State-of-the-Art Report on Temperature-Induced Deflections of Reinforced Concrete Members

By ACI Committee 435

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Synopsis: This report summarizes available methods for calculating deflections of reinforced concrete beams subjected to temperature change. Selection of design temperatures and temperature gradients is discussed as well as the effects of cracking on response in the service load range.

Keywords: beams (supports); cracking (fracturing); deflection; modulus of elasticity; moments of inertia; reinforced concrete; temperature; thermal gradient.

INTRODUCTION

Temperature changes can significantly affect deflections of reinforced concrete building structures. Deflections occur in unrestrained flexural members when a temperature gradient is set up between opposite faces of the member. In cases where deformations due to temperature change are restrained, tensile stresses induced in the member can result in cracking and consequent reduction in flexural stiffness. Because temperature effects do not often affect the ultimate limit state of the structure, effects of temperature on deflection are sometimes not considered in design. However, it has been standard practice to compute thermal stresses and displacements in structures for tall building design. Also, elongation and shortening of bridge superstructures and precast concrete structures are normally computed for support and expansion joint designs.

De Serio (1) states that "Designing for thermal and shrinkage stresses is the most neglected part of today's design practice." With the use of higher strength materials and more refined methods of analysis the need to consider temperature effects is becoming increasingly important.

Most general purpose computer programs have the capability to include temperature changes in the analysis for certain types of members. However, in many cases, particularly those involving relatively simple structures, where computer analysis is not required, or where the computer program does not include the capability for analysis of flexural members subjected to temperature gradient, calculations may be required to investigate the effect of temperature change on serviceability.

The objective of this report is to indicate some of the problems that can result from differential thermal movement and to outline procedures for calculating deflections that result from temperature change. The scope of the report is restricted to performance of structures in service. Temperature effects due to heat of hydration are not considered.

SERVICEABILITY PROBLEMS RELATED TO DIFFERENTIAL THERMAL MOVEMENT

The following examples illustrate potential problems that can result from differential thermal movement in building structures.

Office building - cantilever roof and floor slabs. Daily temperature changes caused deflections of about one inch at the cantilever ends of roof slabs in a four-story building constructed of cast-in-place concrete. The post-tensioned floors and roof were 80 feet by 80 feet in plan. Only four columns were provided and these were 48 feet apart in each direction which resulted in the columns being 16 feet from the edges of the floor and roof slabs in each direction. During construction it was noted that diurnal solar heating caused the roof slab to deflect up and down. The change in elevation was about one inch at the extreme overhanging ends. Fortunately the detail for the connection of the windows, which were located in the perimeter of the floors and roof, provided for sufficient movement that the windows remain attached to the roof when it is its highest position and does not result in the window becoming loaded when the roof is in its lowest position.

Industrial building - cantilever roof slab. Similar temperature-induced movements caused damage at the joint between a cantilever roof slab and exterior wall in a one-storey building that was erected using the lift-slab technique. The post-tensioned waffle roof slabs had significant overhangs at all four exterior walls. The exterior walls were constructed using the tilt-up method. Because of the construction techniques used, the roof was not supported vertically by the exterior walls and the connection of the exterior walls (for horizontal loads) was made when the edges of the roof were in a low position due to the effects of diurnal solar heating. During service the effect of solar heating has been to apply vertical loads to the exterior walls. This has resulted in the vertical cast-in-place concrete closure pours between the wall panels, particularly at the corners of the building, to crack and disintegrate near their bottoms with the passage of time. Reconstructing the destroyed concrete does not solve the problem; it simply disintegrates again. Eventually, attempts to reconstruct the disintegrated concrete were abandoned and steel plates, painted to match the color of concrete, were placed on the affected joints to cover the damage and improve the appearance of the building.

Parking structure - deflection of double tee beams. In a precast parking structure, rotation at bearing ends of beams resulting from deflections caused by diurnal solar heating produced cracking near the ends of the beams. The precast prestressed double-tee beams span about 55 feet. A concrete topping was placed over the top of the double-tee beams. Reinforcing steel was placed in the topping at interior joints in a configuration similar to that used with negative moment reinforcement in cast-in-place construction. At some locations elastomeric bearing pads were placed between the stems of the double-tee beams and their supports; at others, elastomeric bearing pads were not provided under the stems. Measurements showed the deflection caused by diurnal solar heating to be of the order of 0.75 inches at midspan of the double-tee beams. The deflection due to solar heating caused rotations at the ends of the double-tee beams and many of the double-tee beam stems, not provided with elastomeric bearing pads, cracked as a result of the rotation.

