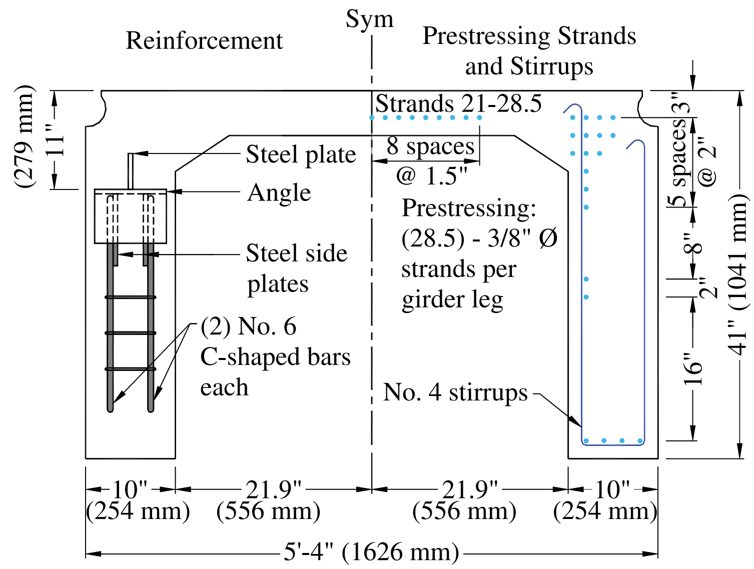

Fig. 8-6: Cross-section at the end of the pier girder shown in Fig. 8-5 *(1" = 25.4 mm)*

1.2.2 Material data

Material data for the drop-in span girder:

Compressive strength of concrete: $f_c' = 6{,}000\ psi$ (41.4 MPa)

Reinforcing steel: $f_y = 40\ ksi$ (276 MPa)

Prestressing steel: 0.5" (12.7 mm) diam. strands, $A_p = 0.153\ in^2$ (98.7 mm^2)

$f_{py} = 270\ ksi$ (1860 MPa), $l_{transfer} = 25.2$" (640 mm)

Structural steel of Cazaly hanger: $f_y = 36\ ksi$ (250 MPa)

Material data for the pier girder:

Compressive strength of concrete: $f_c' = 6{,}000\ psi$ (41.4 MPa)

Reinforcing steel: $f_y = 40\ ksi$ (276 MPa)

Prestressing steel: 0.375" (9.5 mm) diam. strands, $A_p = 0.08\ in^2$ (51.6 mm^2)

$f_{py} = 250\ ksi$ (1724 MPa), $l_{transfer} = 18.9$" (480 mm)

1.2.3 Statement of design problem

This example demonstrates the evaluation of the load carrying capacity of an existing structure using strut-and-tie models. The use of strut-and-tie models for evaluation can be a fast analytical method to determine the load carrying capacity. For an existing structure, the designer cannot arbitrarily choose the strut-and-tie model configuration. The model needs to be adapted to the existing structure, since ties can only be implemented where reinforcement is present in the existing structure. Since the results give lower bound solutions (Schlaich et al. 1987), the carrying capacity of the structure may be underestimated. It is important to choose a strut-and-tie model that is close to the load carrying capacity of the structure in order to avoid unnecessary load restrictions or strengthening of the structure. This paper also demonstrates how a structural steel piece, the Cazaly hanger, can be integrated into the strut-and-tie analysis.

The capacity evaluation of this existing overpass is carried out in accordance with Appendix A of ACI 318-08.

2 Evaluation procedure

ACI 318-08 defines D-regions as the portion of a member within a distance *h* from a force or geometric discontinuity. In this example, the D-regions include the ends of the girders and the support location at the pier. At the pier location, the D-region extends approximately *h* from the edge of the pier in both directions. Since the girders are prestressed, the anchorage zone is also considered a D-region. Clause 2.2 of ACI 318-08 defines the anchorage zone as having an extent equal to the largest dimension of the cross-section; in this case, *h*.

Given that the ends of the girders consist of geometric and load discontinuities and anchorage zones, strut-and-tie modeling is used to evaluate the connection region of the span and pier girders. The remainder of the girder (especially the span girder) could be modeled using beam theory as covered by clauses elsewhere in the code. This example will only focus on the evaluation of the girder end region containing the hanger connection.

