$$\Delta_{mid\,span} = \frac{\phi_{cr} La}{6}\left[La + L_g\right] + \frac{\phi_a}{6}\left[\frac{3L^2}{4} - La^2 - L^2{}_g - LaL_g\right]$$
$$= \frac{\phi_a}{6}\left(\frac{3L^2}{4} - La^2\right) - \frac{\phi_a}{6} L_g\left(L_g + La\right) + \frac{\phi_{cr} La\left(La + L_g\right)}{6} \quad (31)$$

where ϕ_a = is the curvature corresponding to the maximum moment which is calculated by linear interpolation between the cracking curvature(ϕ_{cr}) and the yielding curvature (ϕ_y), as in Fig.1:

$$\phi_a = \frac{\left(\phi_y - \phi_{cr}\right)\left(M_a - M_{cr}\right)}{\left(M_y - M_{cr}\right)} + \phi_{cr} \quad , M_a = \frac{PL_a}{2} \quad (32)$$

Rearranging Equation (31), the maximum deflection is written in terms of the elastic expression, Equation (28), plus an additional deflection term:

$$\Delta_{mid\,span} = \frac{\phi_a}{24}\left(3L^2 - 4La^2\right) + \frac{\left(L_g + La\right)}{6}\left(\phi_{cr} La - \phi_a L_g\right) \quad (33)$$

Equation (33) reduces down to equation (28) when $M_a < M_{cr}$, by substituting ϕ_a for ϕ_{cr} and L_a *for* L_g.

$$\Delta_{mid\,span} = \frac{\phi_a}{24}\left(3L^2 - 4La^2\right) + \frac{\left(La + La\right)}{2}\left(\not{\phi}_a La - \not{\phi}_a La\right) = \frac{\phi_a}{24}\left(3L^2 - 4La^2\right)$$

Post-Yielding Stage:

Upon yielding of the tensile steel, sections in the post yielding stage will nearly be fully cracked. This assumption is verified to be very accurate since the effective I_e of the section beyond yielding, from nonlinear analysis considering tension stiffening, is comparable to or less than that of I_{cr}. The mid-span deflection at any load level after yielding is analytically formulated by determining the moment of the area under curvature distribution in Fig.3c. This expression is written in terms of ϕ_{cr}, ϕ_y and ϕ_a; where the latter is directly related to the load level:

$$\Delta_{mid\,span} = \frac{\phi_a}{24}\left(3L^2 - 4La^2\right) - \frac{\phi_a L_y}{6}\left(L_y + La\right)$$
$$+ \frac{\phi_{cr} L_y}{6}\left(L_y + L_g\right) + \frac{\phi_y}{6}\left[\left(La^2 - L^2{}_g\right) + L_y\left(La - L_g\right)\right] \quad (34)$$

where L_y is the length of the unyielded regions of the beam, Fig.3c.

$$L_y = \frac{2M_y}{P} \quad (35)$$

Rearranging Equation (34) gives:

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$$\Delta_{mid\,span} = \frac{\phi_a}{24}\left(3L^2 - 4La^2\right) + \frac{L_y}{6}\left[\phi_{cr}\left(L_y + L_g\right) - \phi_a\left(L_y + La\right)\right] + \frac{\phi_y\left(La - L_g\right)\left(La + L_y + L_g\right)}{6} \quad (36)$$

where ϕ_a is the curvature corresponding to the maximum moment of the beam, as defined earlier. It is calculated by linear interpolation between the curvature of first yielding and that at ultimate moment, Fig.1:

$$\phi_a = \frac{(M_a - M_y)(\phi_n - \phi_y)}{(M_n - M_y)} + \phi_y \quad (37)$$

where M_y, ϕ_y, M_n, ϕ_n are calculated from equations (16), (7), (21) ,(22) or (26), (27) respectively. Equation (36) can be used as the general equation to calculate mid-span deflection of simple beams subjected to four-point bending. This equation reduces down to equation (33) if $M_{cr} < M_a < M_y$. In this case, $L_y = La$,$\phi_y = \phi_a$ are substituted resulting in:

$$\Delta_{mid\,span} = \frac{\phi_a}{24}\left(3L^2 - 4L_a{}^2\right) + \frac{\left(L_g + L_a\right)}{6}\left(\phi_{cr}L_a - \phi_a L_a + \phi_a L_a - \phi_a L_g\right) \quad (38)$$

which is the same as equation (33).

