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Plain and Reinforced Concrete Arches*

Report of Committee 312

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SYNOPSIS

This report contains a summary of the work done by Committee 312 since the publication of its progress report in 1932, and presents for discussion a suggested specification for reinforced concrete arch design. Further investigation has been made of the effects of shrinkage and plastic flow. Measurements on large arches at Pittsburgh over a period of years are presented and discussed along with the results of other investigations. Recommendations are made as to the calculations of moments and thrusts due to loading and volume changes. A new method is proposed for the design of the arch rib based on ultimate strength formulas and a new approach to the question of factor of safety for members subjected to direct load and flexure.

I—INTRODUCTION

COMMITTEE 312 submitted a progress report in 1932 ([1]) relating to the limitations of the theory of elasticity as applied to concrete arches, and the effect of plastic flow, shrinkage and temperature variations. This second report has been delayed pending the completion of other research which was necessary before satisfactory recommendations could be made for arch design. This research related particularly to the effect of plastic flow and shrinkage and to improvements in the method of giving effect to the thrusts and moments in proportioning the arch sections. Sufficient progress appears to have been made in that direction to warrant the submission of a final report presenting a rational method of arch design.

A series of measurements has been made on a number of large arches in Pittsburgh extending over a period of several years to determine the magnitude of shrinkage, plastic flow, and temperature

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movements. A similar investigation but more in detail has been made on the Arlington Memorial Bridge.[2] Further laboratory investigations have been made on shrinkage and plastic flow[3,4,5,6,7,8] and the comprehensive and very important work of the Special Committee on Concrete and Reinforced Concrete Arches of the American Society of Civil Engineers was brought to a close with the submission of a final report in 1935.[9]

These investigations have served to confirm the fundamental principles expressed in the progress report[1] regarding the effect of plastic flow, shrinkage, and temperature changes. But they have also proven the futility of attempting to predict quantitatively the effect of the induced volume changes on unit stresses. This has lead to the recommendation in this report of a new method of design, abandoning the use of unit stress calculations.

It is proposed that thrusts and moments be calculated by the customary methods of the elastic theory. The sections of the arch are then to be proportioned so that these thrusts and moments do not exceed the useful strength of the sections as determined by ultimate strength calculations. The ultimate strength is determined by simple formulas based on the cylinder strength of the concrete and the yield point strength of the reinforcing steel. The useful strength is a proportion of the ultimate strength depending on the desirable factor of safety and the amount of cracking permissible under extreme conditions.

This method is much more realistic than the calculation of working stresses which are not even approximated under actual conditions and it leads to a much more definite and consistent factor of safety through the full range of eccentricities. It will result in considerable economy in material required in the arches in many cases because it recognizes the real strength of the concrete section under bending and direct stress. Also the calculations are much simpler than they are in the case of the standard straight-line-no-tension theory.

This report discusses under the following sections the determination of thrusts and moments, the design of the sections, and closes with a suggested design specification.

II—DETERMINATION OF THRUSTS AND MOMENTS

1. *Dead and Live Loads*

The reactions of a concrete arch rib and the thrusts and moments at various sections produced by live load can be calculated satisfactorily by application of the theory of elasticity using the transformed moment of inertia of the reinforced sections. This is based on the customary assumptions that stress is proportional to strain, that plane sections remain plane, and that the concrete is uncracked so that the full moment of inertia is used in the calculations. It has been proven that accidental variations in the modulus of elasticity of the concrete or a considerable amount of cracking do not materially effect the elastic action of the rib.(9)

The reactions, moments, and thrusts due to dead load can also be calculated in the same way as those due to live load. Plastic flow under dead load has the effect of increasing dead load rib shortening but also provides sufficient stress relief as explained in the previous report so that the stresses in the concrete are no greater than they would be if it were purely elastic. The dead-load steel stresses are increased by plastic flow of the concrete, but this increment is not important because it has no effect on ultimate strength as it would be eliminated by the large strains occurring before failure. Dead load rib-shortening thrusts and moments can therefore be calculated as though the concrete were elastic. This is based on the assumption that the arch axis will be designed to follow as closely as possible the pressure line of dead loads in order to reduce to a minimum the distortion of the rib due to plastic flow under dead load eccentricity.

Plastic flow can also be neglected in the case of live loads.

