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INTRODUCTION

Reinforced concrete and prestressed concrete structures at service load levels are expected to exhibit essentially linear elastic behaviour in terms of stress-strain response of steel and compressive stress-strain response of concrete. Nonlinear behaviour occurs however if cracking takes place in concrete. The ACI Building Code (1) recognizes this nonlinearity through the use of Branson's effective moment of inertia procedure for calculating beam deflections. Branson and Trost (2) discuss the use of the procedure for both reinforced concrete and prestressed concrete beams. Prestressed concrete beams with large openings in the web require special attention because of the significant discontinuity in beam stiffness that occurs at the opening. At the opening the behaviour more closely resembles that of a Vierendeel truss than a simple beam.

This paper describes an analytical model developed to analyse prestressed beams with large openings. Results of the analysis are compared with results of laboratory beam tests.

ANALYTICAL MODELS

Several methods are available for analysis of concrete structures. For uncracked beams in the service load range, linear elastic beam theory can be used to determine stresses and deflections under load. As mentioned above, effects of cracking can be accounted for by use of the effective moment of inertia concept. In the case of beams with openings, simple beam theory is not strictly applicable because of the Vierendeel truss action at the openings.

Significant advances have been made over the past fifteen years in the application of finite element methods of analysis of concrete structures as outlined in a recent state-of-the-art report. (3) The finite element approach permits detailed stress analysis of reinforced and prestressed concrete structures including effects of cracking and other non-linearities. These refined analytical techniques are however quite complex and may not be warranted for many applications.

An alternative to the use of beam theory or finite element methods is available through the use of truss models. Some of the earliest attempts to model reinforced concrete behaviour, particularly behaviour in shear, were based on truss models. This approach has seen new developments in recent years including work related to shear and torsion, (4) and as an aid to proper detailing. (5) Truss models are attractive in that they are conceptually simpler than the more refined finite element models and at the same time provide more versatility than simple beam theory.

In the following, an analysis procedure based on a truss model for prestressed concrete beams with openings is described. The model provides a means of treating the abrupt change in stiffness that occurs at an opening and allows for cracking through the effective moment of inertia concept. The procedure is therefore valid for loads in the service load range.

METHOD OF ANALYSIS

Figure 1 shows a T-beam with openings and the corresponding truss model. Top and bottom chords, and diagonal web members are proportioned to produce the same flexural stiffness as the beam, as outlined in the next section. To simulate the effect of prestressing, horizontal forces P_T and P_B are applied to the top and bottom chord members respectively, at each end of the truss. Forces P_T and P_B are determined to provide at the end of the truss a total axial force and moment statically equivalent to the actual prestressing force and moment in the beam.

Initially it is assumed that the beam is uncracked and a linear elastic analysis is made for loading due to prestress, dead load and live load. Forces in the truss members are then used to calculate the net axial force and bending moment at the center of each panel of the truss. By comparing the bending moment at sections along the beam with the cracking moment it is possible to scale the applied load to the value at which cracking first occurs. The analysis can then be repeated at selected load levels above the load to cause first cracking to trace the load-deflection response as affected by progressive cracking.

The truss analysis was performed using PFT, a computer program developed at the University of Alberta for analysis of linear elastic plane frames and trusses. Areas of truss members were calculated by hand for each loading stage, although this procedure could be automated if desired.

Calculation of Truss Properties

Truss members are assumed to have a modulus of elasticity equal to that for concrete. It is then necessary to determine areas of truss members to produce a flexural stiffness equivalent to that of the beam. Since it is assumed that deflections due to shear can be neglected the areas of web members are assumed to be unchanged by cracking and areas are determined from the tributary area of concrete based on truss panel spacing as shown in Fig. 1. Top and bottom chord member areas are calculated to provide the same moment of inertia as the beam within each truss panel. The top and bottom chords are assumed to have equal areas.

a) Uncracked Section:

As shown in Fig. 2 the location of the centroid, d_n, of the gross concrete beam section is determined and the gross moment of inertia is calculated. Locations of top and bottom chords are then selected to suit the geometry of the beam, in particular the details of the openings. Areas of all truss members are determined as outlined above.

