According to his proposal the size effect can be described in following exponential form:

$$F_u = k\, d^{\alpha} \tag{4}$$

In this equation the size effect is expresses by the exponent α. In case of no size effect the exponent $\alpha = 1.0$ and the failure load is linearly proportional to the size. Such a relation is assumed in the plasticity theory. Note, that in the plane stress state the size is proportional to the linear dimension d, while in the axisymmetrical stress state the size is proportional to the area d^2.

Using the least square method, the exponents α were derived from the sets of three sizes for each case of shape, constraint and crack model. The exponents are plotted in Fig.14 for the variable span of support. The effect of lateral constraint diminishes with the growing span of support ratio a/d.

The above equation can be refined to include the effect of concrete quality according to the ref.[22] and the effect of the span of support is expressed in the exponential form:

$$F_u = c\sqrt{f_c}\, d^{\alpha}\left(\frac{a}{d}\right)^{\beta} \tag{5}$$

where c, α, β are parameters which should be determined. The formula is composed of three multipliers. The term $\sqrt{f_c}$ coveres the concrete quality (f_c is cube strength), the term d^{α} includes the size effect and the term $(a/d)^{\beta}$ describes the effect of the span of support. The parameter c covers other effects.

Small span of supports has a meaning only in physical laboratory tests. In practical situations the load is typically suspended, and supports are not present at all. Therefore, for design purposes, only large spans should be considered. From these reasons the optimization of the parameters in Eq.(5) is done in two steps. First, the exponents of the size effect are found by least square fit for all spans of one type of constraint as $\alpha = 0.68$ and 0.8 (without and with lateral constraints). Then, the remaining two constants are found as $c = 1.61$ and 1.96, $\beta = -0.28$ and -0.40 (for cases without and with lateral constraint, respectively). The effect of the span of supports vanishes at approximately $a/d = 5.0$, where the force F_u is the same for constrained and unconstrained specimens. In such a case the expression $c\,(a/d)^{\beta}$ is equal to 0.16 for both types of constraint. This corresponds to the practical situation with the large span of support. The exponent for the size effect can be taken as an average from unconstrained and constrained cases as $\alpha = 0.74$. Equation (5) above can be then simplified for large spans as

$$F_u = 0.16\,\sqrt{f_c}\;d^{\,0.74} \tag{6}$$

Failure load F_u is in kN. The embedment length d is in mm. The specimen thickness is considered as $b = 100$ mm. The formula is compared with the simulated tests in Fig.15. An interesting observation is that the exponent

$\alpha = 0.74$ in the formula above is halfway between the values obtained with linear fracture mechanics ($\alpha = 0.5$) and plasticity theory ($\alpha = 1.0$).

The formula designed here was derived on the basis of the computer simulation verified by only one experiment. Further experimental verification is needed before the practical implementation.

A final note is made with respect to the role of the tensile strength in the concrete fracture. The above formula assumes a direct relation of the pull-out force to the tensile strength, which is assumed to be proportional to $\sqrt{f_c}$. However, recent advances in fracture mechanics of concrete indicate [16] that the brittle-type of failure is related to the fracture toughness described by the characteristic length λ in Eq.(1). It has been recognized, that λ is a material property with no direct relation to compressive strength of concrete. Thus a more rational formula could be proposed in the form

$$F_u = c\,\lambda^{\omega}\,d^{\alpha} \tag{7}$$

It would be possible to perform a numerical study similar to the one described in this paper with the aim to derive a formula of the type as suggested in Equation (7). Available computer codes, such as the system SBETA, are suitable tools for these studies.

CONCLUSIONS

The response of the specimens subjected to the anchor loading was simulated by the finite element analysis made with the SBETA computer program, based on the smeared crack approach and the crack band theory of non-linear fracture. In some cases a comparison with microplane analysis was made. The simulations provided a detailed insight in the mechanics of the failure process of the anchors. Furthermore, it provided data for the evaluation of the size effect.

The size effect of the embedment depth d on the pull-out force F_u was expressed in the exponential form of Eq.(6). The value of the exponent $\alpha = 0.74$ is a value halfway between the values obtained with linear fracture mechanics ($\alpha = 0.5$) and plasticity ($\alpha = 1.0$). This means, that the non-linear fracture mechanics analysis based on the concept of toughness gives an intermediate solution between the solution based on linear fracture mechanics and the solution based on plasticity theory.

Lateral constraint has a very large effect on the response of the specimens. The increase-factor of the pull-out force due to lateral constraint ranged from 1.5 to 3.6.

