PAPER SP 18-1 The torsional actions and reactions which are generated in a structure are a function of its configuration and the distribution of the dead and applied live loads. This paper discusses aspects of the torsional problem with emphasis on over-all structural action, as opposed to the consideration of individual members only. Specific reference is made to spandrel beams, edge beams in shells, an orthotropic shell, certain grid systems and curved prestressed concrete bridges. The interrelationships between the applied live loads and torsion are discussed. The need for further research and the necessity to include provisions for torsion into engineering codes of practice is emphasized.

Aspects of Torsion in Concrete Structure Design

By K. G. TAMBERG

☐ Torsion is becoming an increasingly important factor in structural design. Refinements in analysis, the greater use of novel structural forms by architects and the employment of ultimate strength rather than working stress design, have required explicit attention to torsion. Consideration of the combined effects of flexure, shear, and axial load is commonplace under present design methods, but torsion has been treated mostly informally and imprecisely. For example, many engineers plan structural frames in such a manner as to minimize the effects of torsion while others allow for torsion in a discretionary manner. It is, therefore, not surprising that some distress has been experienced.

A recent survey of structures conducted in the United States and Canada by the Portland Cement Association for ACI Committee 438 uncovered some classic cases of torsional distress. Distress occurred in connection with a variety of construction details, and the degree varied from minor to extensive cracking of a serious nature. The maximum cost of repair work encountered was over $50,000 for one structure whose value is in excess of a million dollars. This does not include the legal fees incurred as a result of litigation, the inconvenience and cost of lost space during repairs, or the damage to the professional reputation of the structural engineer. The need to account properly for torsion is thus emphasized.

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Most discussions of combined loadings acting on structural members begin with the assumption that these forces are known. It is clear, however, that the whole structure must be analyzed as a unit in order to determine these forces. The interaction of the torsional and flexural stiffnesses inherent in a structure, and the restraints imposed by component parts upon one another must be considered. Torsion results from the monolithic character of cast-in-place concrete construction, and also occurs when precast elements are employed.

On the other hand, the torsional reactions present in a structure may prove beneficial rather than detrimental in certain cases. The extra strength of structures attributable to torsional stiffness may be best mobilized when applied loads act eccentrically. Design loadings presently in use do not always take advantage of this fact, which is particularly true for bridge construction. If, however, torsional stresses govern the design, a proper assessment of the eccentricity of the applied forces is, needless to say, imperative.

SPANDREL BEAMS

Fig. 1-1 shows cross-sections of spandrel beams which are frequently used in construction. Details a, b and c are suitable for use when precast slab or beam elements are employed, or when the member framing into the spandrel is cast later. Torsion is introduced into the spandrels by the eccentric action of the vertical forces V (details a and b), which are due to members supported as indicated in detail c. Details a and c are used at the