It should be noted that the methods of calculating moments and thrusts are suggested here to be used with the method of design proposed in this report, without consideration as to whether they may or may not be appropriate for use in determining unit stresses. They are considered here entirely from the point of view of their effect on the useful strength of the rib and not on the unit stresses.

The Special Committee of the American Society of Civil Engineers has investigated the action of skewed arches, continuous arches, combinations of arches with deck superstructure and with high piers. Special methods of analysis of such structures is so thoroughly treated in its report (9) that they will not be discussed here.

Tables and diagrams for use in determining dead and live load effects on symmetrical and unsymmetrical arches are available (10,11) as well as for elastic constants to be used for calculating effects of shrinkage, temperature, and elastic supports such as continuous spans or yielding abutments. If the arches are proportioned as directed, these give accurate solutions in accordance with the elastic theory and save a great amount of labor. Live load effects should be calculated with the use of influence lines in order to determine the maximum thrusts and moments due to critical positions of the loads.

Consideration should be given to construction loads in case heavy equipment or materials or other construction conditions may have a more critical effect than the regular live loads for which the arch is designed.

2. Temperature Changes

In addition to the data summarized in the progress report, the following observations have been made on temperature effects on arches.

In the case of one of the arches of the Arlington Memorial Bridge,(2) the initial temperature of the concrete when pouring was begun was about 75° F. After about 25 hours it reached a maximum of 143° F. The evolution of heat of setting continued after the maximum temperature was reached but the temperature of the concrete dropped gradually and reached air temperature in about 15 days. This measurement was made at the springing where the concrete was about four feet ten inches thick. At the crown where the barrel was 27 in. thick, the maximum temperature varied between 90° and 100° F and the concrete reached air temperature in about 8 days after pouring.

This gives an indication of about how long the pouring of transverse keys in the arch should be delayed after the main body has been poured in order to eliminate the effect of temperature rise due to setting. Some of this effect will be eliminated by plastic flow, but there is reason to believe that the rapid setting of the present standard portland cement has effected a reduction of the plastic flow at such early ages that the heat of setting may cause cracking in heavy arches unless precautions are taken.

It was further observed that after the heat of setting was dissipated the mean temperature of the arch closely approximated the mean air temperature for the five previous days. During a period of 27 months the maximum variation in mean concrete temperature was 61° F while the maximum range of air temperature was 99° F. The concrete was made with Potomac River sand and gravel and had a coefficient of thermal expansion of 0.0000065 per degree F. as determined from prisms and measurements of crown deflection. The stiffness of the spandrels and deck reduced the vertical movement of the crown about 13 per cent as compared with the free arch and increased the bending in the arch near the springing. After 25 months the concrete showed a compressive strength of 5400 p.s.i. and a secant modulus of elasticity of 4,200,000 p.s.i.

The concrete of the Rogue River Bridge arches (18) had a strength of about 5000 p.s.i. at 76 to 98 days and an average modulus of elasticity of 4,630,000 p.s.i. The coefficient of thermal expansion was 0.0000053 per degree F. It was made of one part of cement and 2.75 parts of aggregate consisting of high grade local sand and gravel derived from basaltic rock. The water-cement ratio varied from 0.593 to 0.653 and the cement content was 8.48 sacks per cu. yd. Temperatures at the axis of the rib increased to a maximum of about 120° to 140° F 30 hours after pouring with the air temperature at 60° F and dropped to air temperature in about 100 hours after pouring. The rib averaged a little over three feet thick and was six feet wide.

In the case of the Pittsburgh tests to be described later in this report, the movement of the arch crowns appeared to correspond to a thermal coefficient of 0.000006 for the gravel concrete and to 0.000004 for the limestone concrete of the Westinghouse Bridge when the mean air temperature of the previous 5 days was used except in the case of the very thick arches of the Jacks Run Bridge where the 8 day average seemed to give closer results.

It is recommended that for the purpose of design the range of temperature of the arch rib be assumed as 60 per cent of the range between extreme air temperatures at the locality. It may be considered that keying and plastic flow will eliminate the effect of heat of hardening on the rib and that plastic flow will permit the arch to adjust itself to mean temperature so that it can be calculated for equal rise and fall.

The coefficient of thermal expansion may be assumed as 0.0000055 per degree F. unless information can be obtained as to the actual materials and coefficient. It may be 0.0000065 for gravel concrete or as low as 0.000004 per degree F. for limestone concrete. A special investigation of the coefficient should be made for cases where the temperature effects are very important.

