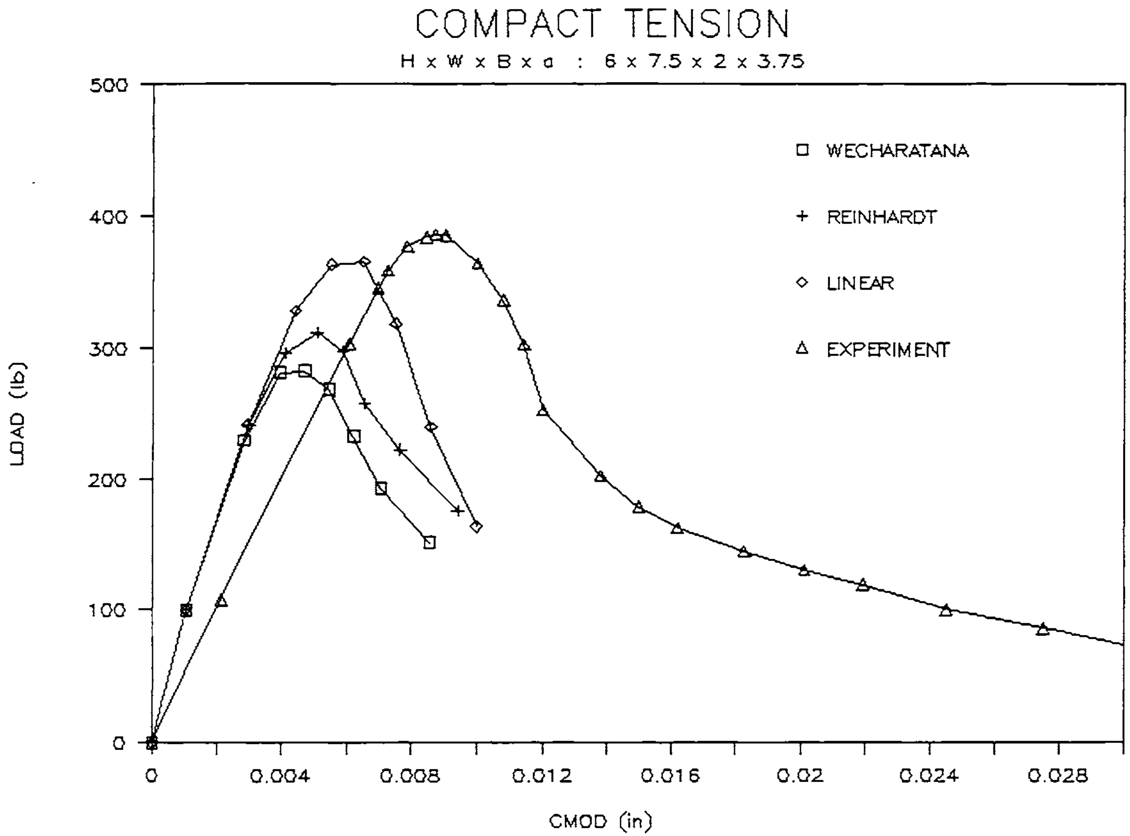

Fig.10b Effect of $\sigma - \omega$ Relations on the Predicted Load-CMOD Curves (Small CT—Present Study)