The strut-and-tie model of this Cazaly hanger connection has two special features when compared to traditional reinforced concrete strut-and-tie models:

1. The girders are prestressed. The prestressing forces are treated as external forces acting on the member.
2. The Cazaly hanger is incorporated in the span side model. The equilibrium of the Cazaly hanger is determined, and the corresponding forces from the Cazaly hanger are introduced as part of the overall strut-and-tie model. The load carrying capacity of the Cazaly hanger can be determined according to steel design/evaluation standards and will not be presented in this paper.

The evaluation/design sequence for the Cazaly hanger connection contains the following steps:

Step 1: Establish a strut-and-tie model based on the location of the existing reinforcement and determine the magnitude of the prestressing forces and the application location.

Step 2: Confirm the adequacy of the existing reinforcement based on the calculated tie forces.

Step 3: Check minimum distributed reinforcement requirements.

Step 4: Verify the nodal stresses and strut capacities.

3 Evaluation of the load carrying capacity

3.1 Special features of the model

The load carrying capacity is only determined for a half-width of the girder (i.e. one web and half the flange) since the girders are symmetrical in cross-section.

In applying the prestressing forces in the models, it is assumed that the prestressing forces are completely transferred at the end of the transfer length and that the force is transferred in a linear manner. Thus, the prestressing forces are introduced into the model at ¼ and ¾ of the transfer length (½ of the prestressing force at each location) as shown in Fig. 8-7. Several prestressing strands are grouped together in order to simplify the model. The transfer length is 18.9" (480 *mm*) for the ⅜" (9.5 *mm*) diameter strands and 25.2" (640 *mm*) for the ½" (12.7 *mm*) diameter strands. The transfer length is taken as 50 times the strand diameter as per ACI 318-08, clause 11.3.5.

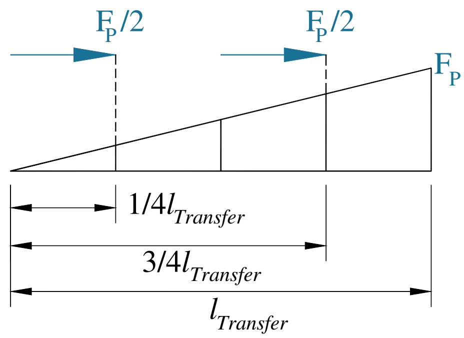

Fig. 8-7: Force transfer from prestressing tendon to concrete over the transfer length, $l_{transfer}$

3.2 Step 1: Establish the strut-and-tie models

3.2.1 Span side model (Cazaly hanger)

The reinforcing scheme and cross-section of the span side model are shown in Fig. 8-3 and Fig. 8-4. The span side has prestressing strands in the longitudinal direction and vertical No.4 stirrups ($A_s = 0.2\ in^2$ ($129\ mm^2$)). The stirrups have two legs in the end zone. Once the web becomes narrower (see extent in Fig. 8-3), the stirrups only have one leg as shown in Fig. 8-4b.

By considering the loads on the Cazaly hanger, equilibrium of the hanger acting as a rigid body can be determined as shown in Fig. 8-8. The horizontal force is assumed to be transmitted through the horizontal plates and into the concrete in bearing or into the reinforcing bars welded to the cantilevered plates when the load is reversed. Note that the cantilevered plates are not only subjected to an axial force, but also to shear and bending. The vertical shear force is transmitted through the vertical steel plates into the concrete through bearing at the base plate. An opposing reaction occurs at the end of the cantilevered plates where a flat plate is welded to the bottom of the two cantilevered plates. At this location, the concrete is also in bearing. The forces from the Cazaly hanger are introduced into the strut-and-tie model as forces acting on the concrete. The two vertical forces are calculated as follows (refer to Fig. 8-8):