b) Cracked Section

Behaviour after cracking can be illustrated by the strain diagram shown in Fig. 3a and the moment-curvature relationship shown in Fig. 3b. The curvature after cracking should be considered as the average curvature in a constant moment cracked zone. Application of the prestressing force produces a curvature ϕ_p corresponding to the moment $M_p = P_e e$. Applied moment of opposite sign to the prestressing moment decreases the net moment till zero moment and zero curvature are present at the cross-section. Further application of moment leads to zero stress at the bottom fiber when the net moment is $M_o = (P_e r^2)/c_2$ with curvature ϕ_o. Application of additional moment $M_{cr} = f_r S_2$ (where $S_2 = I_g/c_2$) produces cracking at the bottom fiber. Further application of load leads to progressive decrease in stiffness as indicated. Unloading of the beam from point D would occur along DBOA in Fig. 3.

In Branson's form of the I_e equation,

$$I_e = \left(\frac{M_{cr}}{M_n}\right)^3 I_g + \left(1 - \left(\frac{M_{cr}}{M_n}\right)^3\right) I_{cr}$$

M_{cr} was taken as $M_{cr} = f_r S_2$. This is identical to the expression used for non-prestressed beams. The corresponding value of M_n is given by $M_n = M_a - P_e\left(\frac{r^2}{c_2} + e\right)$ where M_a is the moment due to applied load.

It should be noted that this procedure for calculating I_e differs from the procedure given by Branson and Trost,(2) who replace M_{cr} by the term $M'_{cr} = M_{cr} + (P_e r^2)/c_2$ with a corresponding modification to M_n. In either case, however, the effective moment of inertia expression provides a gradual transition from I_g to I_{cr} as progressive cracking takes place.

Using an incremental loading procedure, the value of I_e at each panel is determined. At each stage the moment and axial force at a section are obtained from truss member forces. Truss top and bottom chord areas are then adjusted to produce the same moment of inertia, I_e, and the analysis is repeated at the same load level.

COMPARISON BETWEEN ANALYTICAL AND EXPERIMENTAL RESULTS

To assess the accuracy of the analytical model, numerical results were compared with experimental results for three beams. Two of these beams were tested by Alves (6) and one by Barney (7). Beam details and truss models are given in Figs. 4 to 6.

The effective prestressing force used in the analysis of each beam is given below:

Beam B2 : P_e = 14.09 kips
Beam C3 : P_e = 14.03 kips
Beam B7 : P_e = 26.30 kips

In each case, load-deflection readings were taken for loads after prestress and beam self-weight were applied. For purposes of comparison therefore, prestress and beam self-weight effects were subtracted from numerical results before plotting the load-deflection curve. While the beams were all tested to failure, the analysis was terminated when yielding was detected in reinforcement, or fracture detected in prestressing strand.

The results shown in Figs. 7 to 9 indicate reasonably good aggreement between experimental and analytical results.

CONCLUSIONS

The results suggest that the truss model developed in this study can predict reasonably accurately the load-deflection response, within the service load range, of prestressed concrete beams with openings.

The effective moment of inertia concept as applied to the model, adequately models the response after cracking takes place.

It is felt that extension of this modelling technique into the ultimate load range could provide useful insight into the behaviour of prestressed concrete members, particularly in terms of the localized behaviour in highly stressed zones such as in the vicinity of openings.

ACKNOWLEDGEMENT

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NOTATION

A_t, A_b = cross-sectional area of top and bottom chord of truss, respectively

c_1, c_2 = distance of top and bottom fibre, respectively, from centroid

e = eccentricity of prestress force

E_c = modulus of elasticity of concrete

f_r = modulus of rupture of concrete

h = vertical distance between top and bottom chords of truss

I_{cr} = moment of inertia of cracked section transformed to concrete

I_e = effective moment of inertia

I_g = moment of inertia of gross section

M_a moment due to applied loads

M_{cr} = cracking moment

M_o = decompression moment

M_p = moment due to prestress

M_n = difference between applied load moment and the sum of the decompression and prestress moments ($M_o + M_p$)

r = radius of gyration of cross-section

S_2 = section modulus relative to tension fibre

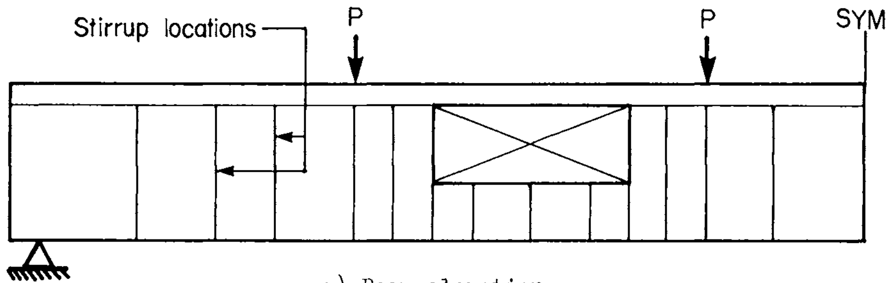
P_e = effective prestress force

P_B, P_T = horizontal forces applied to ends of truss, statically equivalent to prestress force

