Failure Mechanisms and Fracture of Fiber Reinforced Concrete

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Synopsis: Several types of failure mechanisms and fracture of fiber reinforced concrete (FRC) composites are discussed. These include; mulitple fracture of the matrix prior to composite fracture; catastrophic failure of the composite immediately following matrix cracking due to inadequate reinforcing; fiber pull-out following matrix cracking providing significant energy absorption with or without substantial strengthening of the matrix; and fracture of short fibers bridging the matrix crack without multiple fracture of the matrix. Aspects relating to the modelling of the two major causes for nonlinearities associated with fiber concrete composites, namely interfacial bond-slip, and matrix softening are also discussed. Analytical models available for predicting the tensile response of such composite are examined in light of the above mechanisms of failure.

Keywords: bonding; composite materials; cracking (fracturing); crack width and spacing; failure mechanisms; fiber reinforced concretes; mathematical models; pullout tests; strains; tensile strength

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INTRODUCTION

Potentially useful improvements in the mechanical behavior of tension-weak concrete (or mortar) matrices can be effected by the incoportation of fibers. The resulting interest in and research on fibrous concrete composites has led to an improved understanding of the mechanics of composite behavior under various simple modes of loading. Similar to the behavior of plain concrete, composite failure under most types of loading is initiated by tensile cracking of the matrix along planes where normal tensile strains exceed corresponding permissible valves. This may be followed by multiple cracking of the matrix prior to composite fracture, if the fibers are sufficiently long (or continuous). However when short strong fibers (steel, polypropylene, glass etc.) are used to reinforce brittle matrices (concrete, mortar etc.), once the matrix has cracked, one of the following types of failure will occur:

(a) The composite fractures immediately after matrix cracking. This results from inadequate fiber content at the critical section or insufficient fiber lengths to transfer tensile stresses across the matrix crack.

(b) Although the maximum load on the composite is not significantly different from that of the matrix alone, the composite continues to carry decreasing loads after the peak. The post-cracking resistance is primarily provided by pulling out of fibers from the cracked surfaces. Although no significant increase in the composite tensile strength is observed, a considerable increase in composite fracture resistance (or toughness, computed as area under the stress-displacement curve) can be obtained.

(c) Even after matrix cracking, the composite continues to carry increasing tensile stresses; the peak stress and corresponding deformation are greater than those of matrix alone. During the inelastic range (between the matrix first crack stress and the composite peak stress) for type (c) behavior, progressive debonding and frictional slip at the fiber-matrix interface and some additional matrix cracking occur. It is clear that this mode of failure results in an improved performance of both the matrix and the fibers.

Analytical models used for the prediction of the mechanical behavior of fibrous concrete composites may be suitable for one or more of the above types of failure mechanisms. Based in part on the fundamental approach in their formulation, analytical models can be categorized as; models based on the theory of multiple fracture; composite models; strain relief models; fracture mechanics models; interface bond models and micromechanics models.

MULTIPLE FRACTURE OF THE MATRIX

The law of mixtures approach has been used extensively to model the elastic behavior of many types of fibrous composites. Transformed elastic section for the analysis of conventional reinforced concrete is infact based on the law of mixtures. Prior to cracking the effective composite modulus for a composite containing continuous aligned fibers can be expressed as

$$E_c = E_f V_f + E_m V_m \tag{1}$$

where E and V are the modulus of elasticity and volume content of the constituents identified through the subscripts: f for the fiber, m for the matrix and c for the composite. At loads beyond that causing the matrix to crack, relative displacement between steel and concrete is responsible for the crack-widths observed. As a result iso-strain assumptions of the law of mixture are violated. Tensile load transfer through bond stresses from the fiber to the matrix is achieved away from the crack. Due to further increases in load, additional matrix cracks will develop when the tensile strength of concrete is again exceeded. No more cracks can be formed when the distance between the existing cracks is not sufficient to transfer (by bond) the tensile force great enough to exceed the tensile strength of concrete (σ_{mu}). If uniform bond stress (τ) is assumed, then the minimum crack spacing can be calculated as

$$x_{min} = \frac{V_m \sigma_{mu} r}{2 V_f \tau} \tag{2}$$

where r is the radius of the fiber and σ_{mu} is the tensile strength of matrix. The maximum crack spacing is twice the minimum [1]. After multiple cracking has occurred, neglecting the tensile stress carrying capacity of the matrix, effective composite modulus may be obtained as

