Creep and Shrinkage Prediction Model for Analysis and Design of Concrete Structures: Model B3

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Synopsis:
The present paper presents in chapter 1 a model for the characterization of concrete creep and shrinkage in design of concrete structures (Model B3), which is simpler, agrees better with the experimental data and is better theoretically justified than the previous models. The model complies with the general guidelines recently formulated by RILEM TC-107. Justifications of various aspects of the model and diverse refinements are given in Chapter 2, and many simple explanations are appended in the commentary at the end of Chapter 1 (these parts are not to be read by those who merely apply the model). The prediction model B3 is calibrated by a computerized databank comprising practically all the relevant test data obtained in various laboratories throughout the world. The coefficients of variation of the deviations of the model from the data are distinctly smaller than those of the latest CEB model (1990), and much smaller than those for the previous model in ACI 209 (which was developed in the mid-1960s). The model is simpler than the previous models (BP and BP-KX) developed at Northwestern University, yet it has comparable accuracy and is more rational. The effect of concrete composition and design strength on the model parameters is the main source of error of the model. A method to reduce this error by updating one or two model parameters on the basis of short-time creep tests is given. The updating of model parameters is particularly important for high-strength concretes and other special concretes containing various admixtures, superplasticizers, water-reducing agents and pozzolanic materials. For the updating of shrinkage prediction, a new method in which the shrinkage half-time is calibrated by simultaneous measurements of water loss is presented. This approach circumvents the large sensitivity of the shrinkage extrapolation problem to small changes in the material parameters. The new model allows a more realistic assessment of the creep and shrinkage effects in concrete structures, which significantly affect the durability and long-time serviceability of civil engineering infrastructure.

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Chapter 1

Description of Model B3 and Prediction Procedure

1.1 Introduction

During the last two decades, significant advances in the understanding of creep and shrinkage of concrete have been achieved. They include: (1) vast expansion of the experimental data base on concrete creep and shrinkage; (2) compilation of a computerized data bank; (3) development of computerized statistical procedures for data fitting and optimization; and (4) improved understanding of the physical processes involved in creep and shrinkage, such as the aging, diffusion processes, thermally activated processes, microcracking and their mathematical modeling. These advances have made possible the formulation of the present model, which represents an improvement compared to the model in ACI 209. The new model (representing the third major update[2] of the models[3,4] developed at Northwestern University) is labeled Model B3.

In Chapter 1 of this paper the model is formulated succinctly, without any explanations, justifications, extensions and refinements. These are relegated to Chapter 2 and to the Commentary at the end of Chapter 1 of this paper, which does not have to be read by those who merely want to apply the model and do not have time for curiosity about its justification. The methods and typical examples of structural analysis for creep and shrinkage will not be discussed in this paper. They are treated in Chapters 3-5 of ACI 209, other articles and some books.

A background at the level of standard undergraduate courses in mechanics of materials, structural mechanics, engineering mathematics and concrete technology is expected from the user.

The present paper presents an improved alternative to Chapter 2 of ACI 209. Chapters 3-5 of that report, dealing with the structural response, remain applicable to the present paper. The improvement means that the coefficient of variation of the errors of the predictions of creep and shrinkage strains are 23 % for creep (basic and with drying) and 34% for shrinkage for the present model, while those for the model from Chapter 2 of ACI 209 are 58% for basic creep, 45% for creep with drying and 55% for shrinkage. The penalty is modest reduction in simplicity of the model. The user should decide what accuracy he needs depending on the sensitivity of the structure defined in Section 1.2.1 .

1.2 Applicability Range

1.2.1 Levels of Creep Sensitivity of Structures and Type of Analysis Required

Accurate and laborious analysis of creep and shrinkage is necessary only for some special types of structures. That depends on the sensitivity of the structure. Although more precise studies are needed, the following approximate classification of sensitivity levels of structures can be made on the basis of general experience[C1,*].

Level 1. Reinforced concrete beams, frames and slabs with spans under 65 ft (20 m) and heights of up to 100 ft (30 m), plain concrete footings, retaining walls.

