Next, a correlation between fully plastic torsional theories[4,6] and measured data was sought using the equation

$$T^*_{u,\,plastic} = 1.33\, T^*_{u,\,el}(DLF) \tag{12}$$

for specimens of circular cross-section[4] and the equation

$$T^*_{u,\,pl} = \frac{x^2}{2}\left(y - \frac{x}{3}\right)\tau_1 (DLF) \tag{13}$$

for specimens of square cross-section. The computation was repeated using τ_1 equal to direct tension values.

The sixth and seventh columns of Table 9-3 show the values obtained. Although there is a considerably better correlation between the measured and computed values, it can be stated that plain concrete under dynamic torsion (in the range of investigated rise-times) does not behave either linearly elastically nor fully plastically. For this reason, further analytical evaluation of the test results has been accomplished using formulas derived by Dr. T. Hsu,[7] who found that plain concrete members fail by bending about an axis inclined at 45 deg to the longitudinal axis of the member. Based on this failure mechanism, the Hsu equation for determination of ultimate torque for circular sections is:

$$T^*_u = \frac{\pi D^3}{16}\, 0.85\, f_r\, (DLF) \tag{14}$$

where f_r is the modulus of rupture of the concrete and is expressed by:

$$f_r = 7.17 \sqrt[3]{f_t^2}\ 1.45\ (psi) \tag{15}$$

Hsu gives the following equation for determination of the ultimate torque of plain concrete elements of rectangular cross-section:

$$T^*_u = 6\,(x^2 + 10)\,y \sqrt[3]{f'_c}\ (DLF) \tag{16}$$

where

$x = y = 6$ in.

The results of these computations, shown in the eighth and ninth columns of Table 9-3, indicate reasonably good agreement with the measured ultimate dynamic torques.

The agreement was not as good in comparing the measured ultimate angles of twist with those obtained from Hsu's formulas[7] for rectangular sections:

$$\theta_u^* = \frac{0.038}{\beta x}\left(1 + \frac{10}{x^2}\right) \tag{17}$$

and an approximate expression:

$$\theta_u^* = \frac{0.43 \times 10^{-3}}{\sqrt{xy}}\left(1 + \frac{10}{x^2}\right) \tag{18}$$

where, for the case under investigation,

$\beta = 0.141$

$x = y = 6$ in.

Again there were considerable discrepancies between the measured angle-of-twist and the computed values based on elastic theories.[4]

REINFORCED CONCRETE ELEMENTS

In order to obtain information and gain experience in instrumentation for more extensive future testing of reinforced concrete elements, six reinforced concrete elements were tested to failure using dynamic torques with minimum rise-time of 1.25 sec. Fig. 9-11 shows the details of the reinforced test specimens. All six specimens were of $f_c' = 5000$ psi nominal concrete strength. In order to accelerate testing, the specimens were steam cured after approximately 18 hr. The steam was applied gradually, reaching a maximum temperature of 210-215 F after five to six hr. The specimens were cured at this temperature for 14 to 16 hr. Control tests, as previously described, were made from the same mix and given the same curing conditions.

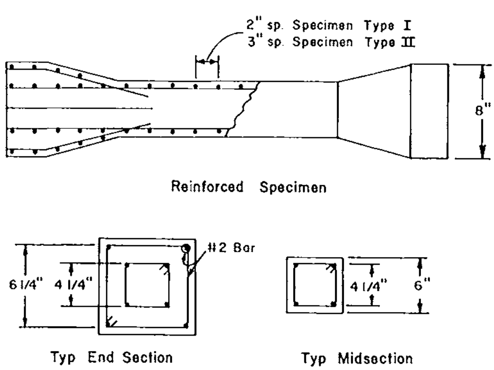

FIG. 9-11 REINFORCED CONCRETE SPECIMENS

In addition to the instrumentation used in testing of plain concrete specimens, 60 strain gages were attached to the longitudinal and transverse reinforcing bars to obtain data concerning stress distribution in reinforcing bars. The conventionally available strain gages were covered with plastic only, which was sealed with epoxy. Few strain gages were lost during the testing. The reinforcing bars used in the test were tested and found to have a yield strength of 56,000 psi.