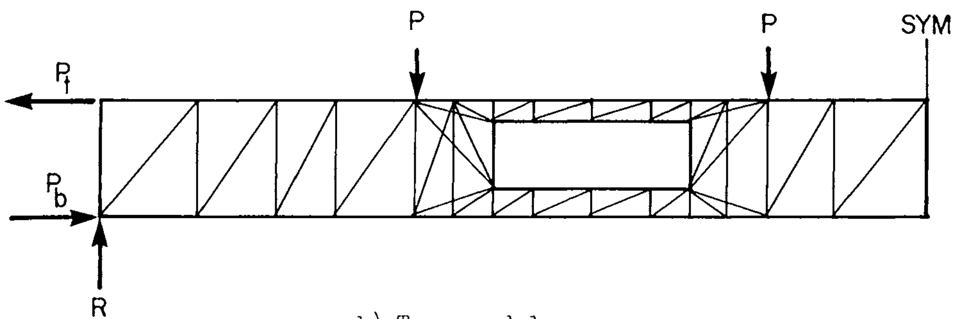
ϕ = curvature

Conversion Factors - SI Equivalents

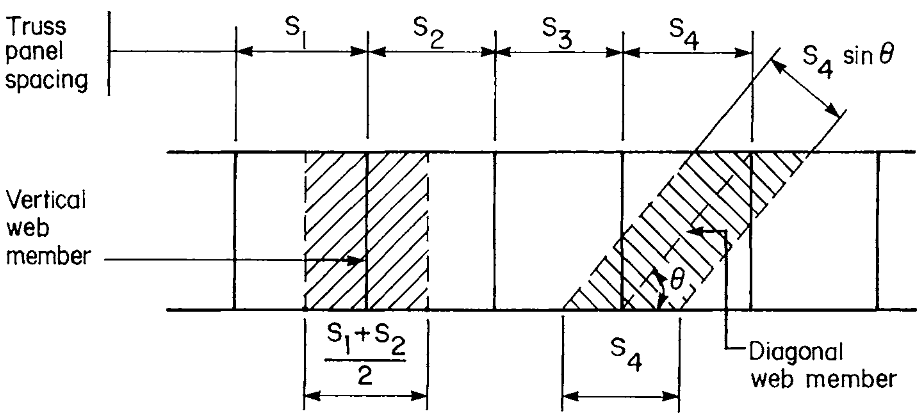
1 in. = 25.4 mm
1 lb (mass) = 0.4536 kg
1 lb (force) = 4.488 N
1 lb/sq in. = 6.895 kPa
1 kip = 444.8 N
1 kip/sq in. = 6.895 MPa
1 in.-kip = 0.1130 N•m



a) Beam elevation



b) Truss model



c) Tributary width for web member area

Fig. 1--Prestressed concrete beam with openings and equivalent truss model.