$$F_1 = \frac{53.7\ kip \times 9.8''}{18''} = 29.2\ kip\ (130\ kN)$$

$$F_2 = \frac{53.7\ kip \times 27.8''}{18''} = 82.8\ kip\ (368\ kN)$$

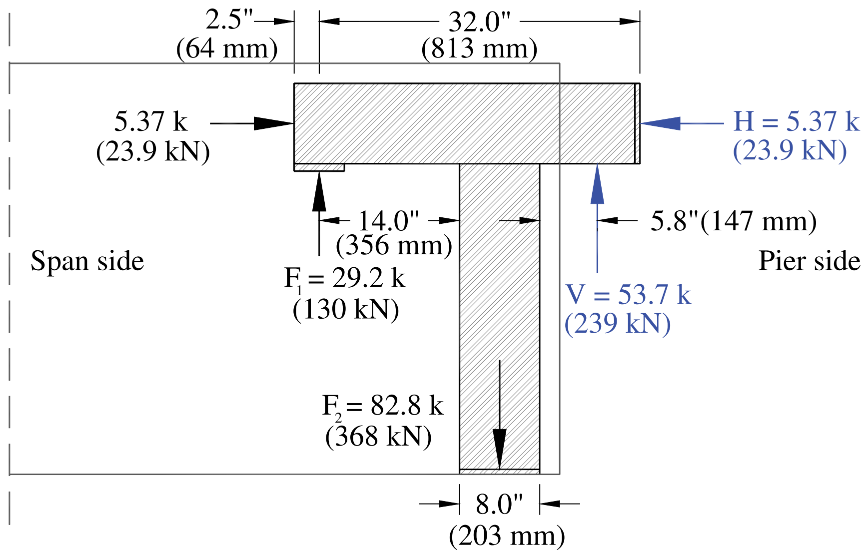

Fig. 8-8: Force equilibrium of the Cazaly hanger

Based on the location of the reactions on the Cazaly hanger, the first nodes in the strut-and-tie model can be placed (nodes D and I). The forces from the prestressing strands are introduced by grouping several strands together and by applying the forces on the concrete at the quarter and the three-quarter points of the transfer length as discussed previously. Ties are situated by considering the location of the stirrups. The bottom chord of the truss is defined by the centerline of the horizontal bottom strands. Since the strut-and-tie model has to follow existing reinforcement, the model in Fig. 8-9 is used to determine the load carrying capacity. The reinforcing steel is given in the background to demonstrate how the ties follow the existing reinforcing steel. Note that there is no vertical tie between nodes I and H or G and H. The vertical force at node I from the Cazaly hanger significantly reduces any required vertical tensile force at this location. Thus, by carefully choosing the location of nodes I and G, the vertical ties become zero force members and all the force is carried by arch action.

The factored prestressing force in each ½" (12.7 *mm*) diameter strand after accounting for all the losses is 20.3 *kip* (90.3 *kN*). Since the force transfer from the strand to the concrete occurs over a length of 25.2" (640 *mm*), half the prestress force was applied at 6" (152 *mm*) and at 18" (457 *mm*). The latter is not quite ¾ of the transfer length but it is within 5%. The reason for the discrepancy is that tie E-F-G must be centered on the five stirrups that are spaced at 3" (76 *mm*). The bottom eight prestressing strands (No. 1-8) are grouped and the prestressing force is applied at nodes D and H. Strands No. 9-12 are grouped and the prestressing force applied at nodes C and G. The prestressing force from strands No.13-16 is applied at nodes B and F while the force from strands No. 17-18 is applied at nodes A and E. Fig. 8-10 shows the location of the prestressing force application and Table 8-2 the magnitude and direction of the forces.

The girder is only modeled to a distance of 74" (1877 *mm*) (i.e. until transition to a B-region). No dead load was applied at nodes A, E, I, or J. The dead load is included in the total load effects (i.e. *H* and *V*) and is conservatively assumed to act to the left of the model. Fig. 8-10 shows the factored truss forces.