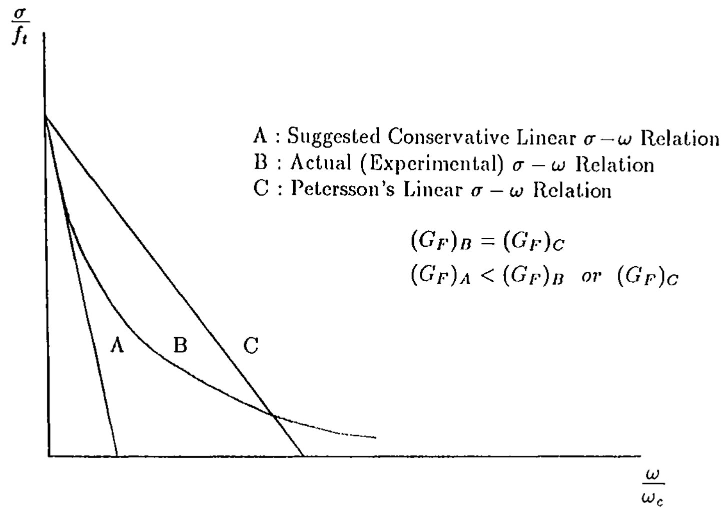

Fig.11 Suggested Normalized Stress-Crack Opening Displacement Relations

Inelastic Constitutive Relations for Concrete

by L. Nobile

Synopsis: The paper focuses on the formulation of a self-consistent model for a compressed concrete containing randomly distributed flat microcracks.
A general formulation of the constitutive law for such material is obtained, finding the overall mechanical response to be strongly nonlinear in the region near the maximum in the stress-strain curve.

Keywords: compression; concretes; damage; energy methods; fracture properties; mechanical properties; microcracking; models; stress-strain relationships

L. Nobile is a research engineering in the Institute of Scienza e Tecnica delle Costruzioni, University of Bari, Italy. The main area of his research is the Mechanics of fracture and the general constitutive behaviour of geomaterials.

INTRODUCTION

The deformation of brittle materials is accommodated primarily by the presence and propagation of microcracks, already present in the virgin material. In the case of concrete such microcracks are localized in aggregate-mortar interfaces and are caused by shrinkage.

The determination of the effect of defects in the structure of a solid on its overall moduli has been the object of numerous studies. In a great majority of these studies (see, for example, (1,2)) the objective was limited to the first stage of inelasticity, when some microcracks slide but don't propagate. However, in many cases of practical interest it is important to formulate a continuum damage model which in addition considers the second stage of inelasticity, when some microcracks propagate (see, for example, (3,4)).

The objective of this paper is to demonstrate the utility of the model in describing deformation and failure of concrete in uniaxial compression and also to check its ability to reproduce the available data. The latter issue is addressed by comparing the test of the model to experimental data. It should be remarked that the significance to be attributed to the available data is mainly one of a qualitative description of the mean features of material behaviour.

STRESS-STRAIN RELATIONS

Consider a body of total volume V and exterior surface S. Let the self-equilibrating overall tractions T_i , prescribed on S, in such a manner that:

$$\bar{\sigma}_{ij} n_j = T_i \tag{2.1}$$

where n_j are the components of the exterior unit normal on S, and:

$$\bar{\sigma}_{ij} = \frac{1}{V} \int_V \sigma_{ij}\, dV \tag{2.2}$$

is the average stress. A fixed rectangular Cartesian coordinate system is used and repeated indices are summed. If the body contains randomly distributed microcracks of the same size, the average strain:

$$\bar{\varepsilon}_{ij} = \frac{1}{V} \int_V \varepsilon_{ij}\, dV \tag{2.3}$$

consists of two parts: the average elastic strain and the average additional strain due to the presence of microcracks.

In the first stage of inelasticity, when some microcracks slide but don't propagate, the average additional strain is (6) :

$$\bar{\varepsilon}^{i}_{ij} = \frac{8}{3\pi}\, \frac{1-\nu^2}{2-\nu}\, \frac{\varrho_o}{E} \int_{\varphi_1}^{\varphi_2} \int_{\alpha_1}^{\alpha_2} (\tau_{ni} n_j + \tau_{nj} n_i)\, d\alpha \cos\varphi\, d\varphi \tag{2.4}$$

where ϱ_o is the initial microcrack concentration parameter:

$$\varrho_o = \frac{N d_o^3}{8V} \tag{2.5}$$

(N is the number of microcracks in V and d_o is the initial microcrack diameter), n_i and n_j are the components of the exterior unit normal to the generic microcrack, τ_n is the shear traction induced on microcrack faces reduced by the amount of Coulomb friction $\mu|\sigma_n|$ (μ is the friction coefficient), E and ν the Young's modulus and the Poisson's ratio for the matrix and the limits of integration $(\alpha_1, \alpha_2), (\varphi_1, \varphi_2)$ determine the set of activated microcracks, for which $\tau_n > 0$.