$$E_c = E_f V_f \tag{3}$$

The composite stress-strain characteristics is thus at this third stage of loading, proportional to that of the fiber. Composite failure occurs when the fibers fracture or yield. Aveston, Cooper and Kelly [1] have proposed a tensile stress-strain relationship similar to that described in Eqns. 1-3, for glass fiber reinforced concrete (GFRC). Assuming uniform distribution of bond stresses and the crack spacing given by Eq. 2, they have proposed the following equation for computing the enhanced matrix cracking strain ε_{mu}

$$\varepsilon_{mu} = \left[\frac{12\ \tau\ \gamma_m\ E_f\ V_f^2}{E_c\ E_m^2\ r\ V_m}\right]^{1/3} \tag{4}$$

where γ_m = surface energy of matrix fracture, and all other quantities are as defined earlier. This equation was obtained by considering the changes in energy that occur when a crack is formed across a tensile specimen. It was assumed that a crack will form when the sum of the work of fracture of the matrix, the energy absorbed due to friction at the fiber-matrix interface, and the increase in elastic energy in fibers is exceeded by the sum of the work done by the applied sresses and the elastic energy released by the matrix. Assumptions with regard to uniform bond, and ignoring the tensile capacity of the the matrix, are not justified for such composites as discussed later. Additionally Eq. 4 is more sensitive to the fiber volume content than observed experimentally [2,3].

Aveston and Kelly [4] have further extended the model of Aveston et. al. [1] to include analysis of continuous fibers which are elastically bonded (non-uniform bond stress) to the matrix. The elastic solution of this problem is similar to solutions obtained by Cox [5] and Piggot [6] (which is discussed later). Also included in the modified model are considerations for the effects of random fiber orientation in 2-D and 3-D spaces. Expressions derived for crack spacing, however, still assume fibers long enough to be treated continuous. The difference in strains predicted by the much simpler model assuming frictional bond is not significantly different than that predicted by the modified model assuming elastic bond, to warrant the more cumbersome computations.

COMPOSITE MODELS

Cox [5] has analyzed the effect of orientation of the fibers on the strength and stiffness of paper (also a fibrous composite). For discontinuous fibers in a 2-D random distribution the composite modulus, E_c, has been determined to be $E_f\ V_f/3$ and the composite strength $\sigma_f\ V_f/3$. For a 3-D random distribution these values are $E_f\ V_f/8$ and $\sigma_f\ V_f/6$ respectively. In Cox's case, the length of the discontinuous fiber was assumed to be longer than the critical transfer length, ℓ_c, whereby composite failure is by fiber fracture rather than by fiber pull-out Fig. 1a.

Jayatilaka [7] has further discussed stress distributions likely if the fiber length is less than, equal to, or larger than the critical fiber length if a purely frictional shear transfer at the interface is assumed (constant τ). For the case in which the fibers deform elastically and the matrix deforms plastically he has obtained an expression for the composite modulus, E_c, using an average fiber stress solution similar to Cox. It should be noted that the composite failure mechanism assumed by Jayatilaka is still by fracture of fibers.

Piggot [6] has extended the work by Cox, to develop fiber and interface stress solutions for fiber reinforced polymers and reinforced ceramics and cements when the interfacial bonding is elastic-frictional (constant τ) and elastic-nonlinear frictional respectively. The expressions for axial fiber stress and interfacial shear stress are more involved than those proposed by Cox, but as the nonlinear aspect of interfacial shear is incorporated in this model, composite tensile stress-strain behavior predicted exhibits nonlinearities observed in practice. The matrix and fibers are treated to be in a purely elastic state until the onset of interfacial bond degradation. Figs. 1b and c illustrate the fiber axial stress and interfacial shear stress distributions obtained by Piggot for the case of fiber reinforced polymers and fiber reinforced ceramics respectively. The phenomenon of slip introduced by Piggot will be useful in modelling the behavior of steel fiber reinforced cement composites, as described later.

Naaman, Moavenzadeh and McGarry [8] have analyzed FRC probalistically for its tensile strength and post-cracking characteristics. Representing a tensile member as a chain link series they have obtained expressions for weakest link as

$$\left.\begin{aligned} \bar{\sigma}_{cc} &= \bar{\sigma}_{mu}\,(1 - V_f) + \alpha\,\bar{\tau}\,V_f\,\frac{\ell}{\phi} \\ \bar{\sigma}_{cu} &= \frac{\bar{\tau}}{\pi}\,V_f\,\frac{\ell}{\phi} \\ \bar{\gamma}_c &= \bar{\gamma}_{mu}\,(1 - V_f) + \bar{\sigma}_{cu}\,\frac{\ell}{12} \end{aligned}\right\} \qquad (5)$$

where all "bar" quantities signify average quantities: $\bar{\sigma}_{cc}$ = composite cracking strength, $\bar{\sigma}_{mu}$ = matrix tensile strength, τ = bond strength from single fiber pull-out test, $\bar{\sigma}_{cu}$ = composite post cracking strength $\bar{\gamma}_c$ = composite surface energy, $\bar{\gamma}_{mu}$ = matrix surface energy ℓ , ϕ= fiber length and diameter, and α = empirical constant.