A value of 4,000,000 p.s.i. is recommended for the modulus of elasticity of the concrete and the calculation of temperature thrusts is to be made as though the rib were elastic and uncracked. The stresses due to difference in thermal coefficients of the steel and concrete may be neglected because they do not effect the useful strength of the arch. A single value of modulus of elasticity is recommended for this purpose because it seems best after consideration of many factors involved. Because of the effect of age, speed of loading and other factors it is difficult to determine the actual modulus of elasticity. Too low a value should not be used because of the importance of temperature thrusts on abutment and piers. It does not appear that a higher value need be used because the temperature effects must be relieved somewhat either by plastic flow ([12]) or a reduction in modulus of elasticity which has been observed under repeated loading.[13] This maximum value also seems justified by the nature of the temperature thrust which is immediately relieved by any cracking or yielding of the rib under high stresses so that it cannot cause failure but is eliminated by the strains occuring long before ultimate load is reached. The purpose of provision for temperature effects is principally to prevent undesirable cracking of the rib in service and this is quite important in the case of arches with a low rise-span ratio.

3. Shrinkage and Plastic Flow

Since 1932, tests on arches and large specimens have indicated that the actual shrinkage and plastic flow movements are smaller than the figures used in the calculations in the progress report. The principles expressed regarding their effects on stresses appear to be correct but a simplified treatment in design of ribs is recommended. The effect of plastic flow on rib-shortening due to load has already been discussed and it is suggested that the effect of shrinkage be reduced to an equivalent elastic shortening and be included with the temperature effect calculation, neglecting the additional effect which it has on the steel stresses.

It is recognized that the drying of large members of concrete is very slow and that the occasional wetting or immersion in fog of arches exposed to the weather will permanently prevent the loss of much moisture from a greater part of the section.

An elaborate investigation of shrinkage was made in connection with the Rogue River Bridge.[18] An 8- by 16-in. cylinder was cast with a telemeter at the center and protected from direct wetting by rain but otherwise exposed to the effects of atmospheric humidity. The total shrinkage over a period of 2425 hours was 0.0007 in. per in. A full size section of the reinforced rib six feet long cast beside the bridge and exposed to the same weather conditions showed a shrinkage of only 0.00015 in the same period. During this period there was no rain but occasionally heavy fogs enveloped the bridge and produced a measurable swelling of the concrete.

The average shrinkage measured at a large number of points on the spans during 100 days after pouring was 0.00011 and the individual measurements varied from 0.00002 to 0.00027 because of varying conditions. Some were wet longer than others and some were exposed to direct sunlight. There was practically no rain during this period and most of the time a stiff breeze was blowing.

During the construction of the Arlington Memorial Bridge in Washington and for a period of about two years after the pouring of the arches in 1929, accurate measurements were made to determine the effects of temperature, shrinkage and plastic flow.(2) The concrete was made of one part portland cement, 1¾ parts sand, and 3½ parts of gravel. It showed a 28 day strength of 3900 p.s.i. The dimensions of Arch 7 on which the observations were made are given in Table 1.

Fig. 1 shows the unit strains in the arch due to shrinkage and plastic flow as nearly as they can be deduced from the movement of the crown of the arch. Corrections have been made for temperature movements, distortion due to deck construction, and deck restraint. These influences introduce some uncertainty and unfortunately no attempt was made to isolate the shrinkage effect by direct shrinkage observations but the results do give a good indication of the magnitude of the combined effect of shrinkage and flow. The upward trend during the winter months is probably due to increased humidity of the air.

The total unit strain at the end of the first two years was about 0.00036. If the shrinkage strain during this period was about 0.00015 as would appear reasonable from other tests, the balance due to plastic flow would be 0.00021 or about 0.00000064 per lb. per sq. in. This unit strain is calculated from the full dead load stress which was not on the arch for the full period so the flow factor must have been somewhat higher.