The effect of span of supports is stronger for constrained specimens and it grows with size d. The pull-out force decreases with the larger span a. For small specimens the effect of span of supports is insignificant.

All analyses were performed with two SBETA material models, namely, fixed and rotated cracks. The fixed crack model gave the greater pull-out forces in cases of unconstrained specimens. In the case of laterally constrained specimens there was no significant difference in responses for small sizes and for large sizes the rotated crack model gave greater force. From the view of the involved stress states, a conclusion can be suggested, that the rotated crack model gives a softer response in cases with predominantly tensile stress fields. In cases when cracking is taking place under interaction with high compression, the rotated cracks can give stronger response. From a limited comparison with experiments it can be concluded that the rotated and fixed analyses give bounds for a real behavior.

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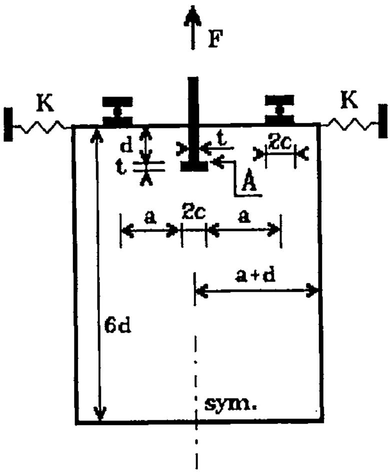
TABLE 1 — SUMMARY OF PEAK LOADS

Peak loads [kN]							
lateral constraint		0			∞		
d [mm] =		50	150	450	50	150	450
a =	material model						
d/2	fixed cracks	31.9	74.3	129.4	49.8	144.6	366.5
	rotated cracks	26.0	54.3	109.0	49.5	142.7	382.7
d	fixed cracks	23.1	47.2	110.2	47.6	130.7	254.9
	rotated cracks	18.8	36.5	81.9	47.1	129.3	276.1
	microplane	17.5	42.7	93.4	-	-	-
2d	fixed cracks	22.2	49.5	94.3	34.6	84.0	145.5
	rotated cracks	16.5	31.1	82.9	34.3	96.8	179.6
	Wittmann's test	-	38.4	-	-	-	-



Geometry:
$d = 50, 150, 450$ mm
$a = d/2,\ d,\ 2d$
$2c = 3d/10,\ t = d/10$
Thickness = 100 mm

Lateral constraint: $K = 0, \infty$

Material properties:
$f_t = 3$ MPa, $f_c = 40$ MPa
$E = 30$ GPa, $\nu = 0.2$
$G_f = 100$ N/m