Depending upon the number of fibers pulling out from a unit area the composite cracking strength is obtained using an α value of 0.122 determined empirically. The model predicts a linear increase in both the cracking stress and post-cracking strength of FRC composites with an increase in the volume fraction and aspect ratio of the fibers. Strains or deformations at cracking and post-cracking strength levels cannot, however, be predicted using the model.

STRAIN RELIEF MODELS

The name for this category of models has been chosen because they essentially assume an elliptical zone around the crack to be partially relaxed. Outside the partially relaxed elliptical zone the material is assumed to be unaffected by the presence of the crack. Uniform uniaxial loading perpendicular to the plane of the crack and parallel to the direction of reinforcement is considered. Only strains parallel to the direction of loading are considered and shear interactions between the adjacent zones together with stress concentrations at the crack tips are neglected. For the unreinforced matrix, a linear change in strain is assumed parallel to the direction of loading from the crack face to the extremities of the elliptical zone. The size of the elliptical zone is chosen such that the strain energy released by the presence of the crack on the basis of the assumptions made, is the same as calculated by Griffith [9] for the classical case of crack in an infinite elastic material.

Korczynskyj, Harris and Morley [10] and Hannant, Hughes and Kelly [11] have extended the idea to study the influence of strong reinforcing fibers on the growth of cracks in brittle matrices. The strain-field in fibrous composites unlike in the unreinforced matrix, is modified by the presence of crack bridging fibers. The essential difference between Korczynskyj et.al. [10] and Hannant et. al. [11] is in the strain-field assumed within the elliptical zones. Fig. 2 shows the strain-field assumed by Hannant et. al. [11] The fibers are assumed to be sufficiently longer than the crack length and are assumed to be uniformly distributed and aligned perpendicular to the crack (parallel to the loading direction). Stress is transferred between the fibers and the matarix via the fiber matrix interface which is assumed to be purely frictional (constant shear transfer). As a result of these assumptions the fiber and matrix strain distributions within a strip of the elliptical zone are as shown in Fig. 2.

Numerically the crack propagation problem is evaluated thus. The elliptical zones are divided into a number of strips, perpendicular to the crack. Only one quadrant of the ellipse is analyzed due to the two axes of symmetry. For a given crack length and farfield tensile strain the energy stored within these strips by the fibers and matrix are easily evaluated once the strain fields are established. The energy released, U_r, due to the presence of the crack can be evaluated by cummulating the contributions of the individual strips. Similarly the energy absorbed by friction, U_f, due to the difference in the matrix and fiber strains within ellipse L_1 can be computed. U_r and U_f are again evaluated for the same farfield tensile strain, for a small increment δc in the crack length, and thus the rate of energy released $\delta U_r/\delta c$ can be computed. If the rate of energy released is not greater than that absorbed, the farfield matrix strain is incremented and the procedure repeated until the condition of catastrophic crack growth is achieved. The castastrophic crack growth condition in the modified matrix is given by

$$\frac{\delta U_r}{\delta c} > \frac{\delta U_f}{\delta c} + G_{cm} V_m \qquad (6)$$

Where G_{cm} is the Griffith's critical strain energy release rate for the unreinforced matrix. The corresponding farfield matrix strain, ε_{mu}, that just satisfies Eq. 6 is the "enhanced matrix failure strain" as a result of incorporation of fibers in the matrix. Constant bond stress and a traction free crack are assumed in the model. It is also very sensitive to the initial flaw size c assumed [2,3].

FRACTURE MECHANICS MODELS

Two broad classes of models can be identified among those developed using fracture mechanics concepts. The more fundamental class of models use concepts of classical linear elastic fracture mechanics (LEFM) or modified LEFM to solve the problem of crack initiation growth and stability under a farfield tensile loading. The others, do not attempt to predict the global composite behavior under tensile loading mode, but rather use its tensile post-peak stress-displacement relationship to model crack growth in the composite under other loading configurations. This class of model is popularly labelled as the fictitious crack model (FCM).