Uniform Load

The same integration above is solved by finding the moment of the area under the curvature distribution in closed form using the parabolic moment expressions of the uniform load case. The general solution can be obtained by summing the deflection contribution of the three regions:

$$\Delta_{mid\,span} = \delta_1 + \delta_2 + \delta_3 = \int_0^{l_g} x\phi_{un}(x)dx + \int_{l_g}^{l_y} x\phi_{cr-y}(x)dx + \int_{l_y}^{\frac{l}{2}} x\phi_{y-n}(x)dx \quad (39)$$

where, $\phi_{un}(x), \phi_{cr-y}(x), \phi_{y-n}(x)$ are the curvature expressions before cracking, after cracking and past yielding.

Precracking Region:

Performing the first integral of Equation (39) analytically:

$$\delta_1 = \frac{qL_g^3}{2EI_g}\left(\frac{L}{3} - \frac{L_g}{4}\right) \quad (40)$$

where $L_g = \frac{L}{2} - \frac{L}{2}\sqrt{1 - \frac{8M_{cr}}{qL^2}}$. In the case of uncracked beam, $L_g = \frac{L}{2}$ leading to:

$$\Delta_{mid\,span} = \delta_1 = \frac{5qL^4}{384EI} \quad (41)$$

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Post Cracking Region:

Integrating the second term of Equation (39):

$$\delta_2 = \frac{\phi_y - \phi_{cr}}{M_y - M_{cr}}\left[\frac{qL_y^3}{2}\left(\frac{L}{3}-\frac{L_y}{4}\right)+\frac{M_{cr}}{2}\left(L_g^2 - L_y^2\right)-\frac{qL_g^3}{2}\left(\frac{L}{3}-\frac{L_g}{4}\right)\right] + \frac{\phi_{cr}}{2}\left(L_y^2 - L_g^2\right) \qquad (42)$$

where $L_y = \dfrac{L - \sqrt{L^2 - \dfrac{8M_y}{q}}}{2}$, *and* L_g is given above.

Post Yielding Region:

Carrying out the third integral of Equation (39):

$$\delta_3 = \frac{\phi_n - \phi_y}{M_n - M_y}\left[\frac{5qL^4}{384}+\frac{M_y}{2}\left(L_y^2 - \frac{L^2}{4}\right)-\frac{qL_y^3}{2}\left(\frac{L}{3}-\frac{L_y}{4}\right)\right]+\frac{\phi_y}{2}\left(\frac{L^2}{4}-L_y^2\right) \qquad (43)$$

In the case of a post yielded beam, L_y, L_g are given above leading to:

$$\Delta_{mid\ span} = \sum_{i=1}^{3}\delta_i = \frac{qL_g^3}{2EI_g}\left(\frac{L}{3}-\frac{L_g}{4}\right)+\frac{\phi_{cr}}{2}\left(L_y^2 - L_g^2\right)$$
$$+\frac{\phi_y - \phi_{cr}}{M_y - M_{cr}}\left[\frac{qL_y^3}{2}\left(\frac{L}{3}-\frac{L_y}{4}\right)+\frac{M_{cr}}{2}\left(L_g^2 - L_y^2\right)-\frac{qL_g^3}{2}\left(\frac{L}{3}-\frac{L_g}{4}\right)\right] \qquad (44)$$
$$+\frac{\phi_n - \phi_y}{M_n - M_y}\left[\frac{5qL^4}{384}+\frac{M_y}{2}\left(L_y^2 - \frac{L^2}{4}\right)-\frac{qL_y^3}{2}\left(\frac{L}{3}-\frac{L_y}{4}\right)\right]+\frac{\phi_y}{2}\left(\frac{L^2}{4}-L_y^2\right)$$

To obtain the deflection prior to steel yielding, the δ_3 term is dropped from equation (44) with L_y replaced by $L/2$. Similarly, the deflection before cracking may be determined by omitting the δ_2 *and* δ_3 terms from equation (44) and replacing L_g with $L/2$.

EXPERIMENTAL VERIFICATION

Generating insights on the applicability of the different procedure for calculating deflections is one the objectives of this study. Different beams with a variety of properties, tested by others, are analyzed here by using the ACI equation (1), Modified ACI Equations (3)-(5) and the present procedure. The mid span deflections predicted by these methods are compared with their corresponding experimental results. The pertinent geometric and material

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properties of the surveyed beams are summarized in Table 1. All beams are simply supported subjected to four-point bending.