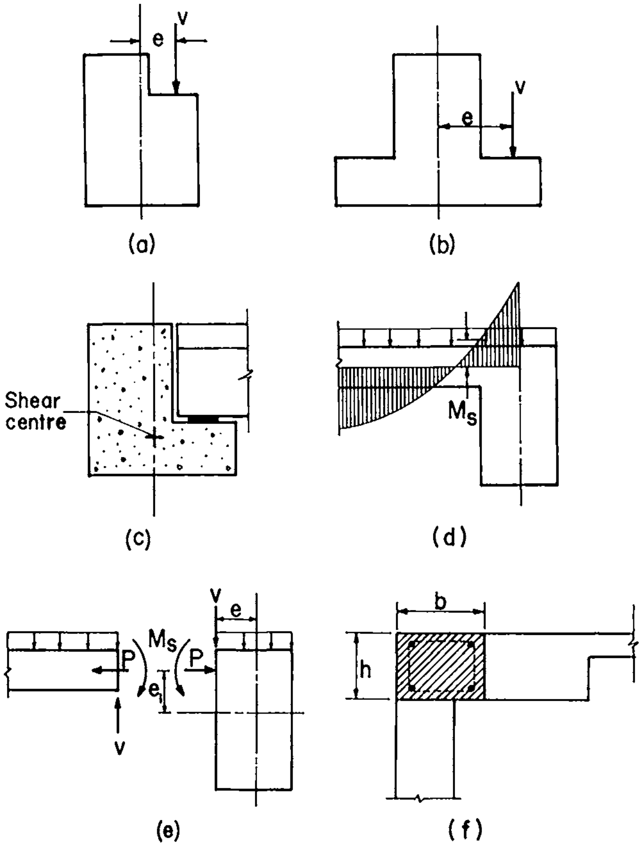

FIG. 1-1 SPANDREL BEAMS

discontinuous edges of floor systems. Detail b is generally used to support precast elements along an interior line of columns, in which case torsional moments may arise because of the presence of asymmetrical loading during erection. It may be said categorically that members under vertical load will be subjected to torsion in addition to shear and bending unless the line of action of the loads passes through the shear center of the cross-section employed. Design procedures should, of course, take this fact into account. It is noted that the shear center does not coincide with the centroid of the cross-section except for bisymmetrical or anti-symmetrical sections.

Cast-in-place spandrel beams which are integral with the floor slab present more complicated problems in design. Detail d shows the cross-section of a spandrel beam cast monolithic with the slab. The ordinate of the typical associated moment diagram, per unit length of the spandrel, is M_S at the inside face of the beam. Detail e shows the resultant forces acting at the vertical interface of slab and beam, per unit length of spandrel. A strict evaluation of these forces requires that the whole structure be treated as a space frame; alternatively, some idea of the magnitudes of the forces involved may be obtained by use of approximate methods.[1-5] If the quantity $M_S + Ve - Pe_1$ is known along the length of a spandrel, the resulting torsional moments acting at any cross-section may be evaluated. If concentrated wall loads are present, the torsional moments are modified accordingly.

Both creep and cracking will change the magnitude of the quantity $M_S + Ve - Pe_1$ arrived at on the basis of an elastic analysis. The presence of creep will result in the reduction of the transverse moment M_S. The state of knowledge with regard to creep in reinforced concrete, however, does not allow a precise analytical evaluation of the resulting change. The presence of cracks due to torsion results in a pronounced reduction of the torsional stiffness of a spandrel. The significance of the consequent redistribution of moments in the structural frame should be investigated, and the advisability of permitting cracks to occur where architectural appearance is not a consideration must be established on the basis of serviceability criteria.

Shoolbred and Holland,[6] employing elastic analysis, have indicated that the influence of the force P in reducing the torsional moments applied to a spandrel is of minor consequence.

It has become a rather widespread architectural practice to use spandrel girders that are wide and shallow, as in detail f of Fig. 1-1, where the width of the spandrel beam is greater than that of the column. The consequent misalignment of the beam and column center lines not only complicates the calculation of torsional moments, but raises questions regarding both the transfer of torsional moments into the columns and the effective width b of the cross-section in torsion.

EDGE BEAMS OF SHELLS

In shells, the presence of edge beams affects the magnitude of the transverse moments, which, in turn, influence the transverse reinforcement and the thickness of the shell. Economy in shells requires the most efficient distribution of the applied forces. For example, the importance of edge beams for cylindrical shells is demonstrated in Fig. 1-2, which is due to Kemp.* The example considered, representative of a number of shells encountered in

*Dr. E. L. Kemp, West Virginia University, by private correspondence.