Fig. 1—Geometry and material properties of the pullout specimens

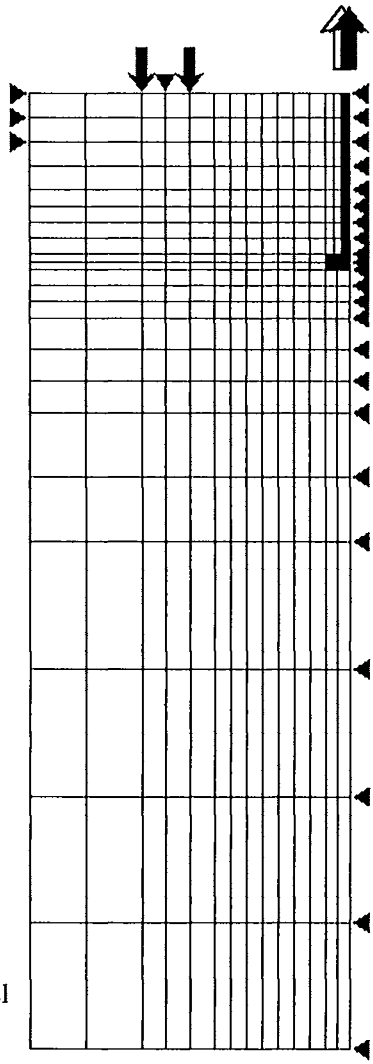


Fig. 2—Finite element model for specimen $a = d$, $K = \infty$

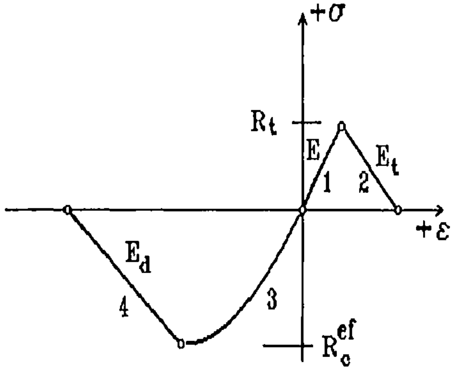


Fig. 3—Effective stress-strain law

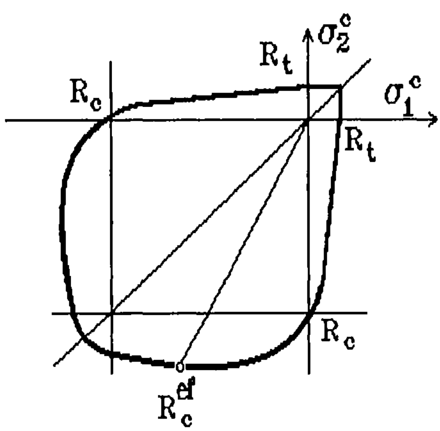


Fig. 4—Biaxial failure function

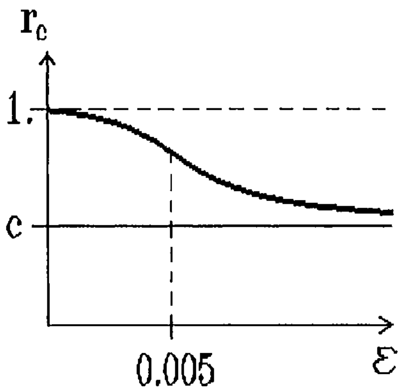