Office building - vertical wall panels. Exterior precast wall panels two stories high were supported at the bottom and supported laterally at the top of the second storey. In some locations the panels were placed adjacent to wall construction supported laterally at the top and bottom of the second storey. Seasonal temperature changes caused bowing of the two storey panels resulting in mid panel deflections up to 3/4 in. Relative deflection between the two-storey panels and adjacent walls supported at the bottom of the second storey caused damage to the caulking material in the vertical joint.

These examples illustrate the need to consider differential thermal movements in design. The following sections outline calculation procedures that can be used to estimate stiffness changes and deflections resulting from temperature change in structural members.

DESIGN TEMPERATURES

Before performing an analysis for temperature effects it is necessary to select design temperatures and temperature gradients for use in the analysis. Martin (2) summarizes design temperatures that are provided in various national and foreign codes. Design information is given in terms of temperature rise ranging from 27 to 40°F and temperature drop ranging from 27 to 45°F. Martin also provides extreme values of normal daily maximum and minimum temperatures as reported by the Environmental Data Service for various locations in the United States. He suggests that design should be based on 2/3 of the difference between extreme values of normal daily maximum and minimum temperatures for each location. Based on this criterion average proposed design thermal differential is 41°F with a minimum of 10 and a maximum of 62°F. Fintel and Khan (3) suggest contacting the local weather bureau to obtain mean temperatures for specific localities. They also point out that surface temperatures can be affected by direct radiation. This may be significant, particularly for thin members. Fintel and Khan provide a method to determine isotherms and temperature gradients for irregular configurations and boundary conditions. They show that nonlinear thermal gradient can induce internal stresses in the member even if the member is free to deform at the ends.

A design thermal gradient used in New Zealand for concrete bridge superstructures is shown in Fig. 1. A fifth power temperature profile is used between the top surface and a point at depth 1200 mm for webs and cantilever decks. For decks above enclosed air-cells, a linear gradient is used. A linear thermal gradient is also used for the bottom 200 mm of a section. For superstructures less than 1400 mm in depth, the top and bottom thermal gradients are superimposed.

Charts showing temperature gradients in slabs of different thickness exposed to sunlight on a summer day in Melbourne, Australia are given in Ref. 4. These charts show approximately linear temperature gradients and a temperature difference of about 40°F over the depth of an 8-in. slab.

Fintel and Khan (3) provide a graphical method for determining temperature gradients for given steady state temperatures. Temperature gradients can also be determined by the finite element method (5), or the finite difference method (6).

TEMPERATURE GRADIENT ON UNRESTRAINED CROSS SECTION

Consider a temperature distribution t(y) on the cross section shown in Fig. 2(a). Thermal strain at distance y from the bottom of the section is given by:

$$\varepsilon_t(y) = \alpha t(y) \tag{1}$$

To restrain movement due to temperature t(y), apply a stress in the opposite direction to $\varepsilon_t(y)$:

$$\sigma(y) = E\alpha t(y) \tag{2}$$

The net restraining axial force and moment are obtained by integrating over the depth:

$$P = \int_A \sigma dA = \int_0^h \alpha E t(y) b(y) dy \tag{3}$$

$$M = \int_A \sigma (y-n) dA = \int_0^h E\alpha t(y) b(y)(y-n) dy \tag{4}$$

To obtain total strains on the unrestrained cross section, as shown in Fig. 2(d), apply P and M in the opposite direction to the restraining force and moment. Assuming plane sections remain plane, axial strain (ε_a) and curvature (ϕ) are given by:

$$\varepsilon_a = \frac{P}{AE} = \frac{\alpha}{A} \int_0^h t(y) b(y) dy \tag{5}$$

$$\phi = \frac{M}{EI} = \frac{\alpha}{I} \int_0^h t(y) b(y)(y-n) dy \tag{6}$$

The net stress distribution on the cross section is given by:

$$\sigma_n(y) = \frac{P}{A} \pm \frac{M(y-n)}{I} - E\alpha t(y) \tag{7}$$

For a temperature gradient varying linearly from 0 to Δt, the curvature obtained by Eq. (6) is given by

$$\phi = \frac{\alpha \Delta t}{h} \tag{8}$$

In the case of a uniform temperature gradient along the length of a member, deflections for simply supported (Δ_{ss}) and cantilever beams (Δ_{cant}) are calculated as

$$\Delta_{ss} = \frac{\phi L^2}{8} = \frac{\alpha \Delta t}{h} \cdot \frac{L^2}{8} \tag{9}$$

$$\Delta_{cant} = \frac{\phi L^2}{2} = \frac{\alpha \Delta t}{h} \cdot \frac{L^2}{2} \tag{10}$$

Deflection to span ratio is given by

$$\frac{\Delta}{L} = \frac{\alpha \Delta t}{k} \cdot \frac{L}{h} \tag{11}$$

where k = 8 for simply supported and 2 for cantilever beams.