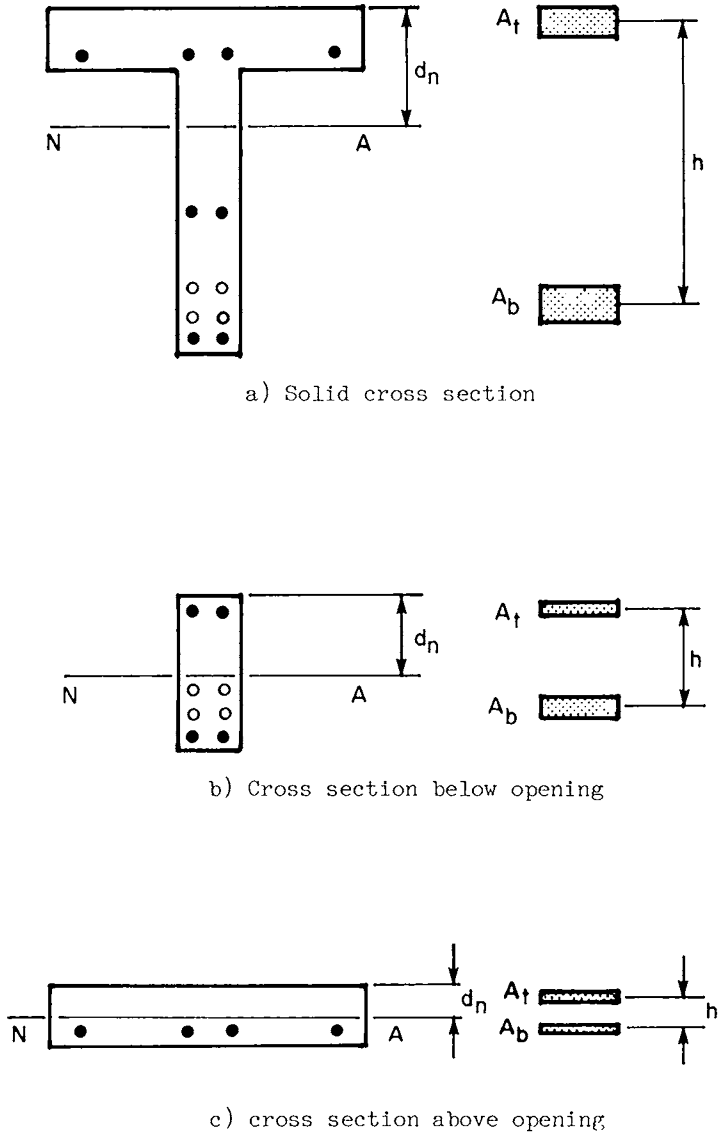


Fig. 2--Beam cross sections and equivalent top and bottom truss chord members.