Level 2. Prestressed beams or slabs of spans up to 65 ft. (20 m), high-rise building frames up to 325 ft (100 m) high.

Level 3. Medium-span box girder, cable-stayed or arch bridges with spans of up to 260 ft (80 m), ordinary tanks, silos, pavements.

Level 4. Long-span prestressed box girder, cable-stayed or arch bridges; large bridges built sequentially in stages by joining parts; large gravity, arch or buttress dams; cooling towers; large roof shells; very tall buildings.

Level 5. Record span bridges, nuclear containments and vessels, large offshore structures, large cooling towers, record-span thin roof shells, record-span slender arch bridges.

The foregoing grouping of structures is only approximate. If in doubt to

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*The superscripts preceded by 'C' refer to the comments listed in the Commentary in Section 1.7.8

which level a given structure belongs one should undertake an accurate analysis of the creep and shrinkage effects in a given structure (such as maximum deflection, change of maximum stress and crack width) and then judge the severity of the effects compared to those of the applied loads.

The full B3 model presented in this paper is necessary for levels 4 and 5. It is also preferable but not necessary for level 3. For level 3 the short form of model B3 presented also in this volume is appropriate. For level 2 and as an approximation also for level 3, simpler models are adequate including the model in Chapter 2 of ACI 209.

A refined model such as that presented here ought always to be used for structures analyzed by sophisticated computer methods, particularly the finite element method (because it makes no sense to input inaccurate material properties into a very accurate computer program). The error in maximum deflections or stresses caused by replacing an accurate analysis of creep and shrinkage effects with a simple but crude estimation is often larger than the gain from replacing old fashioned frame analysis by hand with a computer analysis by finite elements.

The age-adjusted effective modulus method (ACI 209) is recommended for levels 3 and 4. The effective modulus method suffices for level 2. For level 1, creep and shrinkage analysis of the structure is not needed but a crude empirically based estimate is desirable. Level 5 requires the most realistic and accurate analysis possible, typically a step-by-step computer solution based on a general constitutive law, coupled with the solution of the differential equations for drying and heat conduction.

The creep and shrinkage deformations invariably exhibit large statistical scatter. Therefore a statistical analysis with estimation of 95% confidence limits is mandatory for level 5. It is highly recommended for level 4. For lower levels it is desirable but not necessary, however, the confidence limits for any response X (such as deflection or stress) should be considered, being estimated $\bar{X} \times (1 \pm 1.96\omega)$ where $\bar{X}$ = mean estimate of X and ω is taken same as in Eq. (1.25).

Analysis of temperature effects and effects of cycling of loads and environment ought to be detailed for level 5 and approximate for level 4. It is not necessary though advisable for level 3 and can be ignored for levels 1 and 2 (except perhaps for the heat of hydration effects).

1.2.2 Parameter Ranges

The prediction of the material parameters of the present model from strength and composition is restricted to Portland cement concrete with the following parameter ranges:[C2]

$$0.35 \le w/c \le 0.85, \quad 2.5 \le a/c \le 13.5 \tag{1.1}$$

$$\begin{array}{lll} 2500 \text{ psi} \le \bar{f}_c \le 10,000 \text{ psi}, & 10 \text{ lb/ft}^3 \le c \le 45 \text{ lbs/ft}^3 & \text{inch-pound system} \\ 17 \text{ MPa} \le \bar{f}_c \le 70 \text{ MPa} & 160 \text{ kg/m}^3 \le c \le 720 \text{ kg/m}^3 & \text{S.I.} \end{array} \tag{1.2}$$

$\bar{f}_c$ is the 28 day standard cylinder compression strength of concrete (in psi (inch-pound system) or MPa (S.I.) units), w/c is the water-cement ratio by weight, c is the cement content (in lb/ft^3 (inch-pound system) or kg/m^3 (S.I.) units) and a/c is the aggregate-cement ratio by weight. The formulae are valid for concretes cured for at least one day[C3]. If the model parameters are calibrated by tests, the present model is applicable to any portland cement concrete including light-weight and high-strength concrete except that the autogenous shrinkage of the latter may need a more detailed formulation.