The mechanics of crack arrest in concrete reinforced with small diameter steel wires was studied by Romualdi and Batson [12]. Superimposing simplified solutions for a penny shaped crack in an infinite matrix subjected to farfield tension and the equilvalent restraining effect of adajcent fibers (arranged in a square array and aligned in the loading direction) they obtained the effective stress intensity factor for cracks in fibrous composites. Implicit in their analysis is the assumption of a perfectly bonded fiber-matrix interface during the crack propagation process. Applying concepts of linear elastic fracture mechanics and adopting a critical stress intensity factor criterion, they have noted that the cracking stress (stress at which the crack propagates beyond the bounds of the adjacent fibers) is an inverse function of the square-root of fiber spacing. Romualdi and Mandel [13] have verified the fiber spacing concept based on splitting tension and flexural (3 point bending) tests on composites with randomly distributed short steel fibers. The applicability of LEFM using a single parameter fracture criterion for small crack lengths (corresponding to fiber spacing) has been questioned by Shah and Rangan [14] and Jenq and Shah [15].

Kasperkiewicz [16] has proposed a novel approach to modeling the tensile fracture of unreinforced and fiber reinforced cement composites. Matrix inhomogeneity is accounted for in his fracture model by making the surface energy a function of location, thus

$$\gamma_f(x) = \frac{\bar{\gamma} + \underline{\gamma}}{2} - \frac{\bar{\gamma} - \underline{\gamma}}{2} \cos \frac{2\pi x}{\lambda} \tag{7}$$

where $\bar{\gamma}$, $\underline{\gamma}$ are bounds of the variations of the local surface energy, and, λ, a material periodicity parameter to account for a representative volume in the inhomogeneous composite. A tensile specimen of matrix is considered to comprise a series of initial flaws of size c_o. Using the Griffith criterion for stable crack growth in addition to assumptions regarding energy absorption in a cycle of unloading/reloading in concrete materials he has obtained solutions for relating the average tensile stress to crack size and the overall displacement of a finite sized tensile specimen. Softening behavior predictions for the matrix in tension have been accomplished qualitatively using these solutions. For the FRC specimens an additional term to account for energy absorbed during fiber pull-out has been incorporated to yield relationship between average tensile stress and corresponding displacements. The model, it should be noted, has only been used to qualitatively describe trends in softening behavior observed. It is quite sensitive to some of the assumed model parameters.

The fictitious crack models for fiber cement composites have been formulated, using approaches similar to the cohesive force model of Barenblatt [17] or the simpler plastic yielding model of Dugdale [18]. The major differences are eventually in the singularity assumptions at the crack tip, the criteria used for crack initiation and growth, and the stability of crack growth. A Fictitious Crack Model (FCM) has been proposed by Hillerborg [19] for fracture analysis of FRC, Fig. 3. The fracture zone (analogus to the plastic zone for metals) ahead of the real crack is assumed to act as a "fictitious crack" which has the ability to transfer stresses. Instead of conventional fracture parameters, the tensile stress-displacement curve of the composite is introduced to describe the behavior of the fracture zone. Elastic stress-strain relations are used to describe the behavior of zones outside the fracture zone. Incorporating the FCM in a finite element scheme, Hillerborg has shown that general trends observed from experiments on the flexural behavior of notched and unnotched FRC beams can be reproduced. Petersson [20] has used the FCM to study crack growth and development of fracture zones in plain and fiber reinforced concrete notched beams.

Wecharatana and Shah [21,22] have modelled the process zone ahead of the real crack in a similar fashion, with the closing pressure function determined experimentally. Unlike the Dugdale model, however, no assumptions have been made with regard to the singularity at the crack tip. An iterative procedure is set-up to determine the size of the process zone so that the predicted

Crack Tip Opening Displacement (CTOD) at a particular load matches the experimentally observed CTOD at the load level. The strain energy release rate has been modified in their formulation to account for slow crack growth as well as residual deformation (analogous to plastic deformations) on unloading. Resistance or R-curves (plot of strain energy release rate versus crack extension inclusive of the proces zone) thus predicted for concrete and FRC compare well with experimentally obtained R-curves for different specimen sizes and geometries, namely: Double Canatilever Beam (DCB), Double Torsion (DT) and notched beam specimens. While simple to use in numerical computations the model does not incorporate conditions for crack stability.