***Beam B2 by Arduini et. al.* (15):**

Beam B2 was strengthened with a thin CFRP sheet which lead to failure by FRP rupture, Table 1. It is evident from Fig.5 that the deflections from the present procedure are in excellent agreement with the experimental results. On the other hand, the ACI original equation is seen to slightly overestimate the stiffness resulting in less deflection prior to yielding. The ACI original equation is clearly not applicable after yielding, confirming similar conclusions reported earlier[10]. The ACI modified equations by El-Mihilmy and Tedesco overestimate the deflections especially at the post-cracking stage. This may be attributed to the dependence of I_{eff}, in equation (3), on the ratio of $M_{\max}/M_y$. This ratio happens to be 0.51 for the beam at cracking leading to $I_{eff} = 1.13 I_{cr}$ which is substantially lower than I_g ($I_g \approx 2 I_{cr}$). This is reflected in Fig.5 by sudden increase in deflection upon cracking and motivates further investigation of the applicability of equation (3). The close agreement of the experimental response and the present analytical response, despite the analysis assumption of perfect bond, is attributed to the small tensile force transferred to the FRP leading to a low interface shear stresses and negligible bond slip.

***Beam A by Saadatmanesh and Ehsani* (16):**

Saadatmanesh and Ehsani (16) tested a series of doubly reinforced concrete beams strengthened by bonding a GFRP plate. Beam A was selected for comparison. Fig.6 presents the load-deflection curve of beam A. The geometric and material properties of the beam are given in Table 1. It is clear from the plot that the curve of the present procedure is slightly closer to the experimental curve than those of ACI original and modified equations, throughout the whole loading range. The difference in the yielding load between the present and the ACI modified method is attributed to the nonlinear response of the beam, which was accounted for in the present analysis. However, it is evident that the present, ACI original and modified equations yield comparable deflection calculations. These are noticeably lower than the experimental deflections. This is attributed to the bond slip in the adhesive interface. Beam A had a relatively thick and narrow FRP plate. This is expected to give rise to interface shear stresses due to the higher tension force transformed across a smaller width leading to higher bond slip in the absence of additional anchorage systems.

***Beam B3.3 by Spadea et al* (17):**

Fig.7 presents the load deflection response for beam B3.3 strengthened with a CFRP plate and tested by Spadea et.al (17). This beam had steel U-wrap anchorages along the span to control premature shear failure. This anchorage system also controlled the bond slip of the FRP plate as evidenced by the excellent correspondence of the experimental and analytical results, Fig.7. It is clear from Fig.7 that the predictions of the ACI modified equations are in good

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agreement with those of the present procedure while the latter are still closer to the experimental response. This is owing to the small ratio of M_{cr}/M_y bringing I_{eff} in equation (3) closer to I_g at the cracking stage. On the other hand, the higher deflections typically predicted by El-Mihilmy and Tedesco's modified ACI equation after yielding is due to their assumption that the effective section flexural rigidity at mid span is valid for the entire beam, equation (5).

Beam 2 by Blais and Picard **(18)**

Fig.8 shows the load-deflection curve for beam 2 strengthened with a GFRP plate, which was anchored by bolts and tested by Blais and Picard (18), and the corresponding curves predicted by the three analytical procedures. It is clear from the graph that all analytical solutions compare very well prior to yielding. The present calculations appear to be closer to the experimental curve throughout the loading range especially beyond yielding for the reasons mentioned earlier. This comparison confirms that the perfect bond assumption is applicable to beams with FRP plates having additional mechanical anchorage. The softer experimental response in the service load range up to yielding is attributed to some bond slip.

Beams B4 and B10 by Quantrill et al. **(3)**

Quantrill et.al (3) conducted an experimental study on several beams with different anchorage systems and modes of failure. Beam B4 has a bonded GFRP plate without additional anchorage system. The bond slip effect was clear on the experimental response right from the start before cracking, Fig.9. Beam B10 had a CFRP plate under the supports to control premature shear failure but yet the mode failure was described to be of concrete crushing accompanied by plate slip at plate/adhesive interface. The slip represented by softer experimental behavior was evident up to failure, Fig.10. The present and the ACI modified analytical responses are also in good agreement.