1.3 Definitions, Basic Concepts and Overview of Calculation Procedures

The present prediction model is restricted to the service stress range[C4] (or up to about $0.45\bar{f}_c$, where $\bar{f}_c$ = mean cylinder strength at 28 days). This means that, for constant stress applied at age t',

$$\epsilon(t) = J(t,t')\sigma + \epsilon_{sh}(t) + \alpha \Delta T(t) \tag{1.3}$$

in which $J(t,t')$ is the compliance function = strain (creep plus elastic) at time t caused by a unit uniaxial constant stress[C5,C6] applied at age t', σ = uniaxial stress, ϵ = strain (both σ and ϵ are positive if tensile), ϵ_{sh} = shrinkage strain (negative if volume decreases), $\Delta T(t)$ = temperature change from reference temperature at time t, and α = thermal expansion coefficient (which may be approximately predicted according to ACI 209[5]).

The compliance function may further be decomposed as

$$J(t,t') = q_1 + C_0(t,t') + C_d(t,t',t_0) \tag{1.4}$$

in which q_1 = instantaneous strain due to unit stress, $C_0(t,t')$ = compliance function for basic creep (creep at constant moisture content and no moisture movement through the material), and $C_d(t,t',t_0)$ = additional compliance function due to simultaneous drying[C7].

The creep coefficient $\phi(t,t')$, which represents the most convenient way to introduce creep into simple structural analysis, must be calculated as[C8]

$$\phi(t,t') = E(t')J(t,t') - 1 \tag{1.5}$$

where $E(t')$ = (static) modulus of elasticity at loading age t'. Various definitions of E exist. Nevertheless, since only the values of $J(t,t')$ matter for structural analysis, any reasonable estimate of $E(t')$ can be used[C7] provided that $\phi(t,t')$ is calculated from Eq. (1.5)—for example, the result of the ASTM standard test or the value estimated from strength $\bar{f}_c(t')$ at age t' by ACI formula, valid if $\bar{f}_c$ and E are in psi (if in MPa, replace 57000 with 4734):

$$E(t') = 57000\sqrt{\bar{f}_c(t')} \tag{1.5a}$$

The relative humidity in the pores of concrete is initially 100%. In the absence of moisture exchange (as in sealed concrete), a gradual decrease of pore humidity, called self-desiccation, is caused by hydration[C9]. Exposure to the environment engenders a long-term drying process (described by the solution of the diffusion equation), which causes shrinkage and additional creep[C10]. In the absence of drying there is another kind of shrinkage, called autogeneous shrinkage, which is caused by the chemical reactions of hydration. This shrinkage is usually small for normal concretes (not for high-strength concretes) and can usually be neglected[C11].

In the following sections, the expressions for individual terms in Eq. (1.3)–(1.4) will be presented first. The formulae to predict the coefficients of these equations, statistically derived from calibration with the data bank, will be given next. Two examples of the calculation procedure will then be given. Estimation of the statistical scatter of the predicted shrinkage and creep values due to parameter uncertainties will be discussed next. Finally, a method of improving the predictions of the model by extrapolation of short-time test data will be presented.

1.4 Calculations of Creep and Time Dependent Strain Components

1.4.1 Basic Creep (Material Constitutive Property)

The basic creep compliance is more conveniently defined by its time rate than its value:

$$\dot{C}_0(t,t') = \frac{n(q_2 t^{-m} + q_3)}{(t-t') + (t-t')^{1-n}} + \frac{q_4}{t}, \qquad m = 0.5, n = 0.1 \tag{1.6}$$

in which $\dot{C}_0(t,t') = \partial C_0(t,t')/\partial t$, t and t' must be in days, m and n are empirical parameters whose value can be taken the same for all normal concretes and are indicated above; and q_2, q_3 and q_4 are empirical constitutive parameters which will be defined later[C12]. The total basic creep compliance is obtained by integrating Eq. (1.6):

$$C_0(t,t') = q_2 Q(t,t') + q_3 \ln[1+(t-t')^n] + q_4 \ln\left(\frac{t}{t'}\right) \tag{1.7}$$

in which $Q(t,t')$ is given in Table 1.1 and it can also be calculated from an approximate explicit formula given by Eq. (1.35) in the Appendix to this chapter[C13]. Function $Q(t,t')$, of course, can also be easily obtained by numerical integration (see Section 1.8.1 in the Appendix).