In the calculation in the progress report of the effect of flow in relieving the bending moments due to shrinkage the unit shrinkage was taken as 0.0003 and the flow factor as 0.000001. This showed a

TABLE 1—DIMENSIONS OF BRIDGES INVESTIGATED FOR LONG TIME DEFLECTION

Description of Bridge	Arch No.	Span (Ft.)	Rise (Ft.)	Width of Rib (Ft.)	Depth of Section		Area of Long. Steel (Sq. in.)
					Crown (Ft.)	Spring. (Ft.)	
George Westinghouse	2	255	98	14	3	6	43.7
	3	411.4	158	14	5	10	53.0
	4	255	98	14	3	6	43.7
Ohio River Blvd. over Spruce Run	1	150	41.75	10	3	5	37.4
	2	150	41.75	10	3	5	37.4
	3	150	41.75	10	3	5	37.4
Ohio River Blvd. at S. Fremont St.	1	118	41	10	3	5	37.4
Ohio River Blvd. over Jack's Run	1	400	90	10	8	14.25	62.5
Arlington Memorial over Potomac River	7	177.33	24.33	87.33	2.25	5.9	3.00 per ft.

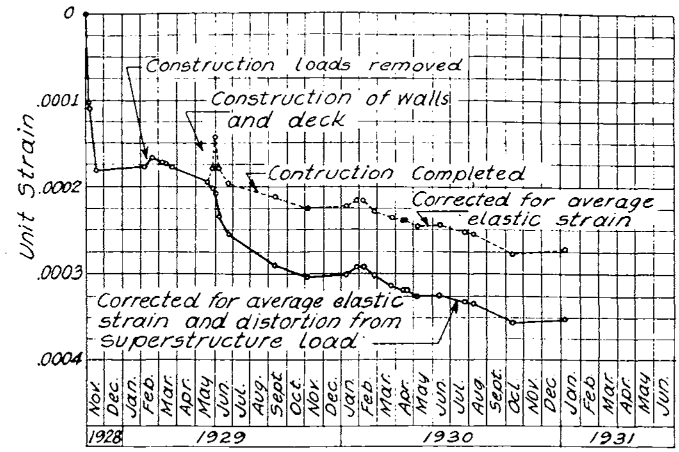

FIG. 1—UNIT STRAIN CAUSED BY SHRINKAGE AND PLASTIC FLOW ARCH NO. 7, ARLINGTON MEMORIAL BRIDGE, WASHINGTON, D. C.

release of about 79 per cent of the shrinkage stress, making the shrinkage effect equal to a drop of about 12° F. in temperature. It appears reasonable to assume now that the actual shrinkage may be about 0.00015 and that the flow factor will be sufficient to release at least 60 per cent of the shrinkage stress. This would reduce the shrinkage effect to that of a unit shortening of 0.00006 which is equivalent to 10° F with a thermal coefficient of 0.000006 or 15° F. if the thermal coefficient is only 0.000004.

Fig. 2 and 3 give the crown deflection measured at the request of Committee 312 on four bridges constructed by Allegheny County* under the supervision of Vernon R. Covell, Chief Engineer of Bridges, and George Richardson in charge of design.(19) Unfortunately it was not possible to make arrangements to start the observations as soon as the arches were constructed. Dimensions are given in Table 1.

The concrete was 1:2:4 mix with an average strength of 2600 p.s.i. Crushed limestone was used for the George Westinghouse Bridge and gravel for the others.

The dotted lines indicate the actual observed elevations (average of opposite curbs) at the center of spans and over the piers. The full lines connect the points which have been corrected for temperature variations as noted in the previous section.

*The measurements were made by Department of Highways, Bridges and Tunnels, Allegheny County, through the courtesy of Charles C. McGovern, Chairman of the Board of County Commissioners and the cooperation of Thomas O. Hasley, and J. F. Laboon, Directors of the Department, Park H. Martin, Chief Maintenance Engineer, and C. K. Harvey and Wm. McClurg Donley, Maintenance Engineers.