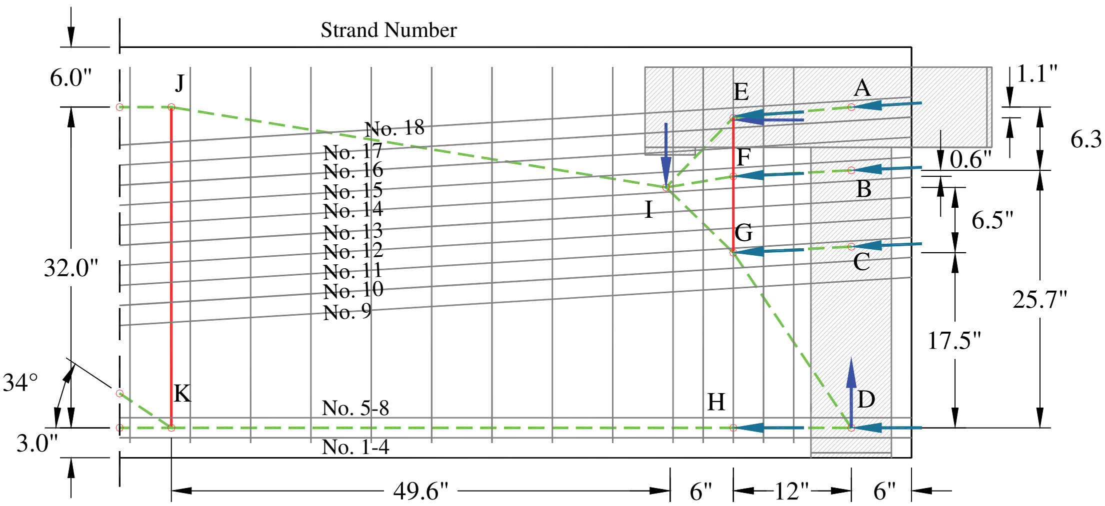

Fig. 8-9: Strut-and-tie model superimposed on the reinforcement layout
(struts are shown as dashed green lines and ties as solid red lines; *1" = 25.4 mm)*

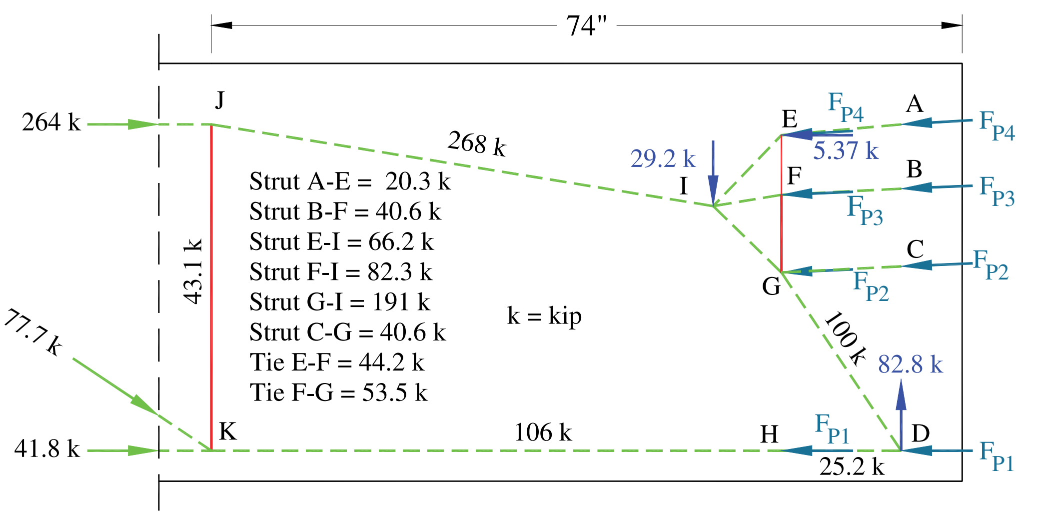

Fig. 8-10: Strut-and-tie model showing the location of the force application and the corresponding factored truss forces
(struts as green dashed lines, ties as solid red lines; *1"=25.4 mm, 1 kip =4.448 kN)*

Table 8-2: Force applied to the concrete from the strands (see Fig. 8-10 for locations)

Force	Node	Strand No.	Angle* (degrees)	Force [kip (kN)]
$\mathbf{F_{P1}}$	D, H	1-8	0.0	81.2 (361.2)
$\mathbf{F_{P2}}$	C, G	9-12	2.4	40.6 (180.6)
$\mathbf{F_{P3}}$	B, F	13-16	2.9	40.6 (180.6)
$\mathbf{F_{P4}}$	A, E	17-18	3.4	20.3 (90.3)

*Angle is measured down from horizontal

3.2.2 Pier side model

The reinforcing scheme and cross-section of the pier girder model are shown in Fig. 8-5 and Fig. 8-6. The span side has 20 prestressing strands in the web and 8.5 strands in half the flange (17 strands total in the flange and only half the flange is modeled, hence 8.5 strands). Shear reinforcement consists of No. 4 stirrups (A_s = 0.2 in^2 (129 mm^2)) having two legs in the half girder. The strut-and-tie model is created based on the location of the existing reinforcement as shown in Fig. 8-11. The top chord is positioned such that it aligns with the centroid of strands No. 13-28.5. The bottom chord is positioned at the location of the centroid of strands No. 1-4, however, between nodes V and X the chord centerline was raised to 3.1" (79 *mm*) from the soffit to ensure strut X-Z will fit within the web. Tie L-T is positioned along the centroid of the corresponding "C"-shaped bars while ties Q-R-S, U-V and W-X are placed at the centroid of the existing stirrup groups.

