior. Figure 5, taken from Ref. 3, shows the similarity between analytically predicted and experimentally observed cracking patterns for a shear panel.

However, local models have several disadvantages. Their accuracy depends on precise multiaxial constitutive relations for all materials involved. Such relations are not available at this time for many common construction materials, particularly unit masonry. Local methods, while a valuable research tool, are too costly and time-consuming for use in analyzing the response of large, complex structures.

ANALYSES USING MACROSCOPIC MODELS

Macroscopic models are useful in analyzing large structures which would otherwise require prohibitive amounts of storage if analyzed microscopically. However, their accuracy is limited by the extent to which they duplicate actual behavior rather than simply fitting an experimental curve. Macroscopic models can generally be used most successfully in idealizing the behavior of elements whose basic models of structural resistance are well known, and which can be designed to respond stably in those resistance modes. In addition to predicting a wide range of experimental behavior, a theoretically sound macroscopic model can give information about the relative importance of various local mechanisms, thus aiding in the improvement of local idealizations.

One specific example of the usefulness of macroscopic models is their potential application to complex, infilled frame structures. Reference 8 discusses a design approach producing engineered infilled frames with stable lateral force-deflection characteristics in the elastic and inelastic ranges. The effectiveness of this design approach was investigated in a series of quasi-static, cyclic load tests on multistory infilled frame subassemblages (Fig. 6). Figures 7-9, taken from Ref. 8, show the lateral load-deflection behavior of those subassemblages. The engineered infilled frames were found to exhibit high strength, stiffness, and energy dissipation capacity. As discussed in that reference, those desirable characteristics were obtained principally by special design for column shear resistance in excess of the infill cracking shear, and should not be attributed to ordinary infilled frames. It was decided to idealize those infilled frames using macroscopic models. The models described herein were developed for use with ANSR-I, a general purpose, nonlinear structural analysis program (11).

Experimental results indicated that engineered infilled frames behaved essentially as a combination of two types of structural components: (1) the frame members themselves; and (2) the infills, which strengthened the frames, stiffened them, and dissipated large amounts of energy through distributed cracking. The presence of the engineered infills changed the basic behavior from that of a bare frame to that of a frame braced by equivalent diagonal compression struts. The process of infill panel degradation greatly influenced the location of critical regions in the frame members, and, consequently, the final mechanism of the infilled frame subassemblage. However, the forces induced by the infills did not significantly reduce the available rotational ductility of the critical regions under cycles of reversed loading.

It was decided that the analytical model of the subassemblage should predict the following aspects of the filled frame behavior:

(1) Initial stiffness
(2) Initial strength
(3) Degrading stiffness and strength behavior, particularly the pinching effect associated with the deterioration of infill stiffness.

It was considered convenient to model the infilled frame subassemblage using two separate types of elements:

(1) The frame elements were modeled using two-component elements, each consisting of a linear elastic member in parallel with an elasto-plastic one (14). Single elements were used for the columns. The beams were modeled using several parallel two-component elements to give a combined moment-rotation curve consistent with the nonlinear moment-curvature curves of each beam.

(2) As shown in Fig. 10, a pair of equivalent diagonal strut elements was used to idealize each infill panel. Three different equivalent strut models were developed. Each successive model involved a slight increase in complexity, and produced results more closely approximating those observed experimentally.

<u>Strut Model #1</u>. It was hypothesized that the strut model should duplicate the following main aspects of the experimentally observed infill behavior:

(1) Initial stiffness and strength
(2) Decreased strength with increased deformation
(3) Decreased stiffness on reloading

A macroscopic equivalent strut element was written with the mechanical characteristics shown in Fig. 11. Note that the complete infilled frame response combined the behavior of the frame members and two equivalent diagonal struts per panel, one in each direction. The following behavior description refers to a single strut only:

(1) Elastic Loading (path OA)

This is defined by

$$S = \frac{EA}{L} v$$

where S is the axial force in the strut; E is Young's modulus for the infill material; v is the axial deformation in the strut, positive values corresponding to extension; L is the length of the strut; taken here as the distance between diagonally opposite nodes; and A is the product of the panel thickness and the effective width of the strut.

(2) Strength Envelope Curve (path AB)

This curve is defined by

$$S = Af_c(e^{\gamma v})$$

where A, S, and v are defined as above, and f_c is the compressive strength as determined from prism tests. The strength degradation parameter, γ, is selected on the basis of experience. A value of 1.0 was used in all analyses described here. The envelope curve was defined by a decaying exponential because this was the simplest class of mathematical expressions reflecting the desired characteristics of decreasing strength with increased deformation. It is probable that some increase in accuracy could be achieved by defining the strength envelope curve in terms of more complicated classes of functions.

(3) Elastic Unloading (path BC)

In this range, the strut unloads elastically, with a stiffness equal to the elastic loading stiffness of path OA.

