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Load and Resistance Factor Design

By Ivan M. Viest

Synopsis: Load and Resistance Factor Design is a major advance toward rational design of steel framed buildings. It combines the limit states of strength and serviceability with a modern probability-based approach to structural reliability. After a historical introduction, the method is defined and a generalized LRFD format developed. The discussion centers on sources of variability in design, on limit states and on probabilistic concepts underlying the numerical values of load and resistance factors. The results of a design evaluation of LRFD are presented and steps are described that are being taken toward future adoption of the method into the AISC Specification.

Keywords: buildings; limit state design; load factors; loads (forces); probability theory; reliability; resistance factors; safety; serviceability; structural design; structural steels.

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Two major structural developments of the last 30 years will have a profound influence on future design: (1) extensive experimental and analytical studies of structural members and connections and (2) probabilistic studies of the safety and reliability of civil engineering structures.

The first development has already resulted in a quantum improvement in the understanding of structural response to loading. Numerous design changes were adopted during the past 20 years. Structures designed today are lighter and more economical than the designs of - say - 1950 vintage.

The second development is less well-known. Probabilistic approaches to structural safety are relatively new. Over the centuries, structural safety was achieved empirically - through trial and error. Present factors of safety seem to have originated with the use of iron in construction, as described in the following quotation from Professor Pugsley's book, The Safety of Structures: (1)

"When one's personal safety is ... at stake in, say, a surgical operation, it is natural to ask that the operation shall be done by a man who has done it many times and with uniform success. And if the operation is an unusual one ..., then it is all the more desirable that the surgeon should be both of high repute in related work and known to be a fine practitioner ... The same sort of approach was from the earliest times applied to the safety of major structures. One sought out ... a designer of proven success ... a Stephenson or a Brunel was chosen and he could do no wrong. But as accidents nevertheless occurred, these leading engineers were forced to call in each other to investigate each other's mistakes, and so there grew up a system of accident inquiries or investigations ... progressively associated with some agent of Government, in the early days the Board of Trade.

"Thus it came about that with the general introduction of wrought iron in structures some officials of the Board of Trade became knowledgeable on matters affecting structural safety ... and began to recommend, and later insist upon,

> certain design features making for safety. In particular ... the Board, in about 1840, insisted upon a limiting working stress in wrought iron of 5 tons/in.^{2}, the forerunner of many such limitations to come ...
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> "The limit ... for wrought iron corresponded to a factor of safety of at least 4, and set a standard that was promptly applied to mild steel structures when they came into general use ...; hence the limit of 7-1/2 tons/in.^{2} ... set by the first London County Council rules for the use of steel in building construction."

Structural engineers have been working with factors of safety ever since. They have adjusted their values from time to time on the basis of increased knowledge and both good and bad practical experience.

A radical change was brought about by the aircraft industry during World War II. The leaders in aircraft design realized that many design parameters - such as material properties and loads - are random. Starting from this premise, these leaders began collecting statistical data for use with mathematical theory of probability in a rational approach to the question of safety.

Two outstanding civil engineers, Professors Freundenthal in the United States and Pugsley in Great Britain, took part in this work. Shortly after the war they introduced these concepts to civil engineering by creating two committees - one in ASCE, one in ICE - to investigate ways of using probabilistic concepts in the design of civil engineering structures.

Both committees completed their work in the mid-50's. Freundenthal's paper, Safety and Probability of Structural Failure, (2) is considered by many the most basic and definitive paper on probabilistic design.

As is often the case, these early studies brought out many new concepts that departed from traditional norms. They also indicated that existing statistical evidence was too meager to permit early adoption of these methods by the practitioner. Another decade of intensive effort followed. Not until the late 1960's did these developments reach a point where one could see the implementation of these new concepts into practical design.