The pier side girder is modeled using the strut-and-tie method up to the pier support. The region from the end of the girder to the pier consists of two D-regions. The end of the beam is a D-region and another D-region occurs at the support location (pier). In Fig. 8-11, node Z is located on the centerline axis of the inclined pier. The segment between the two piers has not been modeled and would be analyzed by either continuing the strut-and-tie model or using sectional design methods.

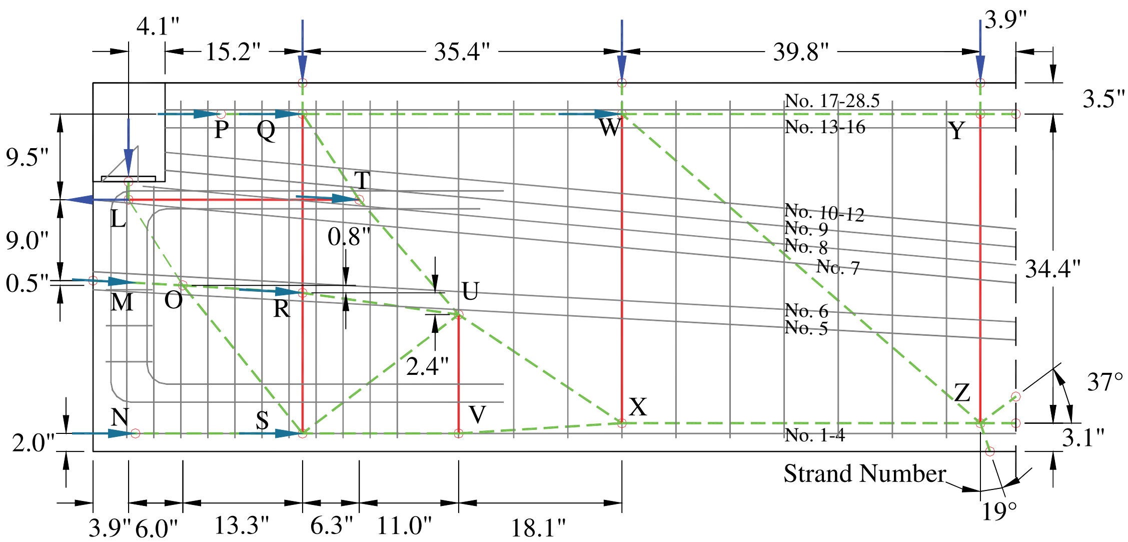

Fig. 8-11: Pier side strut-and-tie model superimposed on the reinforcement layout (struts are shown as green dashed lines and ties as solid red lines; *1" = 25.4 mm*)

The prestressing force from the ⅜" (9.5 *mm*) diameter strands is also applied at ¼ and ¾ of the transfer length. The force in each strand after accounting for all losses is 9.8 *kip* (43.5 *kN*) and half the force is applied to the concrete at 4.7" (120 *mm*) and at 14.2" (360 *mm*). Force from strands No. 21-28.5, which are located in the flange, is assumed to spread towards the web at a 45 degree angle as shown in Fig. 8-12 and is applied in the strut-and-tie model in Fig. 8-11 at 25.6" (650 *mm*) and 35" (889 *mm*) from the girder end. For simplicity, the force is projected to node W (refer to Fig. 8-13) to prevent the addition of two nodes between nodes Q and W. Transverse reinforcing steel is provided in the flange to form the tie shown in Fig. 8-12, however, the design check for the flange will not be shown in this example but a similar procedure would apply as outlined for the web.