In the early part of the testing, the concrete carried all of the applied torque. From the film coverage, taken by high-speed camera, the early crack formation pattern could be established. First cracks developed at the head of the specimens; later cracks appeared over the entire section. The crack formation followed the classical helical pattern with an approximate angle of inclination of 45 deg to the axis of specimens.

Before the first cracks appeared, all of the torsion was being resisted by the concrete; thus no difference was found between the torsional resistance of plain and reinforced specimens. But the behavior of reinforced elements after initial failure was markedly different since, as expected, reinforced specimens displayed considerable torque capacity after initial failure (Fig. 9-12).

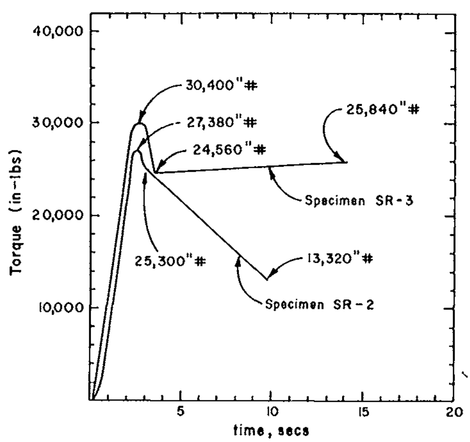

FIG. 9-12 TORQUE-VS-TIME CURVES OF REINFORCED CONCRETE SPECIMENS

The strain gages (Fig. 9-13) indicated that stirrups carry more strain than longitudinal reinforcements. Consequently, with increased transverse reinforcing, the ultimate torsional capacity of the specimens was increased. The longitudinal reinforcement affected the ultimate torque capacity in the specimens in the following way: The torque-vs-time diagram for specimens with four

FIG. 9-13 TYPICAL FAILURE OF REINFORCED CONCRETE SPECIMENS

longitudinal bars went up linearly to the maximum torque capacity; after the first cracks appeared in the concrete, the torque capacity continued to decline (Fig. 9-12). On the other hand, the same type of specimen, with eight longitudinal bars, kept its torque capacity fairly constant after the initial cracks in the concrete, indicating the importance of balanced reinforcing. The analytically obtained ultimate torques[8] showed relatively good agreement with the measured ones.

CONCLUSIONS AND RECOMMENDATIONS

The dynamic torsion tests described in this paper have indicated that with shorter rise-times the plastic range has a tendency to disappear and the torque-vs-twist curve becomes increasingly nonlinear. Although up to the minimum rise-time (0.85 sec) used in the tests Hsu's formulas[7] gave relatively good agreement with the test results, it is expected that using considerably shorter rise-times (in the millisecond range) will show that significant discrepancies will occur. Concrete is a typical viscoelastic material; consequently, the consideration of its rate sensitivity is of prime importance. Thus, it is recommended that new torsional formulas based on the "correspondence principle" of linear visco-elasticity should be developed using "complex" shear moduli of concrete based on a "four-element-Maxwell-Kelvin" model.[9]

While the equations used in determinations of ultimate torque will not describe the material properties correctly analytically in the case of short rise-time, it has been found that they properly consider the effect of variation in the shape of cross-section.

The author feels that the number of reinforced concrete specimens tested is too limited to reach a final conclusion; based on the results, the following tentative conclusions can be made concerning dynamic torsion of reinforced concrete elements up to a rise-time of one sec:

(1) Reinforced concrete specimens behave plastically before failure.

(2) Stirrups carry more load than longitudinal bars, and the concrete cracks.

(3) The ultimate torsional capacity of the specimens is a combination of the resistance of concrete and reinforcing.[10]

Future tests of reinforced concrete elements must include rise-times in the millisecond range to be able to study the true dynamic behavior of reinforced concrete elements subjected to dynamic torsion. At the present, the University of Denver, Denver Research Institute, is studying more extensively the direct tension properties of concrete subjected to dynamic loads.