PARAMETRIC STUDY

The discrepancy between El-Mihilmy and Tedesco's modified ACI equation results and the experimental response in the first example motivated the need to have a comprehensive parametric study conducted with a wide range of material and geometric properties. The steel, FRP plate and B/H ratios and the FRP material type are the main parameters that are expected to have explicit effect on the post-cracking stiffness of the beam. Therefore, the emphasis was put on varying these parameters to assess the estimate of the ACI original and modified equations in comparison with the present analytical procedure. The steel ratio was varied between the minimum and maximum values specified by ACI 318-99 (11). The FRP ratio was changed from zero, unstrengthened beam, to its maximum passing through the ratio that switches the mode of failure from FRP rupture to concrete crushing. The properties of GFRP plate used are E_{GFRP}= 45 GPa and

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f_{GFRP} =400MPa, and those for the CFRP are E_{CFRP} =400GPa and f_{CFRP} =3000MPa, corresponding to the material used by Arduini et al. (15) The other material properties used in this study are those of beam B2 of Arduini et al. (15), Table 1. All the parameters are varied according to Table 2.

Effect of steel ratio:

To study the influence of ρ_s on load-deflection predictions, high, medium and low steel ratios (0.0215, 0.013, 0.0045) are selected for use, along with the rest of the varying parameters, Table 2. In the case of Glass FRP plates, the effect of ρ_s is similar for the different FRP and B/H ratios. To illustrate this effect, B/H =0.75 and ρ_{FRP} =0.00147 are selected as typical values and the variation of the beam effective moment of inertia is plotted using the three different procedures. It can be seen from Fig.11 that the modified ACI equations compare closely to the variation of the present procedure for the high steel ratio Fig.11a. However, as the steel ratio reduces, the modified equation curves show higher discrepancy by sudden reduction in I_{eff} upon cracking due to reducing the yielding moment involved, as explained earlier, see Figs.11b-c. It is worth mentioning that the mode of failure changes from concrete crushing in the case of high steel ratio to FRP rupture for the other two cases.

This behavior is slightly changed by changing the FRP material to Carbon. In this case, the same steel ratios as well as the FRP and B/H parameters above are used. It is evident from Fig.12a that the high steel ratio causes I_{eff} of the modified equations to be higher than that of the procedure proposed herein. This is attributed to the significantly higher CFRP modulus compared to that of GFRP causing less tension force to be carried by the steel, which increases the yielding moment leading to higher estimates of I_{eff} from the empirical modified equation (3). As the steel ratio reduces, the I_{eff} curve of the modified equation gets closer to that of the present procedure then pass it to the softer side exhibiting the very same sudden drop in I_{eff} upon cracking, Figs12b-c. The ACI original equation is seen to underestimate I_{eff} for higher ρ_s values and compare well with the I_{eff} of the present procedure for small ρ_s ratios with ρ_{FRP} =0.00147. It is clear, however, that all the curves of the present procedure shown in Figs.11-12 have a smooth continuous trend similar to that of I_{eff} from the original ACI equation. The comparison of the three procedures for unstrengthened beams is discussed separately below.

Effect of FRP Ratio:

The second parameter studied is the FRP ratio. In the case of GFRP, it is found that the modified procedure by El-Mihilmy and Tedesco (10) generally provides lower estimates of the beam effective moment of inertia I_{eff} than those of the present procedure for lower FRP ratios. These estimates converge towards

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the present I_{eff} curve as the FRP ratio is increased. This is due to the higher tension force carried by the strengthening plate for higher FRP ratios causing M_y to be higher and, thus, I_{eff} for the modified equation to be higher, equation (3). Figs.13 a-g illustrate this clearly for the case of B/H=0.75 and the smallest steel ratio (ρ_s =0.0045) with ρ_{FRP} =0-0.011< $\rho_{FRP\max}$. The discrepancy between the two curves reduces with the increase of ρ_s , as mentioned earlier. The curve of the modified procedure reproduces very closely the present analysis curve for ρ_s =0.0215. On the other hand, the curve of the original ACI code equation appears to be in very good agreement with the present I_{eff} curve up to steel yielding, after which the ACI original equation is not applicable. As mentioned above this agreement prior to steel yielding gets poorer as ρ_s is increased.

The same effect is studied in the case of CFRP indicating different observations, Fig.14. The comparison of the curve of the modified and present procedure is similar to that of GFRP for the small steel ratio. However, the curve of the modified procedure seems to change course to overestimate the I_{eff} for the high steel ratio due to the higher M_y and hence higher I_{eff} from equation(3). Figs.14.a-e show the comparison for B/H=0.75, ρ_s =0.0215 and ρ_{FRP} =0-0.0065< $\rho_{FRP\max}$. It is evident in this case that the two curves match in the case of unstrengthened beam, Fig.14a. However, the curve of the modified equations[10] starts shifting to the right with higher I_{eff} predictions (stiffer beam) as ρ_{FRP} is increased, Fig.14b-e. Significant discrepancy is observed for ρ_{FRP} >0.004, Figs.14d-e.