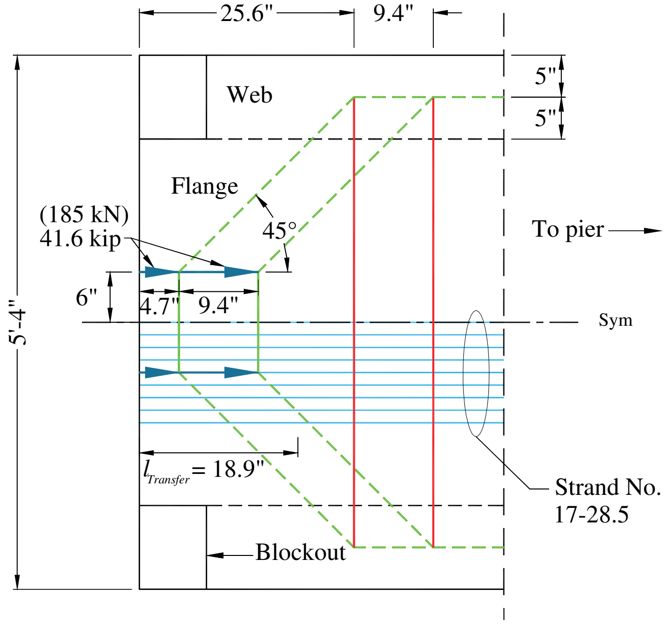

Fig. 8-12: Plan view of the pier side girder showing the spreading of the prestressing force from the top flange into the webs
(struts are shown as green dashed lines and ties as solid red lines; *1" = 25.4 mm*)

Fig. 8-13 shows the location of the load application and Table 8-3 presents the corresponding prestressing forces. The prestressing force is not necessarily applied at ¾ of the transfer length. In this model, it is conservatively applied at the nearest node which is further away than the ¾ point for simplicity. The self-weight of the girder is applied at the nodes using a tributary area approach. The reactions from the span girder are applied at the girder end as shown in Fig. 8-13. A horizontal force of $0.1V$ is again applied to account for friction in the joint. The force acts at the top of the plate, but is applied in this example at node L. The factored truss forces are given in Fig. 8-13.

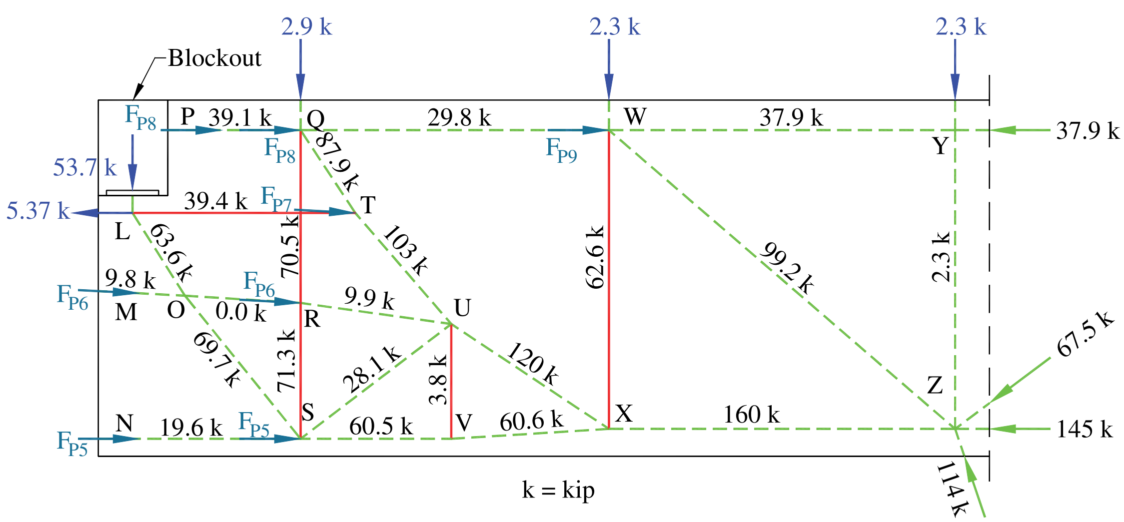

Fig. 8-13: Pier side girder strut-and-tie model showing the location of the force application and the forces in the truss members
(struts as green dashed lines, ties as solid red lines; *1" =25.4 mm, 1 kip = 4.448 kN*)

Table 8-3: Prestressing force applied to the concrete in the pier side girder model (see Fig. 8-13 for locations)

Force	Node	Strand No.	Angle* (degrees)	Force [kip (kN)]
F_{P5}	**N, S**	1-4	0.0	19.6 (87.0)
F_{P6}	**M, R**	5-6	3.0	9.8 (43.5)
F_{P7}	**T**	7-12	4.8	58.7 (261.0)
F_{P8}	**P, Q**	13-20	0.0	39.1 (174.0)
F_{P9}	**W**	21-28.5	0.0	83.3 (371)