Visalvanich and Naaman [23] have used the R-curve concept to model fracture of FRC. They use (i) an empirical stress-displacement relation for the stress transfer during fiber pull-out, ahead of an actual crack, and (ii) a straight line crack profile in this pseudo-plastic region as the basic data needed to generate the R-curve. Crack propagation is said to occur when the crack opening angle reaches a critical value (termed CCOA). The CCOA determined empirically, varies from 0.13^o for plain mortar to 0.29^o for some fibrous composites. The size of the pseudo-plastic zone varies from 6.35 cm - 226 cm. These are undefined functions of the composition of the composite (V_f and ℓ/d of the fibers). The CCOA cannot as a result be prescribed a priori, which might restrict the use of the model.

Stang and Shah [24] have studied the fracture by fiber pull-out of composites made with continuous fibers aligned in the loading direction. The debonded zone is treated as an interfacial crack. Griffith type criterion is used for the growth of interfacial crack, yielding:

$$\frac{1}{2} \frac{\partial C}{\partial b} (P_{cr})^2 = 2\pi a \; (2 \; \gamma_i) \tag{8}$$

where P_{cr} = critical pull-out load for crack growth, γ_i = specific interfacial work of fracture, b = length of the debonded interface, a = fiber radius, and C is the compliance of the single fiber pull-out specimen. Thus knowing the maximum load one can calculate γ_i which is the material property characterizing the interface. The accuracy of the above equation was demonstrated by measuring changes in compliance resulting from artificially introduced debonded zones of different lengths [25]. In addition, methods to include frictional energy and calibrate the model parameter from the pull-out test was also discussed in [25]. This approach provides an alternative method of analyzing the results of the pull-out test in terms of fracture energy rather than bond strength (this method is discussed later).

Readily available assymptotic solutions have been used for the elastic compliance, C, of the fiber pull-out specimen containing (i) rigid fibers ($(E_f/E_m)(a/\ell)^2 \ln(2\ell/a) >> 1$ and $\ell >> a$), and (ii) elastic fibers ($(E_f/E_m)(a/\ell)^2 \ln(2\ell/a) << 1$ and $\ell >> a$), ℓ is the fiber length and E_f and E_m are as defined earlier. The compliance of the single fiber pull-out problem has been built upon to yield the compliance of an idealized tensile specimen containing one major matrix crack, Fig. 4. Interfacial cracks are present at this stage up to a distance b on either side of the matrix crack along the continuous fibers aligned in the loading direction. The criterion for crack growth (Eq. 8) is once again applied to the idealized tensile specimen to yield the composite strength, σ_{cr}

$$\sigma_{cr} = 2 V_f \sqrt{\frac{E_c E_f}{E_c - V_f E_f} \frac{2\gamma_i}{a}} \qquad \text{(a) Elastic fiber case}$$

$$\sigma_{cr} = 2 V_f \left(\frac{\ell - b}{a}\right) \sqrt{\frac{E_m}{1 + \nu_m} \left(\ln\left(\frac{\ell - b}{a}\right) - 1\right)^{-1} \frac{2\gamma_i}{a}} \qquad \text{(b) Rigid fiber case} \qquad (9)$$

where E_c =composite elastic modulus given by the law of mixtures, ν_m = Poisson's ratio of the matrix, and all other terms as defined earlier.

The solution Eq. 9 (b) shows that for stiffer and shorter fibers, the stiffness of the matrix and the length of the fibers become important in determining σ_{cr} and that crack propagation tends to be unstable soon after interfacial cracks are initiated (σ_{cr} maximum for b = 0). On the other hand Eq. 9 (a) is independent of fiber length. The possible model predicted solutions for the composite tensile stress-strain behavior is presented in Fig. 4 (b). It should be noted that the stress transfering capacity of cracked matrix is ignored in the model. Attempts to predict displacements or strains of the tensile specimen at σ_{cr} have also not been made.

More recently, Jenq and Shah [15] have proposed a fracture mechanics based model to predict the crack propagation resistance of fiber reinforced concrete. Fracture resistance in fibrous composites is separated into four regimes which include; subcritical crack growth in the matrix and the beginning of fiber bridging effect; post-critical crack growth in the matrix such that the net stress intensity factor due to the applied load and the fiber bridging closing stresses remain constant (steady-state crack growth); and the final stage where the resistance to crack separation is provided exclusively by the fibers. The model uses two parameters that describe the matrix fracture properties (K^S_{Ic}, modified critical stress intensity factor based on LEFM and the effective crack length, and $CTOD_c$ the critical crack tip opening displacement), and a fiber pull-out stress versus slip (σ