By that time the American Iron and Steel Institute completed the development of load factor design for steel bridges, and then embarked on similar work for steel buildings. A contract was awarded in 1969 to Professor T. V. Galambos of Washington University at St. Louis. In consultation with his advisory committee, Galambos decided to base the new method, the Load and Resistance Factor Design for steel buildings, on probabilistic concepts. The method was first published by AISI in January 1978 as Research Bulletin No. 27. (3) Shortly after, it was described in a general manner in three papers published in the First Quarter 1978 issue of the AISC Engineering Journal. (4-6) A more detailed treatment was presented in a series of eight papers in the September 1978 issue of the ASCE's Structural Division Journal. (7-14)

Independently of these studies, Committee A58 of the American National Standards Institute has been working to place design loads on a consistent statistical basis. These improved load standards will be included in the revision of A58.1-1972 (15) scheduled for 1980. Beyond 1980, a subcommittee established to develop load factors common for all materials is reviewing a proposal drafted by Drs. Cornell, Ellingwood, Galambos and MacGregor.

LRFD Defined

First, what is load and resistance factor design (LRFD)?

Load and resistance factor design is a method of proportioning structural members and connections so that the strength and serviceability limit states are greater than the corresponding factored load combinations.

This definition can be illustrated with the limit state of fracture at a net section of a tension member, under a combination of dead and live loads.

Strength T_u of such a tension member is equal to the area of net section, A_n, times the tensile strength of steel, F_u. The load combination involves axial tensile loads P_D caused by dead load and P_L caused by live load.

Using load factors proposed by Cornell, Ellingwood, Galambos and MacGregor and the resistance factor from Reference (3), the definition of LRFD may be written as

$$0.74\ T_u \geq 1.2\ P_D + 1.7\ P_L$$

where .74 is the resistance factor, 1.2 is the dead load factor and 1.7 is the live load factor.

The expression can be made more general by using symbols for the numerical values of load and resistance factors:

$$\phi\ T_u \geq \gamma_D P_D + \gamma_L P_L$$

ϕ stands for resistance factor, γ_D and γ_L for dead and live load factors corresponding to the load effects P_D and P_L.

These factors account for various uncertainties in design. The resistance factor accounts for uncertainties in the resistance for a particular limit state. Its magnitude depends on variability associated with that particular limit state.

For example ϕ usually accounts for (1) variability in material properties such as tensile strength of steel; (2) variability in member geometry such as cross-sectional area, seldom exactly as given in a handbook because of practical tolerances; and (3) variability in the ratio of the strength used in calculations to the actual member strength represented by available test data.

Load factors account for dead and live load uncertainties. Examples are (1) variations of live load with time; (2) variations caused by idealization of loads (in building design, we usually assume uniform load) and (3) variations in the location of the loads on the structure.

Additionally, load factors account for uncertainties in structural analysis. Generally, we analyze structures as if they were two dimensional rather than three dimensional. We assume connections are either infinitely rigid or with no rigidity at all, whereas every connection has some rigidity, but never infinitely rigid. In analysis we also assume that a member has no depth, while in a structure all members must have some depth.

To make the LRFD equation completely general, general symbols can now be substituted for the limit state and force effects. R_n is used for nominal resistance, that is, resistance computed from the appropriate design formula. Q_i is used for load effects, where i represents various types of loads like dead load, live load, wind, snow, etc. This general format is

$$\phi R_n \geq \Sigma \gamma_i Q_i$$

Load effects Q_i may be axial forces as in the example; or moments, shears or torsions. They are computed from a structural analysis using code specified loads just as today.

Limit States

As to the resistance R_n, it is computed from formulas given in the specifications from various limit states. Limit states can be classified into two groups: strength limit states representing the capacity of a member or connection to survive extreme loads; and serviceability limit states that must be checked to avoid malfunctioning of the structure during routine use.

Some examples of strength limit states are fracture on the net section of a tension member, local buckling of a column with axial load, lateral-torsional buckling of a beam column and maximum plastic strength of shear connectors in a composite beam.

Examples of serviceability limit states are (1) permanent sag or drift due to yielding, (2) excessive elastic sag or drift, (3) major slip in high strength bolted joints and (4) unacceptable vibrations. Vibrating floors can interfere with personnel using a building or with the functioning of machinery. Excessive elastic drift during windstorms or earthquakes may result in cracking of partitions.

Looking back at the general LRFD format, it is apparent that items for design consideration are not much different from those in strength design in concrete today. The difference lies in the manner of deriving specific numerical values of load and resistance factors. This step is, of course, accomplished by specification writers rather than by designers.