EFFECT OF RESTRAINT TO THERMAL MOVEMENT

If a member is restrained from deforming under the action of temperature changes, internal stresses will be developed. Cracking that occurs when tensile stresses exceed the concrete tensile strength reduces the flexural stiffness of the member and results in increased deflections under subsequent loading. Temperature effects should therefore be taken into account when determining member stiffness for deflection calculation.

Priestly (7) points out that an iterative solution is necessary to solve for the final strain distribution in cracked sections since the extent of cracking is a function of the thermal load. However for design, he suggests that unrestrained thermal curvature be calculated on the basis of the uncracked section, and that thermal continuity moments and secondary thermal stresses be calculated on the basis of moment of inertia associated with the expected distribution of cracking at service loads.

Mentes et al. (8) presented a method for estimating reduced stiffness due to thermal effects. The method is based on Branson's effective moment of inertia and an iterative procedure. Branson (9) has presented a method for calculating differential temperature effects in composite construction.

For nuclear power plant structures, ACI Committee 349 (10) recommends that analysis for mechanical loads be based on uncracked stiffness and that analysis for thermal effects be based on cracking due to mechanical loads. ACI 349 calculation procedures for thermal stresses are based on I_g for uncracked sections and I_{cr} for cracked sections rather than effective moment of inertia I_e as outlined in the ACI Building Code. For deflection calculations, the effective moment of inertia approach is generally more appropriate.

Figure 3 shows a two span one-way slab under gravity loads. The bending moment diagram for this slab indicates that only in the negative moment region does the moment exceed the cracking moment M_{cr}. If however, a temperature gradient is set up in the slab an additional bending moment is induced in the slab due to continuity. In this case the bending moment distribution due to thermal effects increases the positive moment in both spans. Summing moments due to gravity loads and thermal effects results in a bending moment diagram where the cracking moment M_{cr} is exceeded in the positive moment region. Calculation of effective moment of inertia should be based on maximum moment conditions in all regions of the slab.

In cases where stresses are developed in the member due to restraint to axial deformations, the induced stress due to axial restraint should be included in the calculation of cracking moment in a manner analagous to that for including the prestressing force in prestressed concrete beams as outlined for example in Ref. 11.

DESIGN EXAMPLES

The following examples illustrate the calculation procedures outlined above.

Example 1 - Simply Supported Vertical Wall Panel - Linear Temperature Gradient

$\Delta t = 40^0\,F$

$\alpha = 0.0000055$ in./in./^{0}F

$h = 4$ in.

(a) Single storey span: L = 12 ft

$$\Delta = \frac{0.0000055 \times 40 \times 144^2}{4 \times 8} = 0.14 \text{ in.}$$

(b) Two storey span: L = 24 ft

$$\Delta = \frac{0.0000055 \times 40 \times 288^2}{4 \times 8} = 0.57 \text{ in.}$$

Example 2 - Simply Supported Tee Section - Linear Temperature Gradient over Depth (Fig. 4b)

$\Delta t = 40^0\,F$

$\alpha = 0.0000055$ in./in./^{0}F

$h = 36$ in.

$L = 60$ ft (simply supported)

$$\Delta = \frac{0.0000055 \times 40}{36} \times \frac{720^2}{8}$$

$$= 0.40 \text{ in.}$$

Example 3 - Simply Supported Tee Section - Constant Temperature over Flange Depth (Fig. 4c)

$I = 69319$ in.^{4}

$n = 26.86$ in.

$\Delta t = 40^0$ F

$\alpha = 0.0000055$ in./in./^{0}F

$h = 36$ in.

$L = 60$ ft

$$\phi = \frac{\alpha}{I} \int_0^{36} t(y)b(y)(y-n)dy$$

$$= \frac{\alpha}{I} \int_{33}^{36} 40 \times 96(y - 26.86)dy$$

$$= \frac{88{,}013 \times 0.0000055}{69319}$$

$$= 0.00000698$$

$$\Delta = \frac{\phi L^2}{8} = \frac{0.00000698 \times 720^2}{8}$$

$$= 0.45 \text{ in.}$$

Example 4 - Continuous One-Way Slab-Linear Temperature Gradient (Fig. 3)

$h = 6$ in.
$w_D = 75$ psf
$w_L = 100$ psf
$\Delta t = 40$ F
$\alpha = 0.0000055$ in./in./°F
$L = 16$ ft.
$A_s = 0.465$ in.^{2}/ft. top and bottom
$I_g = 216$ in.^{4}/ft.
$f'_c = 4000$ psi
$f_y = 60{,}000$ psi
$I_{cr} = 59$ in.^{4}/ft.
$M_{cr} = 2.85$ ft.kips/ft.