In the second stage of inelasticity, when some microcracks propagate, the average additional strain can be decomposed into the sum of the average strain caused by frictional slidings, eq.(2.4), and the average strain caused by propagation, calculated by methods of analysis based on energy variations associated with self-similar growth of the microcracks (6):

$$\bar{\varepsilon}^{i}_{ij} = \frac{\delta}{\delta \bar{\sigma}_{ij}} \left[\frac{16(1-\nu^2)}{3E(2-\nu)}\, \varrho(d) \int_{\varphi_1'}^{\varphi_2'} \int_{\alpha_1'}^{\alpha_2'} \tau_n^2\, d\alpha \cos\varphi\, d\varphi \right] \tag{2.6}$$

where $\delta/\delta\bar{\sigma}_{ij}$ denotes partial derivative with respect to $\bar{\sigma}_{ij}$, $\varrho(d)$ is the microcrack concentration parameter related to the current value of the diameter. The limits of integration $(a_1', a_2'), (\varphi_1', \varphi_2')$ determine the set of activated microcracks for which $\tau_n > \tau_{no}$, where τ_{no} is the value of the above mentioned shear traction corresponding to the beginning of microcrack propagation.

The above derived formulae for the additional average strain, taken in a finite form, are valid in the range of path-independence (that is, for example, when τ_n increases monotonically). They should be taken in an incremental form when path-dependent.

Since the limits of integration in eqs. (2.4) and (2.6) change in function of the applied load, a stress induced anisotropy develops.

As indicated experimentally (7), a microcrack will grow along the aggregate-mortar interface in a self similar fashion and the critical stress-intensity factor for the interface increases as the load is slowly incremented, probably due to the presence of asperities. Besides, the critical stress-intensity factor for the aggregate is much larger than the one for the mortar, which is, in turn, two times larger than the critical stress-intensity factor for the interface.

Assuming in a simple manner that the aggregate has a regular shape (that is, for example, a regular polyhedron), stable microcrack growth ends as soon as the edge of the most favorably oriented microcrack is inscribed in the regular face polygonal of the polyhedron.

UNIAXIAL COMPRESSION

For the uniaxial compression test the only non-zero components of the additional average strain during the first stage of inelasticity, can be derived from (2.4):

$$(3.1)\quad \bar{\varepsilon}^{i}_{11} = \bar{\varepsilon}^{i}_{22} = \frac{16}{3}\,\frac{1-\nu^2}{2-\nu}\,\frac{\sigma}{E}\,\varrho_o\left[\frac{1}{3}(\mu\cos^3\varphi + \sin^3\varphi) + - \frac{1}{5}(\mu\cos^5\varphi + \sin^5\varphi)\right]$$

$$(3.2)\quad \bar{\varepsilon}^{i}_{33} = -\,2\,\bar{\varepsilon}^{i}_{11}$$

The range of activated microcracks is defined by the inequality

$$(3.3)\quad \sigma(\sin\varphi\cos\varphi) - \mu\,\sigma\sin^2\varphi > 0$$

whose solution, $0 < \varphi < 59°$, is load independent. Thus the overall mechanical response will be still linear.

The overall moduli are:

$$(3.4)\quad E' = \frac{E}{1 + \frac{32}{3}\frac{1-\nu^2}{2-\nu}\varrho_o A_o}$$

$$(3.5)\quad \nu' = \frac{3(2-\nu)\nu + 16(1-\nu^2)\varrho_o A_o}{3(2-\nu) + 32(1-\nu^2)\varrho_o A_o}$$

with

$$A_o = \Big[A(\mu,\varphi)\Big]_{\varphi_1}^{\varphi_2},$$

where

$$(3.6)\quad A(\mu,\varphi) = \tfrac{1}{3}(\mu\cos^3\varphi + \sin^3\varphi) - \tfrac{1}{5}(\mu\cos^5\varphi + \sin^5\varphi)$$