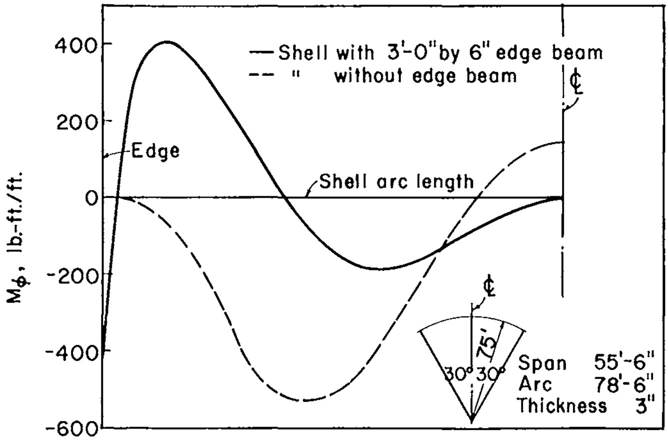

FIG. 1-2 TRANSVERSE BENDING MOMENTS FOR A BARREL SHELL

practice, is a 3-in. thick barrel shell of 55-ft span; the magnitude and distribution of transverse moments are compared for the case of no edge beams and for the case of edge beams 3 ft deep and 6 in. wide. Totally different moment distributions pertain for the two cases, and the absolute maximum moment is reduced by more than 20 percent when edge beams are provided.

Some edge beams thus may be subjected to torsional moments unless care is taken to reduce or eliminate the torsion by proper alignment of the shell reactions.

Fig. 1-3, for example, shows a typical cross-section through a tension ring of a circular dome which is assumed to be supported on a line of columns cast monolithically with the ring. The forces T_V and T_H, respectively, are the vertical and horizontal components of the force T, per unit length of tension ring, which the shell exerts on the ring. At the interface of ring and shell, there are moments M, per unit length of ring. Torsion in the ring is produced by the eccentric action of the forces T_H and T_V, moment, M, and the self-weight of the ring. If prestressing is introduced, the magnitude of the net torque is modified by the vertical and horizontal components P_V and P_H of a general prestressing force P, acting per unit length of edge beam. It follows that the net torsional moment introduced at

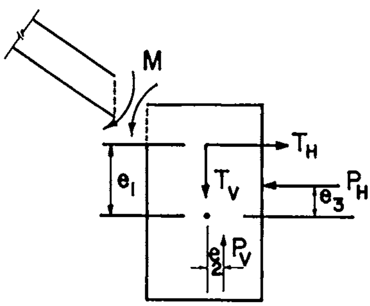

FIG. 1-3 TENSION RING OF SPHERICAL DOME

any section is then primarily a function of M, the forces T and P and their eccentricities, the distributed vertical load along the beam center line, the flexibility and spacing of columns and the diameter of the dome. The magnitude of these torsional moments may be effectively controlled by proper location and dimensioning of the edge support, and by the magnitude and location of the prestressing.

ORTHOTROPIC SHELLS

The torsional analysis of members with open, thin-walled cross-sections, such as cylindrical shell structures, is more complex in principle than the analysis of ordinary linear members, such as rectangular beams. Fig. 1-4 illustrates plain and monotropic cylindrical shells, both of which will deform excessively when subjected to twisting about the longitudinal axes because of low torsional stiffness. The addition of transverse ribs to the