Table 1.1: Values of function $Q(t,t')$ for $m = 0.5$ and $n = 0.1$

	log t'								
log (t-t')	**0.0**	**0.5**	**1.0**	**1.5**	**2.0**	**2.5**	**3.0**	**3.5**	**4.0**
-2.0	0.4890	0.2750	0.1547	0.08677	0.04892	0.02751	0.01547	0.008699	0.004892
-1.5	0.5347	0.3009	0.1693	0.09519	0.05353	0.03010	0.01693	0.009519	0.005353
-1.0	0.5586	0.3284	0.1848	0.1040	0.05846	0.03288	0.01849	0.01040	0.005846
-0.5	0.6309	0.3571	0.2013	0.1133	0.06372	0.03583	0.02015	0.01133	0.006372
0.0	0.6754	0.3860	0.2185	0.1231	0.06929	0.03897	0.02192	0.01233	0.006931
0.5	0.7108	0.4125	0.2357	0.1334	0.07516	0.04229	0.02379	0.01338	0.007524
1.0	0.7352	0.4335	0.2514	0.1436	0.08123	0.04578	0.02576	0.01449	0.008149
1.5	0.7505	0.4480	0.2638	0.1529	0.08727	0.04397	0.02782	0.01566	0.008806
2.0	0.7597	0.4570	0.2724	0.1602	0.09276	0.05239	0.02994	0.01687	0.009494
2.5	0.7652	0.4624	0.2777	0.1652	0.09708	0.05616	0.03284	0.01812	0.01021
3.0	0.7684	0.4656	0.2808	0.1683	0.1000	0.05869	0.03393	0.01935	0.01094
3.5	0.7703	0.4675	0.2827	0.1702	0.1018	0.06041	0.03541	0.02045	0.01166
4.0	0.7714	0.4686	0.2838	0.1713	0.1029	0.06147	0.03641	0.02131	0.01230
4.5	0.7720	0.4692	0.2844	0.1719	0.1036	0.06210	0.03702	0.02190	0.01280
5.0	0.7724	0.4696	0.2848	0.1723	0.1038	0.06247	0.03739	0.02225	0.01314

The terms in Eq. (1.7) containing q_2, q_3 and q_4 represent the aging viscoelastic compliance, non-aging viscoelastic compliance, and flow compliance, respectively, as deduced from the solidification theory[6].

1.4.2 Average Shrinkage and Creep of Cross Section at Drying

Shrinkage

Mean shrinkage strain in the cross section:

$$\epsilon_{sh}(t,t_0) = -\epsilon_{sh\infty}\, k_h\, S(t) \tag{1.8}$$

Time dependence:

$$S(t) = \tanh\sqrt{\frac{t-t_0}{\tau_{sh}}} \tag{1.9}$$

Humidity dependence:

$$k_h = \begin{cases} 1 - h^3 & \text{for} \quad h \le 0.98 \\ -0.2 & \text{for} \quad h = 1 \text{ (swelling in water)} \\ \text{linear interpolation} & \text{for} \quad 0.98 \le h \le 1 \end{cases} \tag{1.10}$$

Size dependence:

$$\tau_{sh} = k_t(k_s D)^2 \tag{1.11}$$

where v/s = volume to surface ratio of the concrete member, $D = 2v/s$ = effective cross-section thickness which coincides with the actual thickness in the case of a slab, k_t is a factor defined by Eq. (1.20) and k_s is the cross-section shape factor:

$$k_s = \begin{cases} 1.00 & \text{for} \quad \text{an infinite slab} \\ 1.15 & \text{for} \quad \text{an infinite cylinder} \\ 1.25 & \text{for} \quad \text{an infinite square prism} \\ 1.30 & \text{for} \quad \text{a sphere} \\ 1.55 & \text{for} \quad \text{a cube} \end{cases} \tag{1.12}$$

The analyst needs to estimate which of these shapes best approximates the real shape of the member or structure. High accuracy in this respect is not needed and $k_s \approx 1$ can be assumed for simplified analysis.