Today, structures are designed for a particular load by using particular single-valued expressions for strength. In reality, however, neither item can be determined with such singular certainty. Test results of 185 beams failing in elastic lateral-torsional buckling, shown in Figure 1, illustrate this point. Figure 1 is a histogram, a plot of the ratio of test to predicted values of strength against the number of tests or frequency of any of these ratios. For example, there were 14 tests in which the ratio of test to predicted value was 1.0. For most of the 185 tests the ratio was close to 1.0, while extreme values of 0.72 and 1.34 were obtained only once each.

Structural Reliability

For the purposes of analysis, the histogram of Figure 1 may be replaced by the smooth curve shown in the top half of Figure 2.

However, variation occurs not only in resistance but also in load effects, as illustrated in the lower portion of Figure 2.

Each of these distributions can be characterized by its mean value; R_m for mean resistance and Q_m for mean load effect, and with standard deviation σ_R or σ_Q to measure horizontal spread. Approximately 95% of all events fall between the vertical lines bounding the cross-hatched areas.

In Figure 3, resistance and load effect are plotted on the same graph. As long as load effect Q is represented by a point on the Q-curve to the left of point A, and resistance is represented by a point on the R-curve to the right of point A, the requirements of LRFD are satisfied and the structure is safe. On the other hand, when the load effect falls beyond point A and the resistance falls short of point A, failure occurs. The cross-hatched area under point A is related to the probability of failure.

It can further be observed in Figure 3 that probability of failure is inversely proportional and, therefore, safety of a structure is proportional to the difference between R_m and Q_m. In other words, the area under A is decreased and the safety of a structure is increased by spreading the Q and R curves further apart. One can also decrease the area under A to increase safety by decreasing variability, that is, by making one or both curves steeper. In other words, the less variability in Q and R, the smaller the area under point A and, therefore, the higher the safety.

These two observations can be written in the form:

$$\beta = \frac{R_m - Q_m}{\sqrt{\sigma_R{}^2 + \sigma_Q{}^2}}$$

where σ_R represents the standard deviation of R and σ_Q represents the standard deviation of Q. This equation was derived from the theory of probability; it is known as the safety index.

Safety index β is related to the probability of failure, the cross-hatched area under point A. As an example, for normal distributions, $\beta = 5$ represents a failure probability of about 3 times 10^{-6}, or that failure would be expected to occur in three out of one million cases.

However, two rather important points must be emphasized. First, probability of failure depends very much on distribution curve shape. Sufficient data is not available to determine these shapes accurately; accordingly, the probability just cited may be substantially in error. Second, while our knowledge of the behavior of individual members and connections alone is quite accurate, our knowledge of the behavior of the same members and connections in a finished structure needs considerable additional studies. Generally speaking, the strength of individual members and connections furnishes minimum values such that current knowledge combined with LRFD concepts results in safe design. Considerable further research will be needed before one can design structures that are utilized to their capacity limits. Probabilities of failure, associated with various β values, must be judged in this context.

Load and resistance factors are not only functions of β, but of coefficients of variation in resistance V_R, engineering analysis V_E, and the load effects V_1. Additionally, they are functions of the ratio of mean to nominal load or resistance. Galambos expressed these functions in closed forms; for example, his equation for resistance factor is:

$$\phi = \frac{R_m}{R_n} e^{-0.55\beta V_R}$$

On the other hand, the method proposed to ANSI A58 involves an optimization procedure that leads to invariant values of load and resistance factors.

One can now choose a β; then compute the numerical values of factors ϕ and γ.

This has been done by Galambos, who evaluated various resistances as well as various coefficients of variation. He then set out to select a particular value of β for his design criteria.

One could select β on the basis of a chosen probability of failure. But in view of current uncertainties concerning relationships between β values and actual probability of failure, it seems preferable to follow the path elected by Galambos. He calibrated the new design against current designs. Or, expressed in another way, he selected a representative safety index value that characterizes the level of safety inherent in current design practice. On this basis, he decided to use $\beta = 3$ for members and $\beta = 4.5$ for connections.

The resulting resistance factors ϕ are shown in Table 1. They vary from .65 for buckling of long columns to .88 for yielding on the gross section of a tension member.

Galambos' resistance factors were rounded off to two decimal places in order to preserve apparent differences for specification writers. But for practical design, ϕ values are likely to be rounded off to one decimal or the nearest .05. It can be readily seen that such rounding will result in very few different resistance factors.