Frictional sliding starts on the microcrack with the orientation

$$\varphi_o = \tfrac{1}{2}\arctan\mu^{-1}$$

In the second stage of inelasticity the only non zero components can be derived from eqs.(2.4),(2.6):

$$(3.7)\quad \bar\varepsilon^{\,i}_{11} = \bar\varepsilon^{\,i}_{22} = \frac{16}{3}\frac{1-\nu'^2}{2-\nu'}\frac{\sigma}{E'}\Big[\varrho_o(A_1+A_2)+2\pi\,\varrho(d)C_o\Big]$$

$$(3.8)\quad \bar\varepsilon^{\,i}_{33} = \frac{32}{3}\frac{1-\nu'^2}{2-\nu}\frac{\sigma}{E'}\Big[\varrho_o(A_1+A_2)+2\pi\,\varrho(d)B_o\Big]$$

with

$$A_1 = \Big[A(\mu,\varphi)\Big]_{\varphi_1}^{\varphi'_1},\quad A_2 = \Big[A(\mu,\varphi)\Big]_{\varphi'_2}^{\varphi_2},$$

$$B_o = \Big[B(\mu,\varphi)\Big]_{\varphi_2}^{\varphi'_2},\quad C_o = \Big[C(\mu,\varphi)\Big]_{\varphi'_1}^{\varphi'_2},$$

where

$$(3.9)\quad B(\mu,\varphi) = \tfrac{1}{3}(\mu\cos^3\varphi + \sin^3\varphi) - \tfrac{1}{5}(\mu\cos^5\varphi + \sin^5\varphi) + \tfrac{1}{5}\mu^2\sin^5\varphi$$

$$(3.10)\quad C(\mu,\varphi) = 2(2\mu^2-1)\left(\frac{\sin^3\varphi}{3} - \frac{\sin^5\varphi}{5}\right) + -2\mu\left(\frac{\cos^3\varphi}{3} - 2\,\frac{\cos^5\varphi}{5}\right)$$

In eqs.(3.7),(3,8) we have replaced E and ν with E' and ν' to account, albeit approximately, for microcrack interaction.

Based on energy balance, the critical value of σ, at which stable microcrack growth will start, can be calculated from:

$$(3.11)\quad \tau_{no} = \sqrt{\gamma^{i}\, G\, \frac{\pi(2-\nu')}{(1-\nu')}} = \sqrt{\frac{\pi(2-\nu')}{4d_o}}\, K^{i}_{IC}$$

where τ_{no} is the above mentioned shear traction induced on the most favorably oriented microcrack and γ^{i} is the surface energy of a unit of free surface, expressed in function of K^{i}_{IC} (critical stress-intensity factor for the interface corresponding to the opening mode of fracture, typically determined experimentally).

Assuming a linear relation between K^{i}_{IC} and d, the limits for $K^{i}_{IC}(d)$ are:

$$(3.12)\quad K^{i}_{IC}(d_o) \leqslant K^{i}_{IC}(d) \leqslant K^{i}_{IC}(D)$$

where $K^{i}_{IC}(D)$ is the critical stress-intensity factor corresponding to the upper limit of growing microcrack diameter, for which the edge is still inscribed in the regular face polygonal of the aggregate.

Thus the critical value of σ, at which unstable microcrack propagation will start, can be calculated from:

$$(3.13)\quad \sigma = \sqrt{\frac{\pi}{2D}}\, \frac{K^{m}_{IC}}{A(\mu,\varphi)}$$

where K^{m}_{IC} is the critical stress intensity factor for the mortar and $A(\mu,\varphi) = \sin^{2}\varphi\cos\varphi - \mu\sin^{3}\varphi$.

As indicated experimentally this limit precedes the development in the mortar of branching cracks, parallel to the direction of maximum compression.

Figs.1 and 2 show total axial and lateral strain as function of σ up to instability, for the following values of parameters:

$E=37GPa$, $\nu=0.15$, $\varrho_o=0.1$, $d_o=2mm.$, $D=4mm.$, $K^{i}_{IC}(d_o)=0.2MPa\sqrt{m}$, $K^{i}_{IC}(D)=K^{m}_{IC}=0.4MPa\sqrt{m}$.

Fig.3 shows total volumetric strain as function of σ up to instability. It should be noted that the dilation is the consequence on the macroscopic behaviour of the microcrack energy release rate associated with self-similar growth.