Time-dependence of ultimate shrinkage:

$$\epsilon_{sh\infty} = \epsilon_{s\infty}\frac{E(607)}{E(t_0 + \tau_{sh})}; \qquad E(t) = E(28)\left(\frac{t}{4 + 0.85t}\right)^{1/2} \tag{1.13}$$

where $\epsilon_{s\infty}$ is a constant (given by Eq. 1.19). This means that $\epsilon_{s\infty} = \epsilon_{sh\infty}$ for $t_0 = 7$ days and $\tau_{sh} = 600$ days[C14].

Additional Creep Due to Drying (Drying Creep)

$$C_d(t, t', t_0) = q_5\left[\exp\{-8H(t)\} - \exp\{-8H(t'_0)\}\right]^{1/2}, \quad t'_0 = \max(t', t_0) \tag{1.14}$$

if $t \ge t'_0$, otherwise $C_d(t, t', t_0) = 0$; t'_0 is the time at which drying and loading first act simultaneously; and

$$H(t) = 1 - (1 - h)S(t) \tag{1.15}$$

Fig. 1.1 shows the typical curves of basic creep, shrinkage and drying creep according to the present model.

1.4.3 Prediction of Model Parameters

Some formulae that follow are valid only in certain dimensions. Those are given both in inch-pound system units (psi, in.) and in SI (metric) units (MPa, m). The units of each dimensional quantity are also specified in the list of notations (Appendix to Chapter 1)[C15].

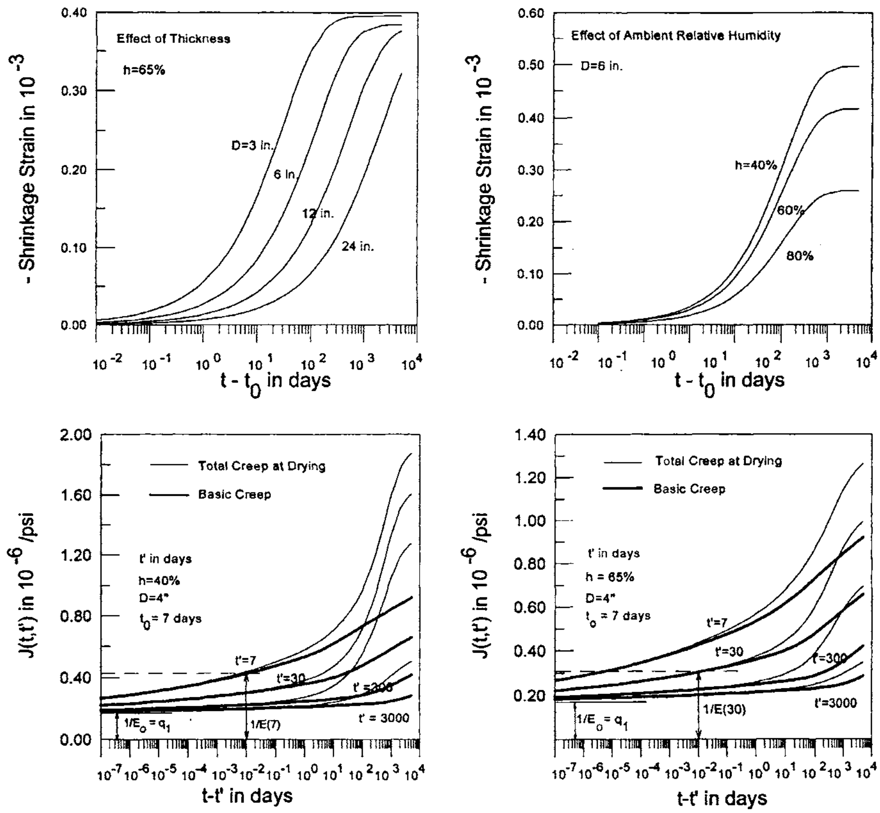

Figure 1.1: Typical creep and shrinkage curves given by Model B3