The load factors γ proposed to ANSI A58 are shown in Table 2 for seven combinations of dead, live, wind, snow and earthquake loads. It should be noted that the resistance factors in Table 1 are not strictly compatible with the load factors shown in Table 2, since the two sets of numbers were computed from different theories. However, preliminary studies have shown that resistance factors compatible to the load factors in Table 2 are essentially the same as those shown in Table 1.

Comparative Designs

Galambos selected β values by calibration of the new design against current designs. While this was done first on a theoretical basis, a practical check followed. This was performed by two well known consulting firms; Sverdrup & Parcel of St. Louis and LeMessurier Associates of Boston. They selected four buildings for redesign of certain members by LRFD.

One building was a large, basically one-story aircraft repair facility in Texas. The redesign included a typical roof purlin, two trusses on a 50-ft span, and one interior and one exterior column.

All steel was ASTM A36. Live loads were 20 psf for the roof, 50 psf for the mezzanine floor, and 70 mph wind. Crane and monorail loads were included. Both columns resisted some wind forces by strong axis bending.

Another building was a brewery with a heavily loaded brewhouse floor framed with steel beams and plate girders. Floor dead load was 135 psf with a live load of 300 psf. Comparative designs were made for a typical simply supported filler beam and a plate girder.

Still another building was a six-story passenger car parking garage. Floor live load was 50 psf. Beam, girder and column elements were redesigned.

The last structure was a 16-story office building. Floor framing in this study supported a 3-1/4 in. lightweight concrete slab over a 1-1/2 in. steel deck. Comparative designs were made for typical 32-ft filler beams and 40-ft girders for three different live loads: 50 psf LL with 20 psf partitions, 100 psf LL and 100 psf reduced LL.

Wind forces were resisted in the longitudinal direction by moment resisting framing while, in the short direction, simple connections were used and wind was resisted by bracing in the elevator walls shaft. Design live load was 50 psf, reduced, plus 20 psf partition allowance. Wind loading was 90 mph. ASTM A572 Grade 50 steel was chosen whenever practicable. Columns and girders of an interior and an exterior moment-framed wind bent, with wind bracing, were redesigned.

Results of these studies, based on load factors proposed originally by T. V. Galambos rather than on the load factors proposed to ANSI, are summarized in Figure 4. For each redesigned element, section weight obtained by AISC working stress design (1978 Specification) (16) are plotted against weights according to LRFD. (3) All were adjusted for design efficiency, i.e., the weights were multiplied by the ratio of applied force effect to the resistance of the cross-section. The 45^{o} line represents cases where LRFD and AISC designs give the same steel section. For points falling below the line, LRFD is more economical. For points above the line, LRFD requires larger sections.

On the basis of Figure 4, it is reasonable to conclude that Galambos succeeded in making load and resistance factor design essentially equivalent to AISC design.

Concluding Remarks

The project at Washington University was completed in 1976. The proposed method was transmitted in 1977 to the American Institute of Steel Construction for consideration in future revisions of its Specification. AISC's Specification Committee divided the document among a number of task forces who are now reviewing it. The questions of accuracy and simplicity of use, as well as the twin questions of safety and economy, are important in this review. Assuming that the task forces will come up with favorable recommendations, it will then be up to the whole Committee to decide whether to include LRFD in future editions of the Specification.

It has been often said that one truly understands a technical relationship only when it can be reduced to a mathematical expression. This condition has existed in structural mechanics for over a century. But a rational approach to safety was not available;

safety considerations have remained strictly empirical.

A rational consideration of safety is now a distinct possi-ility. Its adoption, in combination with design based on limit states, will result in two accomplishments: first, it will open the way to significant improvements of both safety and economy; and second, it will give the designer a far better understanding of the meaning of the design process.

And finally, with LRFD, one can see for the first time a clear path toward fully rational structural design.

Acknowledgments

This presentation is based on the references listed below and draws heavily on material in two papers presented at the 1977 AISC National Engineering Conference in Washington, D.C. (References 4 and 6). Figure 1 was taken from Reference 8 and Table 1 from Reference 5.

The phenomenological explanation of the meaning of the safety index β is an outgrowth of a discussion with Professor S. J. Fenves of Carnegie-Mellon University.

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