*Angle is measured down from horizontal

3.3 Step 2: Confirm that the reinforcement arrangement is adequate

3.3.1 Span side model

The nominal strength of a tie is given by ACI 318-08 clause A.4.1 as:

$$F_{nt} = A_{ts} f_y$$

Therefore, the required steel area is given by:

$$A_{ts} = \frac{F_{xx}}{\phi \cdot f_y}$$

where F_{xx} is the factored tie force from Fig. 8-10. For example, tie F-G must resist a factored force of 53.5 *kip* (238 *kN*) and requires a steel area of:

$$A_{ts} = \frac{53.5\ kip}{0.75 \cdot 40\ ksi} = 1.78\ in^2\ (1150\ mm^2)$$

Five two legged stirrups spaced at 3" (76 *mm*) are centered on tie F-G (Fig. 8-9) which corresponds to $A_{s,provided} = 2 \cdot 0.20 in^2 \times 5 = 2.0 in^2\ (1290 mm^2)$. Therefore, tie F-G has adequate reinforcement. Similarly, tie J-K requires $1.44\ in^2\ (927\ mm^2)$ of reinforcing steel. However, at this location, the No. 4 stirrups only have one leg ($A_s = 0.20\ in^2$ *or* $129\ mm^2$). Tie J-K has a tributary width of 48" (1219 *mm*) and stirrups are spaced at 6" (152 *mm*) (Fig. 8-3). Eight stirrups are within the tributary width providing a steel area of $A_{s,provided} = 0.20\ in^2 \times 8 = 1.6\ in^2\ (1032\ mm^2)$ which is greater than the amount of reinforcement required for strength. Therefore, the existing stirrup configuration provides adequate strength capacity.

3.3.2 Pier side model

The pier girder contains No. 4 stirrups with two legs throughout the modeled area. Tie R-S requires 2.4 in^2 (1533 mm^2) to carry 71.3 *kip* (317 *kN*). From Fig. 8-11, eight stirrups giving $A_{s,provided} = 2 \cdot 0.20\ in^2 \times 8 = 3.2\ in^2\ (2065\ mm^2)$ are centered about tie R-S which is greater than the required amount. Tie W-X requires 2.09 in^2 (1346 mm^2) and stirrups spaced at 6" (152 *mm*) are provided. Tie W-X has a tributary width of 36" (914 *mm*) which results in six stirrups being contained within this width corresponding to $A_{s,provided} = 2 \cdot 0.20\ in^2 \times 6 = 2.4\ in^2 (1548\ mm^2)$.

In addition, the horizontal tie L-T must also be checked. The force in this tie is 39.4 *kip* (175 *kN*) and the required fully developed steel area is 1.31 in^2 (847 mm^2). Four No. 6 "C"-shaped reinforcing bars are welded to the side plates that are connected to the bearing seat providing full development at node L. The provided steel area is $4 \cdot 0.44\ in^2 = 1.76\ in^2$ ($1135\ mm^2$) which is greater than the required amount. However, the development length of the "C"-shaped reinforcing bars at node T must be checked to ensure the bars are sufficiently developed to the left of node T. The development length of the No. 6 bars is calculated according to ACI 318-08 clause 12.2.2:

$$l_d = \left(\frac{f_y \psi_t \psi_e \lambda}{25\sqrt{f_c'}} \right) d_b$$

where ψ_t is taken as 1.3 since more than 12" (305 *mm*) of concrete is cast below the bar, ψ_e is 1.0 and λ is 1.0. The development length of a No. 6 bar is:

$$l_d = \left(\frac{40000\,psi \times 1.3 \times 1.0 \times 1.0}{25\sqrt{6000\,psi}} \right) \times 0.75" = 20" \ \ (508\ mm)$$

From Fig. 8-14, only 16.3" (414 mm) of extension is provided beyond the node. ACI 318-08 clause 12.2.5 also allows the required development length to be adjusted by the ratio $A_{s,required}/A_{s,provided}$. From this ratio the fully developed reinforcement area at node T can be calculated as:

$$A_{s,developed} = \frac{16.3"}{l_d} \times A_{s,provided} = \frac{16.3"}{20"} \times 1.76\ in^2 = 1.43\ in^2 \ \ (925\ mm^2)$$

The developed steel area is greater than the required steel area of 1.31 in^2 (847 mm^2). Therefore, the pier girder model has adequate reinforcement.