(a) D + L

$$\text{Maximum Negative Moment} = \frac{(w_D + w_L)L^2}{8}$$

Maximum Moment = 5.60 ft.kips/ft.

$$I_e = \left(\frac{M_{cr}}{M}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M}\right)^3\right] I_{cr}$$

$$= \left(\frac{2.85}{5.60}\right)^3 (216) + \left[1 - \left(\frac{2.85}{5.60}\right)^3\right] (59)$$

$$= 79 \text{ in.}^4$$

$$\text{Maximum Positive Moment} = \frac{9}{128}(w_D + w_L)L^2$$

$$= 3.15 \text{ ft.kips/ft.}$$

$$I_e = \left(\frac{2.85}{3.15}\right)^3 (216) + \left[1 - \left(\frac{2.85}{5.60}\right)^3\right] (59)$$

$$= 174 \text{ in.}^4$$

$$\text{Average } I_e = (79 + 174)/2 = 127 \text{ in.}^4$$

$$\Delta_{max} = \frac{wL^4}{185\ EI} = \frac{(0.175)(16^4)(12^3)}{(185)(3600)(127)}$$

$$= 0.23 \text{ in.}$$

(b) D + L + T

At interior support, positive moment induced by temperature gradients is given by

$$M_T = \frac{3}{2} EI\phi \quad \text{where } \phi = \frac{\alpha \Delta t}{h}$$

(i) Thermal moments based on I_g

$$M_T = \left(\frac{3}{2}\right)(3600)(216)\left(\frac{0.0000055 \times 40}{6}\right)$$

$$= 7.13 \text{ ft.kips/ft.}$$

Note that this moment exceeds the negative moment due to dead + live load.
At location of maximum positive moment due to dead + live load (i.e. 3/8 L from end of span)

$$M_T = 3/8 \times 7.13 = 2.67 \text{ ft.K}$$

Max. positive moment for D + L + T

$$= 3.15 + 2.67 = 5.82 \text{ ft.K/ft.}$$

$$I_e = \left(\frac{2.85}{5.82}\right)^3 (216) + \left[1 - \left(\frac{2.85}{5.82}\right)^3\right](59)$$

$$= 78 \text{ in.}^4$$

$$\text{Average } I_e = (79+78)/2 = 78.5 \text{ in.}^4$$

Deflection due to D + L

$$\Delta_{max} = \frac{(0.175)(16^4)(12^3)}{(185)(3600)(78.5)} = 0.37 \text{ in.}$$

i.e. Temperature gradient results in 60% increase in deflection under dead + live load

(ii) Thermal moments based on I_e calculated for service loads

$$M_T = 7.13 \times \frac{127}{216} = 4.19 \text{ ft.kips/ft.}$$

$$M_T \text{ at } 3/8 \text{ L} = 1.57 \text{ ft.kips/ft.}$$

Therefore maximum positive moment for D + L + T = 3.15 + 1.57 = 4.72 ft.kips/ft.

$$I_e = 93 \text{ in.}^4$$

$$\text{Average } I_e = (79+93)/2 = 86 \text{ in.}^4$$

Deflection due to D + L

$$\Delta_{max} = 0.33 \text{ in.}$$

SUMMARY AND CONCLUSION

Deflections caused by temperature change can have adverse effects on serviceability of concrete structures. Measures should be taken at the design stage to alleviate these effects, usually by providing adequate joint details to permit relative movement where required. Calculation procedures to estimate changes in stiffness and temperature-induced deflections are outlined in the report.

For uncracked members, effects of temperature can be included in deflection calculations in a relatively straightforward manner. For statically indeterminate systems after cracking, the deflections, stiffness, and temperature are inter-related, and an iterative procedure is required for a correct solution. However a simplified direct procedure is given in the report for calculation of temperature-induced deflections after cracking occurs.

Temperatures and temperature gradients for use in design are specified in several codes. Suggestions for selecting these design quantities are given in the report for cases where the temperatures to be considered are not specified by the governing code.

Some data are available in the literature concerning field measurements of temperatures in structures. Additional data would be of value in developing appropriate temperature gradients for standard design situations.

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Conversion Factors - Inch-Pound to SI

1 in. = 25.4 mm
1 lb (mass) = 0.4536 kg
1 lb (force) = 4.488 N
1 lb/sq in. = 6.895 kPa
1 kip = 444.8 N
1 kip/sq in. = 6.895 MPa
1 in.-kip = 0.1130 N•m

NOTATION

A = cross-sectional area

b = width of beam

D = dead load

E = modulus of elasticity

h = depth of member, blacktop thickness

I = moment of inertia

I_{cr} = moment of inertia at cracked section

I_e = effective moment of inertia

I_g = moment of inertia of gross concrete section