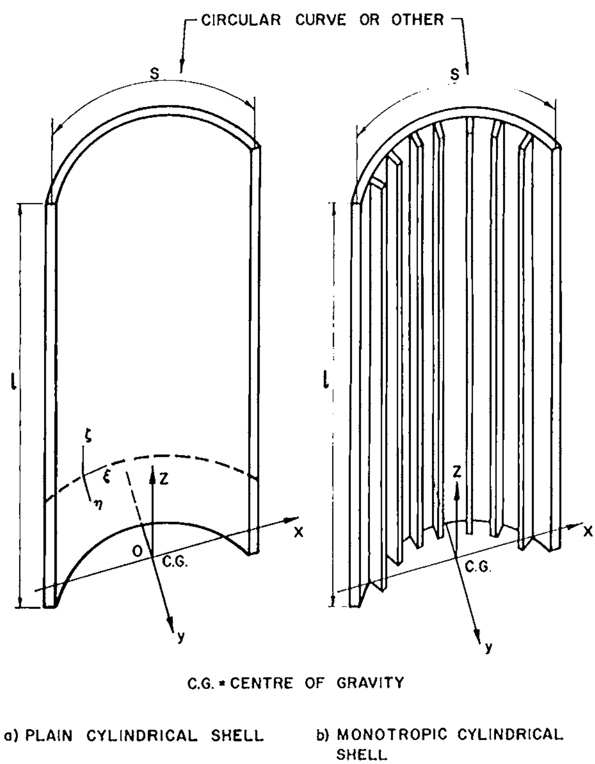

FIG. 1-4 CYLINDRICAL SHELLS WITH CARTESIAN COORDINATE SYSTEM THROUGH CENTER OF GRAVITY OF SECTION

monotropic shell converts it into an orthotropic shell and produces restraints which force warping of the cross-section to occur. This warping, which does not occur in the plain and monotropic shells, is responsible for substantially increased torsional stiffness and, moreover, makes it feasible to design entire structures of a shell-like shape. These structures may be subjected to twist by resultant transverse wind or earthquake forces which do not act through the flexural center.

A particularly interesting example involving both of these features is the Toronto City Hall (Fig. 1-5), selected to present a realistic notion of the stress and deflection characteristics of such a structure against the background of the main analytical expressions.[7]

The magnitudes of the stresses arising from torsion, bending and shear due to self-weight, superimposed live load and wind forces, are compared at the third-floor (podium roof) level (Fig. 1-6). A typical cross-section through the East Tower is shown in Fig. 1-7. The buttresses, walls, and columns are the longitudinal ribs, and the concrete floor slabs are the transverse ribs. The over-all structural action of the building must be considered when establishing the stress distributions involved. For design calculation purposes, the tower is treated as a cantilever, and may be considered fixed at the third-floor level (see Reference 7). The general theory of shells,[8,9] modified for the special boundary conditions resulting from the presence of the diaphragms, must be used for rational determination of the internal stresses, strains, and deformations resulting from the action of external forces.

The following basic assumptions pertain:

(1) Plane sections normal to the middle surface of the shell remain plane.
(2) Stresses acting in a direction perpendicular to the generatrix of the shell may be neglected.
(3) The middle surface of the shell is inextensible in the transverse direction.

The basic eighth-order differential equation which establishes the relationship between the external forces and the internal stresses and strains for a shell is:

$$\Omega\Omega\phi(z,s) + \frac{12}{h^3} \cdot \frac{\partial^4 \phi(z,s)}{\partial z^4} = P(z,s) \qquad (1)$$

where

$$\Omega = \frac{\partial^2}{\partial s^2}\left(R\,\frac{\partial^2}{\partial s^2}\right) + \frac{\partial}{\partial s}\left(\frac{1}{R} \cdot \frac{\partial}{\partial s}\right)$$

FIG. 1-5 TORONTO CITY HALL

$\phi(z, s)$ = stress function
h = thickness of shell
z and s = coordinates of a point on the middle surface of the shell (z is directed along the generatrix, s is directed along the contour of the shell)
$P(z, s)$ = a given load function expressed through the components P_z, P_s, P_n, of the vector of intensity of external forces acting on the shell
$R = R(s)$ = radius of the middle surface of the shell

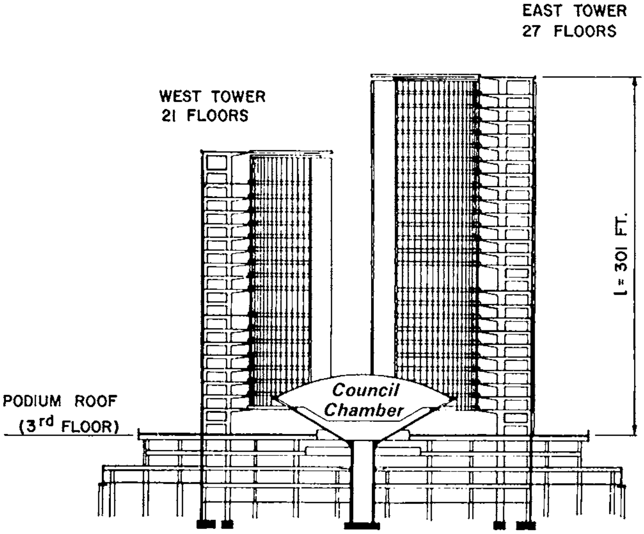

FIG. 1-6 EAST-WEST SECTION OF CITY HALL

The stress function $\phi(z,s)$ which satisfies the differential Eq. (1) and the boundary conditions provides the solution. If the stress function $\phi(z,s)$ is known, the longitudinal normal stresses $\sigma(z,s)$ and transverse bending moments $G(z,s)$ may be determined from the following formulae:

$$\sigma(z,s) = \frac{12}{h^3} \cdot \frac{\partial^2 \phi(z,s)}{\partial z^2} \tag{2}$$

$$G(z,s) = \Omega \phi(z,s) \tag{3}$$

Now, it has been found that Eq. (1) is equivalent to a system of two symmetrical differential equations:

$$\frac{\partial^2 [\sigma(z,s)h]}{\partial z^2} + \Omega G(z,s) = P(z,s) \tag{4}$$

$$\Omega \sigma(z,s) - \frac{12}{h^3} \cdot \frac{\partial^2 G(z,s)}{\partial z^2} = 0 \tag{5}$$

Eq. (4) may be derived directly by considering the equilibrium conditions of a shell element.

Eq. (5) may be derived directly by considering the compatibility of deformation of the middle surface of the shell; namely, from the condition that

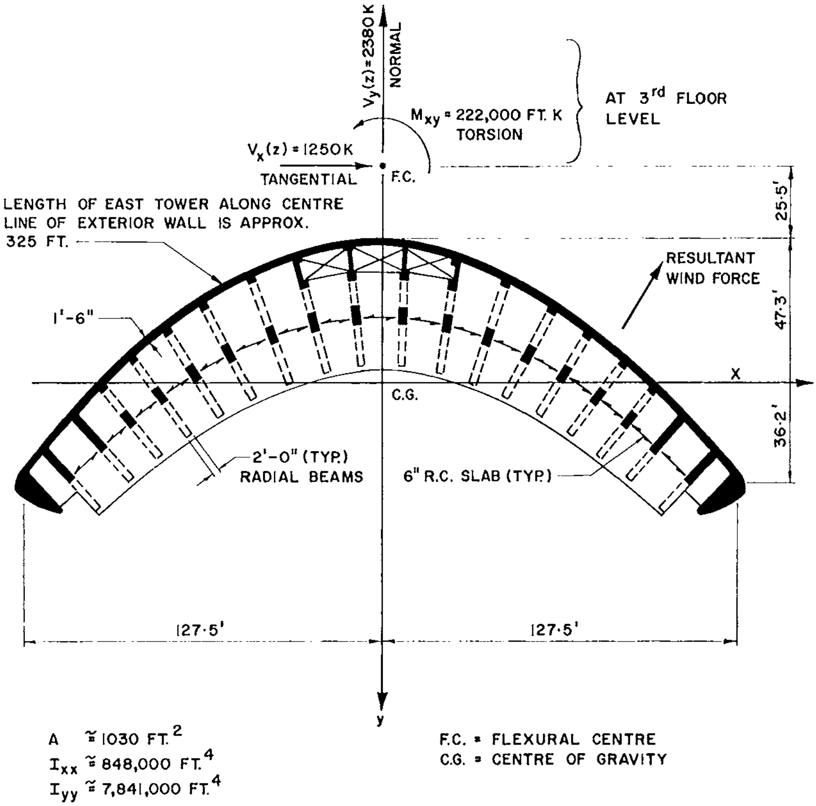

FIG. 1-7 CROSS-SECTION THROUGH EAST TOWER

$$\Omega\,\varepsilon(z,s) + \frac{\partial^2 \chi}{\partial z^2} = 0 \tag{6}$$

and from the well-known equations of elasticity:

$$N(z,s) = E\,h\,\varepsilon(z,s); \quad G(z,s) = -\frac{Eh^3}{12}\,\chi(z,s) \tag{7}$$

where

$N(z,s)$ = normal force

E = modulus of elasticity

$\varepsilon(z,s)$ = longitudinal tensile strain

χ = transverse bending strain

The important feature to note is that Eq. (1) can be derived by way of Eq. (4) and (5) as well as by first principles.