The ACI original I_{eff} equation compares well with the curve of the present procedure for the low FRP ratios Fig.14a-b but diverge towards the stiffer side for higher ρ_{FRP} ratios way prior to steel yielding Fig.14c-e. This is attributed to the limitation of the ACI original equation to the lower bound prediction of $I_{eff} = I_{cr}$. The high steel ratio, high FRP ratio and high FRP modulus delay the yielding of steel engaging the nonlinear response of concrete in compression prior to the occurrence of yielding. This causes the beam I_{eff} to reduce noticeably below I_{cr} upon steel yielding.

Effect of section Aspect Ratio (B/H):

The effect of the section aspect ratio is also studied. B/H of 0.5, 0.75and 1 are used in the comparison for both the GFRP and CFRP plate. It is observed that this parameter has a negligible effect on the correspondence of the three I_{eff} curves in the case of Glass and Carbon FRP, Figs. 15a-c and Fig. 16a-c. This is attributed to the fact that the I_{cr}/I_g ratio and the M_{cr}/M_y ratio are almost constant for the different B/H ratios when ρ_s and ρ_{FRP} are held constant.

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Effect of FRP Material:

The effect of changing the FRP material type from Glass to Carbon is very important. It may be easily observed from the graphs discussed above that the use of Carbon FRP tends to make the modified equation (3) proposed by El-Mihilmy and Tedesco overestimate I_{eff} while it tends to underestimate I_{eff} for Glass FRP with lower ρ_s ratios, Figs.13-14. This is caused by the significantly higher stiffness of Carbon FRP, for the same ρ_s and ρ_{FRP}, causing less force to be carried by the steel leading to higher M_y and I_{eff} estimate, see equation (3). The opposite is true for Glass FRP.

Effect of Unstregthened Beams:

The parametric study covers unstrengthened beams as well. It is obvious from Figs. 17 and 18 that the modified ACI equations correlate well with the present analysis results for high ρ_s and significantly underestimate I_{eff} for low ρ_s. This is attributed to the lower M_y values causing larger M_{cr}/M_y ratios leading to unrealistically low I_{eff} estimates at cracking in cases of low ρ_s ratios. On the other hand, the ACI original equation underestimates I_{eff} for high ρ_s and overestimates I_{eff} for low ρ_s values in the service load range. This is due to the empirical exponent selected for equation (1).

***Practical Deflection Calculations*:**

Along with the variation of the parameters mentioned in Table 2, different La/L ratios (0.275, 0.35, 0.44), different f_c' (30, 35, 45 MPa) and f_y values (340, 400, 450 MPa) are used to cover a wider range of parameters for load-deflection calculations based on the present rational procedure. As shown in Fig. 19, a linear relationship between the normalized cracked section moment of inertia (I_{cr}/I_g) and the normalized effective beam moment of inertia at first yielding (I_{eff_y}/I_g) exists regardless of the linear or nonlinear critical section response at yielding. Similarly a linear relationship between section and beam moments of inertia at ultimate load (I_n and I_{eff_n}) is obtained when these parameters are properly normalized by ($I_g.\rho_s$), Fig. 20. Accordingly, the ACI original equation will be improved here to reproduce more closely the post-cracking and post yielding response predicted by the present rational procedure. The first improved equation will be used to calculate the deflection prior to first steel yielding:

$$I_{eff} = \left(\frac{M_{cr}}{M_{\max}}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_{\max}}\right)^3\right] \overline{I}_{eff\,y} \tag{45}$$

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$$\text{where } \bar{I}_{eff\,y} = \frac{I_{eff\,y} - \left(\dfrac{M_{cr}}{M_y}\right)^3 I_g}{1 - \left(\dfrac{M_{cr}}{M_y}\right)^3} \tag{46}$$

Equation (46) makes I_{eff} from equation (45) converge to $I_{eff\,y}$ at first yielding. Similarly, an improved I_{eff} equation is proposed for the post-yielding region:

$$I_{eff} = \left(\frac{M_{cr}}{M_{\max}}\right)^3 I_{eff\,y} + \left[1 - \left(\frac{M_{cr}}{M_{\max}}\right)^3\right] \bar{I}_{eff\,n} \tag{47}$$

$$\text{where } \bar{I}_{eff\,n} = \frac{I_{eff\,n} - \left(\dfrac{M_y}{M_n}\right)^3 I_{eff\,y}}{1 - \left(\dfrac{M_y}{M_n}\right)^3} \tag{48}$$

The improved equations (45) and (49) will be used for practical design calculations with $I_{eff\,y}$ and $I_{eff\,n}$ computed from the equations in Figs. (19-20).