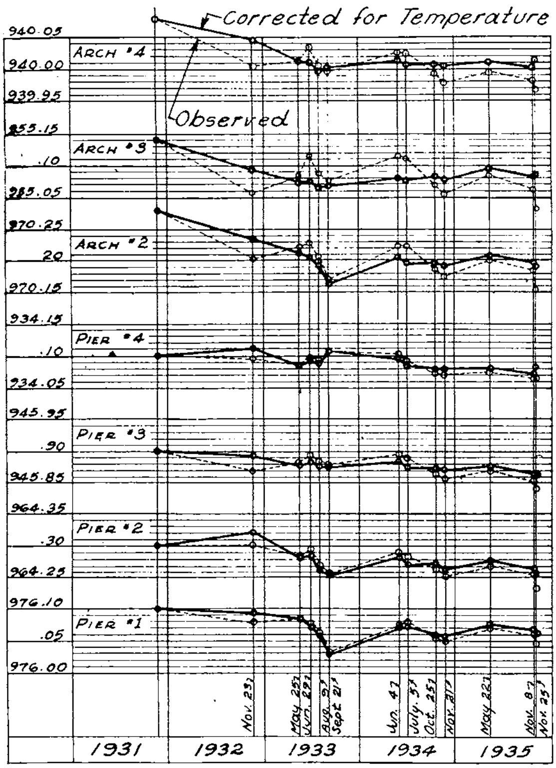

FIG. 2—DEFLECTION OF GEORGE WESTINGHOUSE BRIDGE, PITTSBURGH, PA.

The first elevations shown for the George Westinghouse Bridge are the plan elevations at which the curbs were built.

According to the best information available, the bridge was actually constructed to these levels but this is not certain. A further uncertainty regarding total crown movement of the arches is due to the fact that the movement caused by distortion of the ribs, when the columns and deck were constructed, was not definitely determined. Because these observations were started so late, it is possible to draw only general conclusions but these are important. There was no indication of appreciable shrinkage or flow after the second year of life of the arches although the crowns apparently respond to temperature and probably moisture changes. In general, the levels at the tops of the piers (which in some cases are very high and would not be much effected by plastic flow) indicated a total shrinkage of about 0.00015.

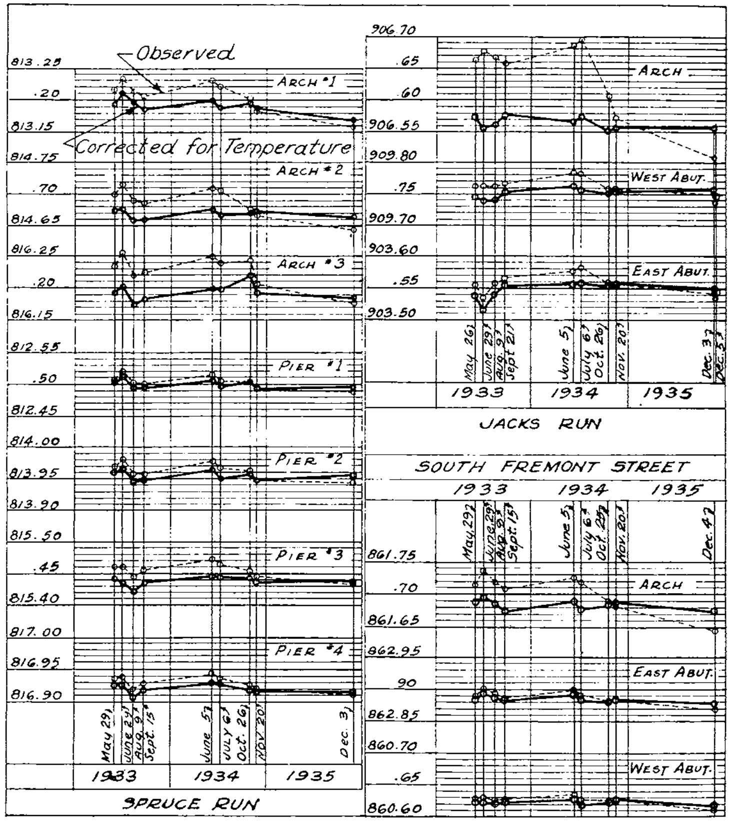

FIG. 3—DEFLECTION OF OHIO RIVER BOULEVARD BRIDGES, PITTSBURGH, PA.

One other interesting fact was noted in the case of these arches as well as the Arlington Memorial Bridge. When the superstructure was built on the completed arches, the crowns rose as the deck was placed over the haunches and came back practically to the original position when the center portion was completed. This must mean that the distortion of the ring due to placing the columns or cross walls and the deck forced the crown up in these cases enough to compensate for the effect of rib shortening. Of course, this action is effected by many factors and might not occur in other cases.

In view of these data it is recommended that the effect of shrinkage be considered in design as equivalent to the effect of a drop in temperature of 15° F. For calculating the shrinkage deflection of arches