(4) Tension Curve (path CC′D)

Initially, an actual equivalent diagonal strut has some tensile resistance, due to the tensile strength of the panel material (usually very low) and the action of the panel steel (also low). Tensile cracking of the strut causes this tensile resistance to drop immediately. The remaining tensile resistance is due to the action of the panel steel alone. In developing Strut Model #1, it was decided that the complexity necessary to model this drop in tensile resistance was not justified in view of the generally minor effects of infill tensile strength. The idealized tension curve was defined by

$$S = Af_t$$

where S and A are defined as above, and f_t is a constant nominal resistance whose value is based on the observed tensile resistance of the panel reinforcement. The strut models permit specification of arbitrary values of f_t. However, panel tensile resistance was not observed to have any significant effect on the behavior of the experimental models, and f_t was therefore assigned a zero value in all the analyses described herein.

(5) Reloading Curve (path DE)

The experimental models were observed to exhibit decreased stiffness upon reloading. Therefore, this reloading curve was defined by a straight line connecting the point on the tension curve corresponding to maximum positive deformation (point D), with the point on the strength envelope curve corresponding to maximum negative deformation (point B).

(6) Further Cycles (path EFGG$'$E)

Strut Model #1 was defined to exhibit elastic loading and unloading during further cycles within the area defined by the strength envelope curve, the tension curve, and the reloading curve. For example, the strut unloads elastically from point E until reaching the tension curve at point F. Decreasing deformation causes movement along the tension curve from point F to point G. Reloading in compression causes the strut to reload elastically until reaching the previously defined reloading curve at point G$'$. The strut then continues to reload along this curve until reaching point E again. Strut extensions beyond the deformation corresponding to point D (for instance, to point D$'$), cause the reloading curve to be redefined in terms of the coordinates of points B and D$'$. A similar redefinition takes place following compressive deformations in excess of the value corresponding to point B.

Using this Strut Model #1, the entire infilled frame subassemblage was modeled as shown in Fig. 12. This model comprised 46 elements, 18 nodes, and 40 degrees of freedom. The experimental results from all three infilled frame tests were qualitatively similar with respect to strength and stiffness degradation. It was decided to compare the analytical results with those obtained experimentally in Test #2, carried out on the clay-infilled bare frame. That test used a tip displacement history consisting of reversed cycles to increasing maximum amplitudes of approximately equal magnitude in each direction, and was therefore judged slightly more convenient for analytical comparison purposes than the other two tests, whose displacement histories were skewed in one direction. The following material parameters were used for the equivalent strut elements:

E = 8290 MPa (1200 ksi), as determined from prism tests on clay blocks loaded perpendicular to the bed joints

f_c = 24.1 MPa (3500 psi), as determined from prism tests on clay blocks loaded perpendicular to the bed joints

f_t = 0.0, as explained previously

A = 12900 mm^2 (20 in.^{2}). This figure was obtained by multiplying the nominal thickness of the panel (51 mm, or 2 in.) times the equivalent strut width (Appendix I and Ref. 10)

γ = 1.0, as explained previously

The entire model was subjected to a cycle of reversed loading, and the results shown in Fig. 13 were obtained. Because the analytical idealization was designed to exhibit decreasing strength for increasing deformations (beyond those required for panel cracking), a program of applied loads would clearly result in overall instability of the model after panel cracking. The ANSR-I computer program did not allow the model to be loaded by a program of specified tip displacements. To simulate displacement controlled loading, the structure was laterally restrained by a relatively stiff horizontal spring at the point where lateral loads were applied. In the analytical procedure, therefore, the applied load was taken primarily by the spring and partly by the structure. Lateral instability of the structure decreased the lateral stiffness of the spring-structure assemblage, but the combination remained stable. In Figs. 13, 15, and 18, the lateral force H is the load resisted by the structure alone, obtained in the analytical procedure by subtracting the resisting spring force from the total applied load.

Figure 13 also shows the corresponding experimentally obtained load-deflection curves for Test #2. Comparison of these two curves shows that use of Strut Model #1 produced a good representation of the experimentally observed initial stiffness and strength. However, the degrading behavior was not modeled correctly. In the analytical procedure, panel damage from post-cracking excursions in one direction did not reduce the maximum panel resistance available in the opposite direction.

Strut Model #2. To correct the above-noted deficiency of Strut Model #1, this model was refined to exhibit the mechanical behavior shown in Fig. 14 and described below:

(1) through (4) same as Strut Model #1 (path OABCC′D)

(5) Reloading Curve (path DB or D′B′)

Strut Model #2 defines the reloading curve in a manner slightly different from that of Strut Model #1, to reflect more accurately the effect of previous damage history on panel strength and stiffness. Experimentally, it was observed that after reaching a given resistance level in one direction, an infilled frame model was not able to develop more than this resistance in the other direction upon load reversal. Physically, this can be explained by the fact that the two equivalent diagonal compression struts share the portion of material at the center of the panel and are therefore not independent. When a single panel infilled frame, whose panel is idealized by two equivalent struts, is loaded laterally in the positive direction, one strut will be placed in compression, and the other one will be placed in tension. The compression strut will load elastically, reach the strength envelope curve, and suffer increasing damage as it moves along the path AB of Fig. 14. The tension strut will offer some nominal tensile resistance, and will intersect and move along the tension curve on a path such as OG′D. When the direction of the lateral load is reversed, the strut which was originally in compression will unload and go into tension along a path such as BCC′FGD. The strut which was originally in tension will reload in compression.