ACKNOWLEDGMENTS

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The testing of the first 15 specimens was carried out by Mr. J. Moore, Research Engineer, Denver Research Institute, whose Interim Report has been partially used in preparation of this paper. Mr. E. Haugan, civil engineering student, and Mr. O. Wallevik, graduate research assistant, both of the University of Denver, have prepared the tables and drawings and assisted in the testing.

REFERENCES

1. International Association for Earthquake Engineering, Proceedings of Third World Conference on Earthquake Engineering, held in New Zealand, Jan. 22 - Feb. 1, 1965 (in print).

2. Norris, Ch. V., et al, Structural Design for Dynamic Loads, McGraw-Hill, New York, 1959.

3. Timoshenko, S., Strength of Materials, V. II, Third Edition, McGraw-Hill, New York, 1956.

4. Kemp, E. L., Sozen, M. A., and Siess, C. P., "Torsion in Reinforced Concrete," C. E. Studies, Structural Research Series No. 226, University of Illinois, Urbana, Ill., 1961.

5. Szabó, I., Höhere Technishe Mechanik, Third Edition, Springer-Verlag, Berlin, 1960.

6. Nadai, A., Theory of Flow and Fracture of Solids, McGraw-Hill, Second Edition., New York, 1950.

7. Hsu, T. T. C., "Torsion of Structural Concrete - Plain Concrete Rectangular Sections," Torsion of Structural Concrete, Special Publication No. 18, American Concrete Institute, Detroit, 1968.

8. Mattock A. H., "How to Design for Torsion," Torsion of Structural Concrete, Special Publication No. 18, American Concrete Institute, Detroit, 1968.

9. Bland, R., The Theory of Linear Viscoelasticity, Pergamon Press, New York, 1960.

10. Szilard, R., and Haugan, E., Report on Study No. 129, "Dynamic Response of Reinforced Concrete Elements to Torsion," Denver Research Inst., 1966.

NOTATION

D = diameter

DLF = Dynamic Load Factor

d = subscript denoting direct properties

e = subscript denoting equivalent system

f'_c = compressive strength of concrete

f_r = modulus of rupture

f'_t = tensile strength of concrete

G_o = static shear modulus

$\bar{I}_j$ = torsional moment of inertia of lump masses

I_p = polar moment of inertia of cross-section

i_m = mass inertia per unit length

k = spring constant

m = mass

p = circular frequency of torque

T = torque

t = time

u = subscript denoting ultimate strength

x, y = dimensions of rectangular section

θ = angle of twist

θ_i = angular displacement in vibration

ω_e = natural frequency of equivalent system

$[\rho_{ij}]$ = stiffness matrix

τ_1 = tensile strength of concrete

PAPER SP 18-10 An extensive laboratory investigation of plain and reinforced concrete members subjected to torsion is being reported in a series of papers. This paper reports tests of 53 reinforced concrete beams which were subjected to pure torsion to investigate the effect of eight variables. The behavior before and after cracking was extensively studied. Provisions of foreign codes and theories for reinforced concrete design in torsion were evaluated. Design equations for ultimate torque, cracking torque, stiffness before and after cracking, angle of twist at ultimate torque, and at cracking torque and other provisions are given.

Torsion of Structural Concrete— Behavior of Reinforced Concrete Rectangular Members

By THOMAS T. C. HSU

☐ Experimental and theoretical studies of structural concrete members subject to torsion began at the PCA Laboratories in 1962. A torsion test rig[1] was designed and constructed for a maximum torque of one million in.-lb to accommodate test beams up to 15 x 20 in. Three types of tests have so far been completed, all involving rectangular members subjected to pure torsion only: (1) plain concrete members, (2) reinforced concrete members, and (3) plain and reinforced hollow members. Future investigations will concern nonrectangular cross-sections (L-, T-, and I-beams) and members subjected to combined torsion, bending, and shear.