Fig. 5—Compressive strength reduction of cracked concrete

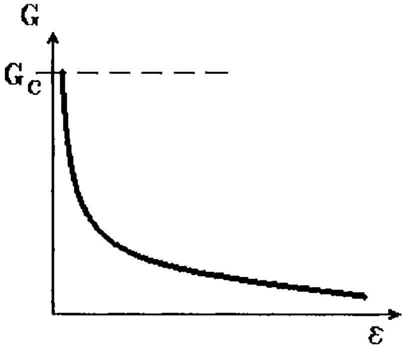


Fig. 6—Shear stiffness of cracked concrete

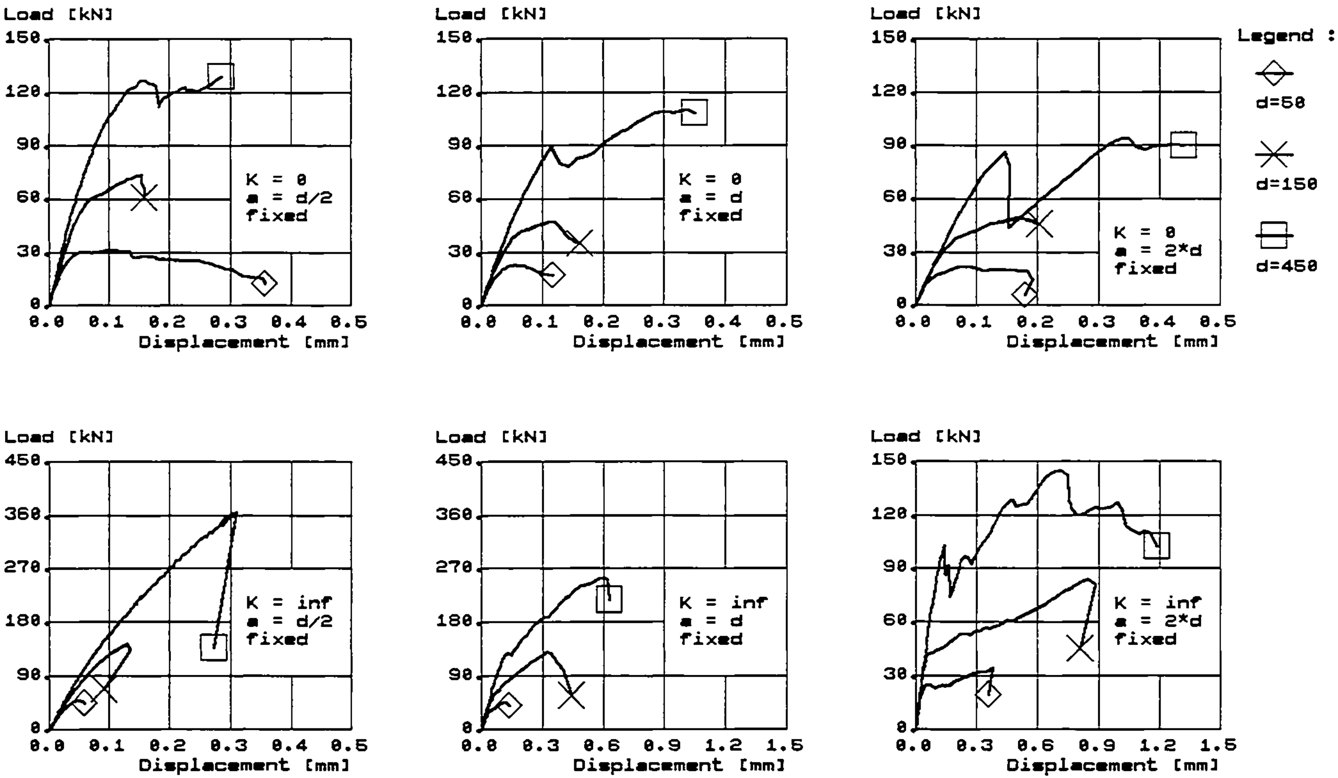


Fig. 7—Review of load-displacement diagrams obtained by SBETA analysis with fixed cracks

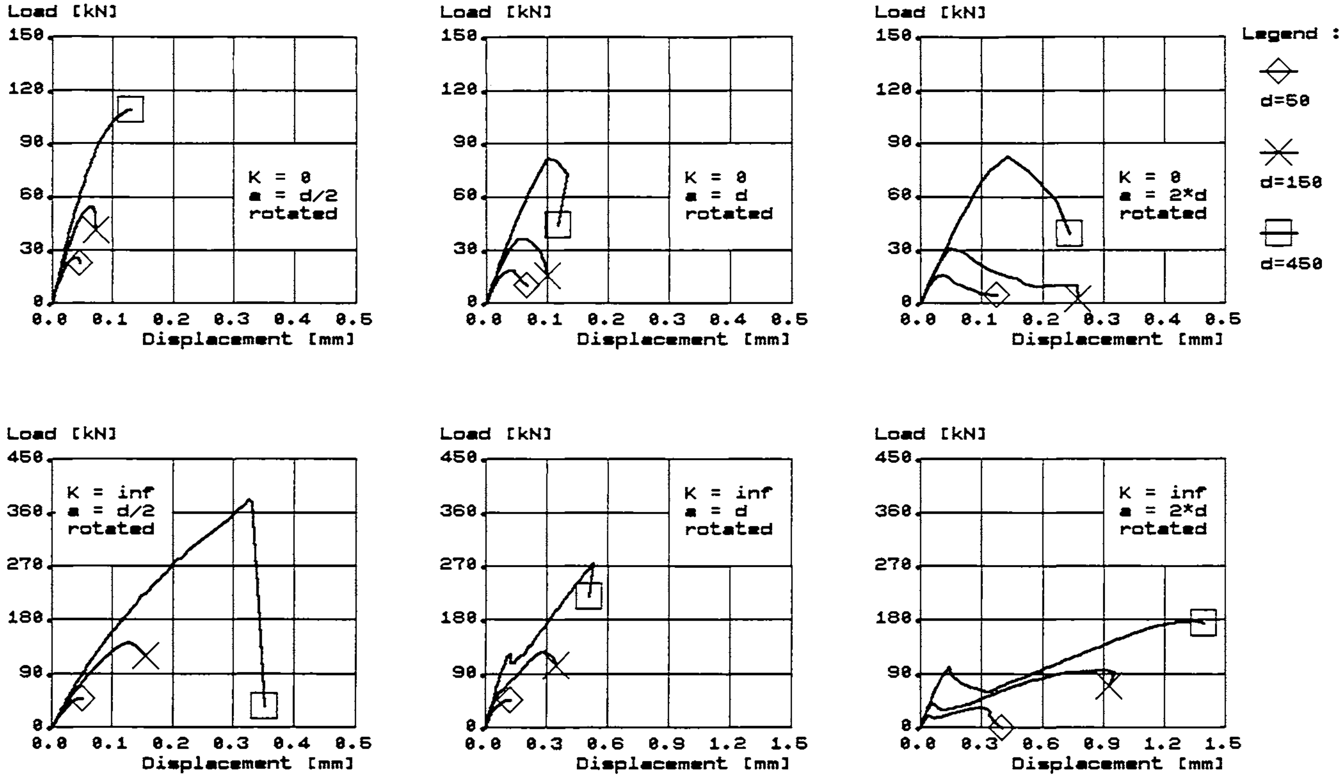


Fig. 8—Review of load-displacement diagrams obtained by SBETA analysis with rotated cracks