Five examples representing cases of extreme discrepancies in the parametric study are selected to examine the predictions of the improved ACI equations (45) and (47) proposed herein. The first example has B/H=0.75 and a low reinforcement stiffness ($\rho_s = 0.0045$, GFRP with $\rho_{FRP} = 0.00147$). Fig. 21a shows the modified ACI equations to significantly underestimates I_{eff} while the ACI original equation slightly overestimates it. On the other hand, the improved equations show excellent agreement with the rational procedure, Fig. 21a. The second example has B/H=0.75 and a very high reinforcement stiffness ($\rho_s = 0.0215$, CFRP with $\rho_{FRP} = 0.0065$). Fig. 21b has both the modified and original ACI equations overestimate I_{eff} while the improved ACI equations compare closely to curve of the rational procedure. The third example has B/H=0.75 and an intermediate reinforcement stiffness ($\rho_s = 0.0215$, CFRP with $\rho_{FRP} = 0.00147$). Fig. 21c shows both the improved and original equations to closely match and slightly underestimate I_{eff} while the modified equations compare very well to the rational procedure. However, this is not a typical comparison for other intermediate reinforcement stiffness values. The last two examples are solved for unstrengthened beams. The fourth one has B/H=0.5 and a low reinforcement ratio ($\rho_s = 0.0045$). Fig. 22a shows excellent agreement between the ACI equations improved here and the rational procedure, which is not the case for the other equations. The fifth example has B/H=1 and a medium reinforcement ratio ($\rho_s = 0.013$). Fig. 22b shows the improved equations to provide no advantage over the ACI original equation since the effective beam moment of inertia at yielding is almost equal to I_{cr}.

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SUMMARY AND CONCLUSIONS

In this study, a rational procedure was developed to analytically predict the load-deflection response of reinforced concrete beams strengthened with FRP plates. This procedure is based on the assumption of trilinear moment-curvature relationship, verified to be very accurate by experimental results. The three key section parameters, (M_{cr}-ϕ_{cr}, M_y-ϕ_y and M_n-ϕ_n), are determined from detailed section analysis. The moment-curvature relationship is used to express the curvature distribution along the beam in three distinct regions, the pre-cracking, the post-cracking and the post-yielding respectively. This curvature distribution is integrated analytically for closed form mid-span deflection expressions in the case of four-point bending and uniform load.

The results of the present procedure, along with those of the ACI original I_{eff} equation and a modified version of it, are compared for a wide range of experimental load-deflection curves showing excellent agreement of the results of the present procedure for properly anchored FRP plates.

A parametric study is conducted to qualify the applicability of the ACI original and modified I_{eff} equations relative to the results of the present procedure for a wide spectrum of geometric and material properties.

The results of the parametric study show that the higher the reinforcement stiffness (i.e. ρ_{steel}, ρ_{FRP}, and E_{FRP}) the higher the I_{eff} estimate by the modified equations leading to stiffer deflection predictions and vice versa. The higher reinforcement stiffness also leads to stiffer I_{eff} by the ACI 318 equation in the range of service to yielding load level due to its lower bound limit of I_{cr}. Low steel ratios cause the ACI 318 equation to accurately predict I_{eff} while high steel ratios cause it to underestimate I_{eff} for unstrengthened beams between the cracking and service loads.

Improved versions of the ACI 318 equation are proposed to closely reproduce the results of the rational procedure using practical calculations. These are based on the linear correlation of the I_{cr} and $I_{eff\,y}$ at yielding as well as I_n of the critical section and $I_{eff\,n}$ of the beam at failure obtained from the parametric study.

Notation:

A'_s area of compressive steel.
A_s area of tensile steel
A_f FRP plate area.
Ec initial modulus of concrete.
f_y yielding stress of the steel (MPa).
f_f stress in the FRP plate when tensile stress yields(MPa).
h height of the section
I_{eff} beam effective moment of inertia.
$I_{eff\,y}$ beam effective moment of inertia at yielding based on rational procedure.
$I_{eff\,n}$ beam effective moment of inertia at ultimate based on rational procedure.

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