Comparison of such theoretical prediction with experimental data indicates that the proposed model provides a realistic description of inelastic behaviour of uniaxial compressed concrete.

REFERENCES

(1) Budiansky B. & O'Connell R.:"Elastic Moduli of a cracked solid",Int.J.Solids and Structures,12,81, 1976.

(2) Horii H. & Nemat-Nasser S.:"Overall Moduli of Solids with Microcracks:load induced anisotropy" , J.Mech.Phys.Solids,31,2,155,1983.

(3) Kachanov M.L.:"A microcrack model of rock inelasticity",Mech.of Materials,1,19,1982.

(4) Krajcinovic D. & Fanella D.:"A micromechanical damage model for concrete",Eng.Fracture Mech.,25, 585,1986.

(5) Costin L.S.:"Damage Mechanics in the post-failure regime",Mech.of Materials,4,149,1985.

(6) Nobile L.:"A microcrack model of concrete behaviour under applied loads",Fracture Toughness and Fracture Energy of Concrete,ed.Wittmann F.H.,Elsevier Science Publ.B.V.,Amsterdam,1986.

(7) Pak A.P. & Trapeznikov L.P.:"Experimental investigations based on Griffith-Irwin theory processes of the crack development in concrete",Advances in Fracture Research,ed.Francois D.,5th Int.Conf. on Fract.,4,1531,1981.

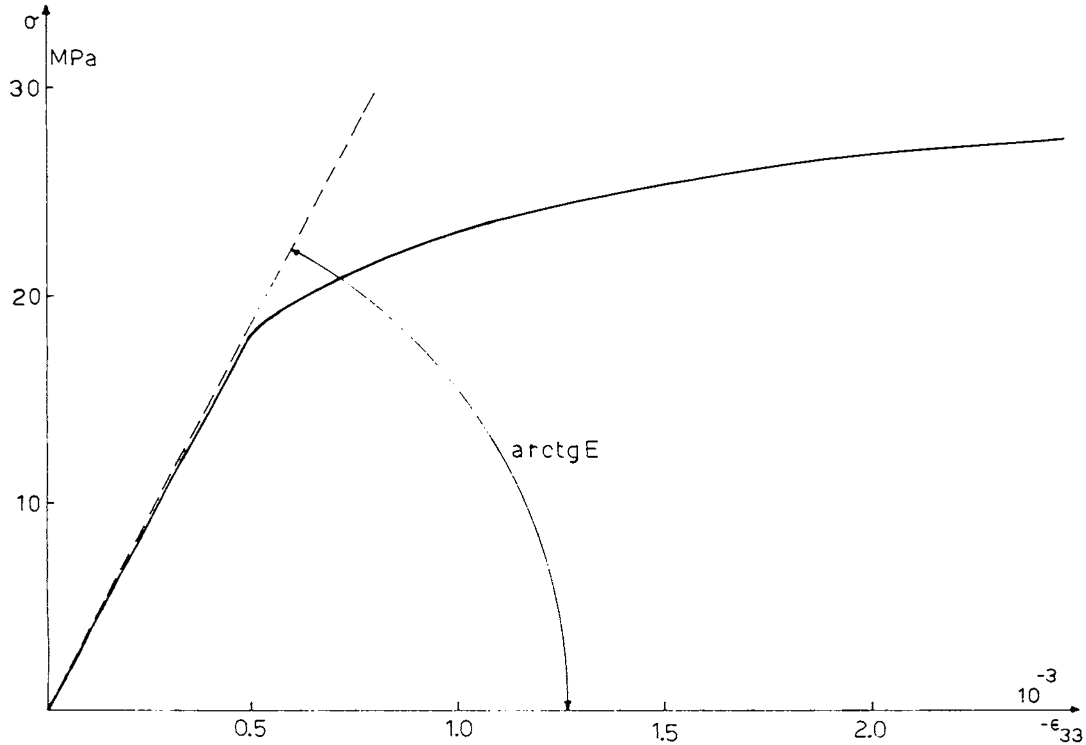

Fig. 1--Predicted total axial strain as function of axial stress