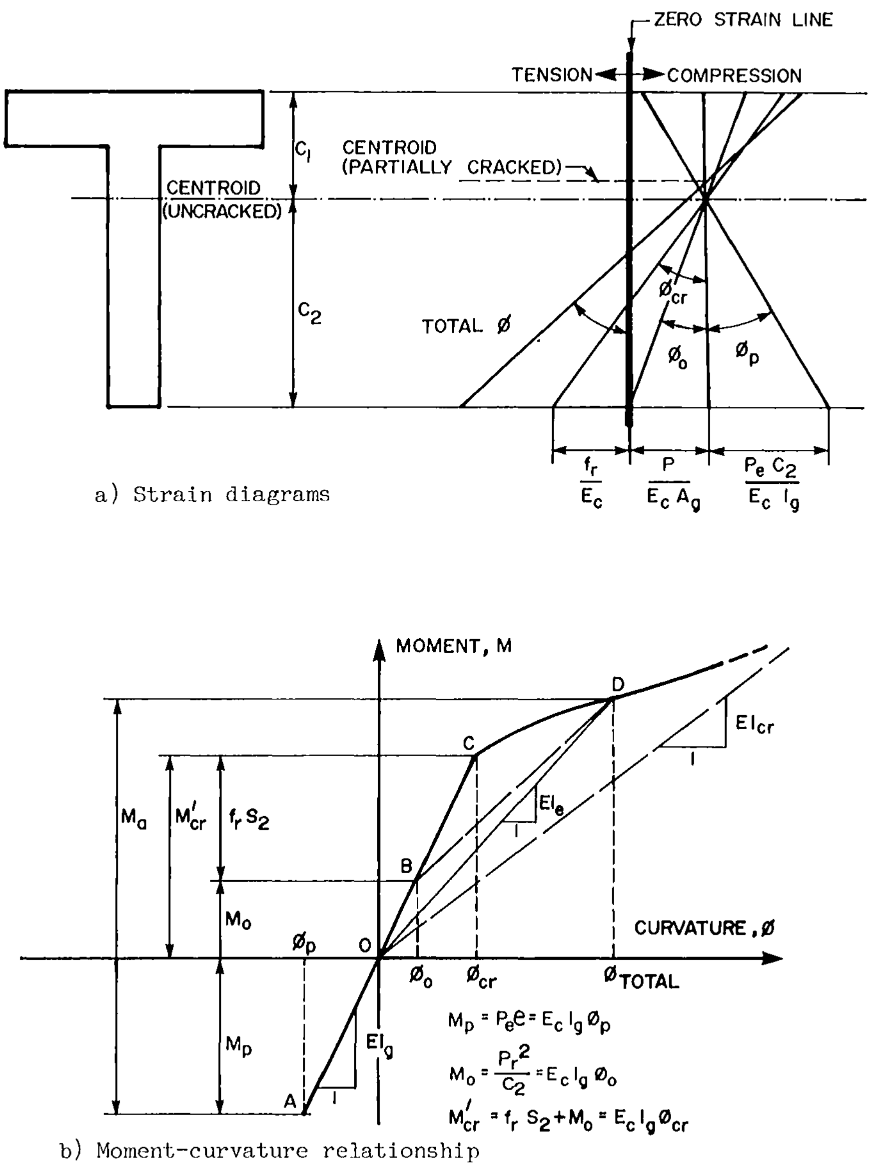
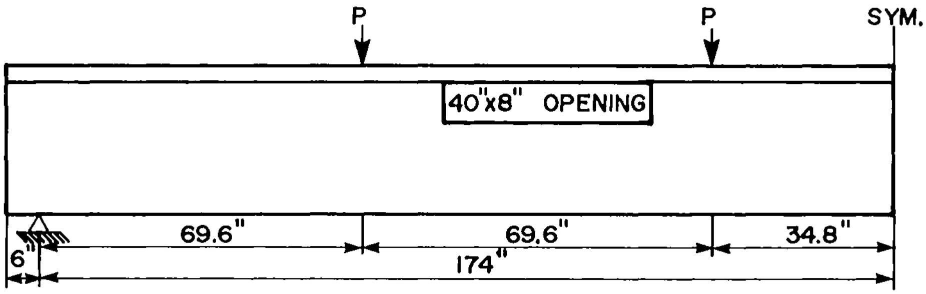
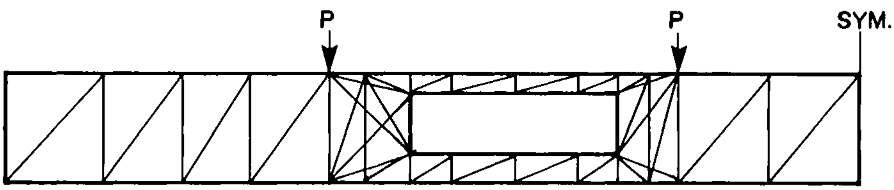


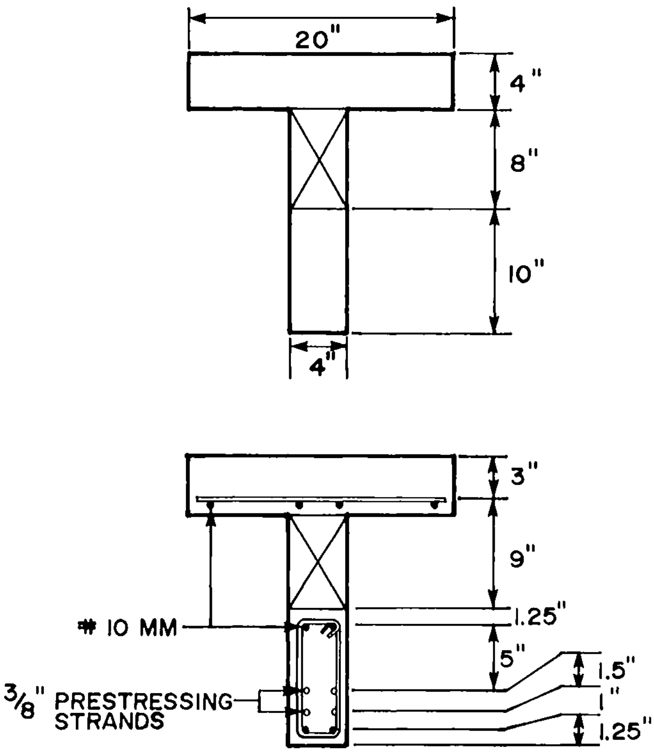
Fig. 3--Strain diagrams and moment-curvature ralationship.



a) Beam elevation



b) Truss model



c) Cross section details

Fig. 4--Details of beam B2 (Ref. 6).