TORSIONAL STRENGTH OF PLAIN CONCRETE

The investigation of plain concrete rectangular members is reported in Reference 2 (see Paper SP 18-8 herein). It was found that plain concrete rectangular members subjected to torsion fail mainly by bending about an axis parallel to the wider cross-section face and inclined at 45 deg to the longitudinal axis. Based on this failure mechanism, new expressions for the ultimate torque were derived:

$$T_{up} = 2\ (x^2 + 10)\ y\ \sqrt[3]{f_t^{\,2}} \tag{1}$$

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where

T_{up} = ultimate torque of plain concrete members, in.-lb

x = smaller dimension of cross-section, in.

y = larger dimension of cross-section, in.

f_t = uniaxial tensile strength of concrete, psi

When only the cylinder compressive strength, f'_c, is known, f_t can be taken as $5\sqrt{f'_c}$ and Eq. (1) becomes.

$$T_{up} = 6\ (x^2 + 10)\ y\ \sqrt[3]{f'_c} \qquad (1a)$$

The angle of twist at failure was found to be

$$\theta_{up} = \frac{0.0038}{\beta\, x}\left(1 + \frac{10}{x^2}\right) \qquad (2)$$

in which

θ_{up} = angle of twist at failure of plain concrete members, expressed in deg/in.

β = coefficient given by Saint-Venant's theory as a function of y/x.

The investigation of plain concrete rectangular members was accompanied by an investigation of reinforced concrete rectangular members. The behavior of the reinforced rectangular members is the subject of this paper.

TESTS OF REINFORCED RECTANGULAR BEAMS

To study the behavior under pure torsion of reinforced concrete beams with rectangular cross-sections, 53 beams were tested, involving the following eight major variables:

(1) Amount of reinforcement

(2) Solid beams versus hollow beams

(3) Ratio of volume of longitudinal bars to volume of stirrups

(4) Concrete strength

(5) Scale effects

(6) Depth-to-width ratio of cross-section

(7) Spacing of longitudinal bars

(8) Spacing of stirrups.

Nine series of beams were tested as outlined in Table 10-1. Only one type of reinforcing steel, intermediate grade, was used in this investigation.

TABLE 10-1 OUTLINE OF TEST PROGRAM

Beam Series	Cross-Section in. x in.	Target f'_c psi	m	Variables Isolated							
				p_t	Solid Versus Hollow	m	f'_c	Scale Effect	$\frac{y}{x}$	Spacing, Longitudinal Bars	Spacing, Stirrups
B	10 x 15	4000	0.205-4.97	o	x	x	x		x		o
D	10 x 15 hollow	4000	1.0	o	x						o
M	10 x 15	4000	1.5	o		x					o
I	10 x 15	6500	1.0	o			x				o
J	10 x 15	2000	1.0	o			x				o
G	10 x 20	4000	1.0	o				x	x	o	o
N	6 x 12	4000	1.0	o				x	x	o	o
K	6 x 19.5	4000	1.0	o					x		o
C	10 x 10	4000	1.0	o					x		o

Note: o = comparison within each series; x = comparison between series.

Test Specimens

A typical test beam is shown in Fig. 10-1. The length of all beams was 122 in., except those of Series N. A length of 14 in. at each end of the beam was threaded into the clamping heads of the torsion test rig, through which the torsional moments were applied. The clear span subjected to torsion was 94 in. To avoid local failure close to the clamping heads due to stress concentration, a length of 25 in. at each end of the beams was reinforced with about 30 percent additional stirrups. The effective length of each beam was therefore reduced to about 72 in. The cross-sections used in each beam series are shown in Fig. 10-2.

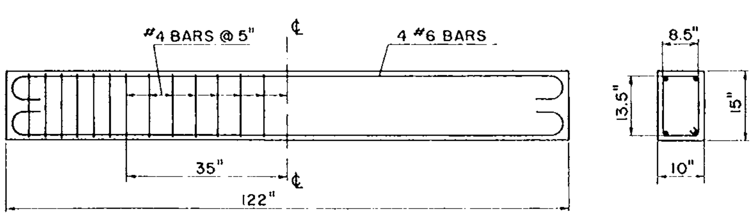

FIG. 10-1 TYPICAL TEST BEAM, B3