Experimental tests showed that the infilled frame developed, upon reversal of loading, a lateral resistance equal at most to the degraded resistance in the original direction. In terms of the strut model, this implies that the strut which was originally in tension (at point D, say) will not reload in compression directly to point A (Fig. 14), but rather to point B along a line DB. The reloading curve for a given strut should be defined to connect the point on the tension curve corresponding to that strut's maximum positive deformation (point D), with the point on the strength envelope curve corresponding to the maximum <u>negative</u> deformation of the <u>opposite</u> strut in the same panel. However, because of the way in which element data are stored during execution of the ANSR-I program, this type of behavior was very difficult to prescribe, and an alternative procedure was therefore devised: in the range of panel deformations associated with significant panel cracking, experiments showed that the most significant panel deformation was in shear. When a panel idealized by two equivalent compression struts deforms in shear, the axial deformations of the struts are equal in magnitude but opposite in sign, and the maximum negative (compressive) deformation of a given strut is equal in magnitude to the maximum positive (tensile) deformation of the other strut. Therefore, it was possible to define the reloading curve in the following manner: for the single cycle of reversed loading considered in this example, the reloading curve for a given strut should be defined to connect the point on the tension curve corresponding to that strut's maximum positive deformation (point D), with the point on the strength envelope curve corresponding to the maximum <u>positive</u> deformation of the <u>same</u> strut. This definition of the reloading curve was much easier to incorporate into the analytical model. Finally, consider extending this definition to loading programs other than the single cycle of reversal considered above: suppose an equivalent strut has been loaded onto the strength envelope curve (along path OAB, say), and then unloaded to the tension curve but without significant reversal. When that strut is reloaded, its strength will clearly not be governed by the negligible amount of damage (compressive deformation) suffered by the opposite strut. The strength of the reloaded strut will depend on the damage that it itself has suffered, i.e., on its <u>own</u> maximum <u>negative</u> (compressive) deformation. For the case of a general lateral load program, the reloading curve for a given strut was, therefore, defined to be the straight line connecting the point on the tension curve corresponding to that strut's maximum positive (tensile) deformation, with the point on the strength envelope curve corresponding to the maximum (absolute value) deformation--positive <u>or</u> negative--previously experienced by that <u>same</u> strut. Referring to Fig. 14, consider the following two examples: First, suppose that a strut has been loaded following the path OABCC′FGD. Because the maximum compressive deformation (point B) is greater in magnitude than the maximum tensile deformation (point D), the former will govern, and the strut will have a reloading curve defined by the straight line DB. Physically, this would represent a case in which the panel had been subjected to some load reversal, but not enough to damage the opposite strut more than the strut under consideration.

Therefore, the damage in the reloading strut (a function of the maximum compressive deformation of that strut) would control. Second, suppose that a strut has been loaded following the path OABCC′FGDD′. Because the maximum compressive deformation (point B) is smaller in magnitude than the maximum tensile deformation (point D′), the latter will now govern, and the strut will have a reloading curve defined by the straight line D′B′, where B′ and D′ are located at equal distances but opposite directions from the vertical axis of Fig. 14. Physically, this would represent a case in which the panel had been subjected to severe load reversal, damaging the opposite strut. The damage to that opposite strut would control. Such damage would be a function of the maximum compressive deformation of that opposite strut, which in turn would be essentially equal (as explained above) to the maximum tensile deformation of the strut under consideration.

(6) Further Cycles (path EFGG″E)

Strut Model #2 was defined identically to Strut Model #1 in this range. Because of the change in definition of the reloading curve between Strut Models #1 and #2, however, redefinitions of the reloading curve could occur following increases in maximum strut deformation in either sense. For example, referring to Fig. 14, the reloading curve DB would be redefined after strut deformations along the path DBB′, or after strut deformations along the path GDD′.

With the same element properties and loading program as before, the use of Strut Model #2 produced the results shown in Fig. 15. That same figure also shows the experimental behavior observed in Test #2. Because only the reloading curve had been changed from Strut Model #1 to Strut Model #2, the analytically predicted initial strength and stiffness continued to agree well with the experimental results. Strength degradation under monotonic load was also reproduced well. However, it may be seen that upon reversal of loading, Strut Model #2 did not produce the observed pinching effect associated with the opening of cracks in the panels.

Strut Model #3. To correct this deficiency, it was decided to introduce some additional refinements into the reloading behavior of Strut Model #2. As discussed in the previous subsection, this strut model exhibited linear reloading behavior. Actually, experimental observations showed that reloading behavior consisted of two distinct phases. In the first phase, the previously formed vertical panel cracks close. Until this closure is complete, the panel's lateral strength and stiffness are essentially zero. Closure occurs when the panel is returned to its undeformed configuration (or, in terms of the equivalent strut idealization, when deformations in the equivalent struts are zero). In the second phase, following panel crack closure, the panel reloads, but with reduced stiffness and strength compared to the virgin elastic behavior. In terms of the equivalent strut