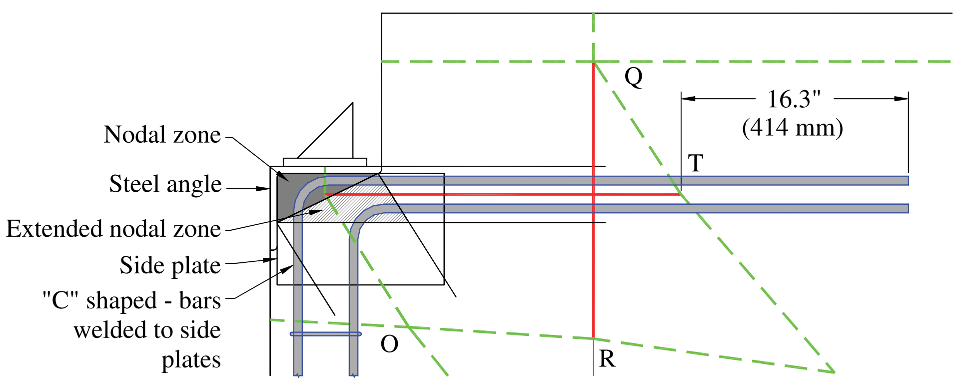

Fig. 8-14: Node L showing the "C" shaped bars and the bar extension past node T (struts are shown as green dashed lines and ties as solid red lines)

3.4 Step 3: Minimum distributed reinforcement

The minimum reinforcement requirement is given by ACI 318-08 clause A3.3 as:

$$\sum \frac{A_{si}}{b_s s_i} \sin \alpha_i \geq 0.003$$

In the span girder, the least amount of reinforcing steel occurs across strut I-J. The No. 4 stirrups consisting of a single leg are spaced at 6" (152 *mm*) and the angle between the stirrups and the strut axis is 81 degrees. Without including the prestressing steel, the reinforcement ratio is:

$$\sum \frac{A_{si}}{b_s s_i} \sin \alpha_i = \frac{0.20\ in^2}{10''\times 6''} \sin 81° = 0.0033$$

which is greater than the minimum requirement of 0.003.

The least amount of reinforcing steel in the pier side model occurs across strut W-Z. The angle between the strut axis and the stirrups is 49 degrees. Since the two legged No. 4 stirrups are spaced at 6" (152 *mm*), the reinforcement ratio is:

$$\sum \frac{A_{si}}{b_s s_i} \sin \alpha_i = \frac{2\times 0.20\ in^2}{10''\times 6''} \sin 49° = 0.005$$

which is greater than the minimum requirement of 0.003.

Therefore, the minimum steel requirements are met for both models.

3.5 Step 4: Verification of the nodal stresses and strut capacities

3.5.1 Nodal zones

The nominal compressive strength of a nodal zone is given in ACI 318 clause A.5.1 as:

$$F_{nn} = f_{ce} A_{nz}$$

where A_{nz} is the area of the face of the nodal zone perpendicular to F_u and f_{ce} is given by ACI 318 clause A.5.2 as:

$$f_{ce} = 0.85 \beta_n f_c'$$

β_n = 1.0 for a nodal zone bounded by struts or bearing areas (CCC nodes), and β_n = 0.80 for nodal zones anchoring one tie (CCT nodes).

In the span side model, node D must be checked to ensure the bearing area is sufficiently large. From Fig. 8-10 it would appear that node D is a CCC node. It is important to realize that the forces from the hanger were directly applied to the concrete at node D. The hanger can be seen as equivalent to a tie between nodes A and D. Therefore, node D must be modeled as a CCT node with tie A-D having the same width as the hanger (i.e. 8" or 203 *mm*). Since the centerline of strut D-H was chosen to be 3" (76 *mm*) from the soffit of the girder, the width of strut D-H is 6" (152 *mm*). The perpendicular dimensions for strut D-G, l_{DG}, shown in Fig. 8-15a can be calculated based on the hanger plate size and horizontal strut D-H dimension:

$$l_{DG} = 8''\sin(56°) + 6''\cos(56°) = 10.0''\ (254\ mm)$$

Even though the web has a thickness of 10" (254 *mm*), the nodal zone is only 7" (178 *mm*) wide as this is the effective transverse dimension of the bearing plate on the Cazaly hanger (Fig. 8-15b). Therefore, the stress on the face DG is $100\ kip/(10\ in \times 7\ in) = 1429\ psi$ (9.9 *MPa*). The stress on the face DG is well below the maximum stress limit of: $\varphi f_{ce} = 0.75 \times 0.85 \times 0.80 \times 6000 = 3060\ psi\ (21.1\ MPa)$. The rest of the stress calculations are tabulated in Table 8-4.