

Fig. 8.4.11.1—Reinforcement in T-beam flanges.



Fig. 8.4.12—Skin reinforcement for girders, beams, and joists with h > 36 in. (900 mm).

exposure, and $d_c = 2.8$ in. (70 mm) for exterior exposure. For joists, $d_c = 2$ in. (50 mm) for interior exposure, and $d_c = 2.4$ in. (60 mm) for exterior exposure.

8.4.14 Positive moment reinforcement

8.4.14.1 *Description*—Positive moment reinforcement should be provided in the girder, beam, or joist section, as indicated in Chapter 8, and should comply with 8.4 and the particular limitations for each member type in 8.6 or 8.7.

8.4.14.2 *Location*—Positive moment reinforcement should be placed longitudinally in the girder, beam, or joist. Positive moment reinforcement should be located as close to the bottom surface of the girder, beams, or joist as practicable following the concrete cover of **5.4**. Where girders, beams, or joists support other girders, beams, or joists, the positive moment reinforcement of the supported member should be placed above the positive moment reinforcement of the supporting member.

8.4.14.3 *Cutoff amount*—No more than one-half the positive moment reinforcement at midspan may be cut off at the locations indicated in 8.6.5 or 8.7.5.

8.4.14.4 *Reinforcement splicing*—The remaining positive moment reinforcement from 8.4.14.3 may be lap spliced between the cutoff point and the opposite face of the support.

8.4.14.5 *Embedment at interior supports*—Positive moment reinforcement terminated at an interior support should be continued to the opposite face of the support plus the lap-splice distance of 5.8.2.

8.4.14.6 *End anchorage of reinforcement*—At the end of the girder, beam, or joist, the positive moment reinforcement should extend to the edge and end in a standard hook.

8.4.15 Negative moment reinforcement

8.4.15.1 *Description*—Negative moment reinforcement should be provided in the girder, beam, or joist section and

at edges and supports as indicated in Chapter 8, and should comply with 8.4 and the particular provisions of 8.6 or 8.7.

8.4.15.2 *Location*—Negative moment reinforcement should be provided at all supports and located as close to the upper surface of the girder, beam, or joist as practicable following concrete cover of 5.4. At supports where girders or beams intersect, the negative moment reinforcement of the member with the longer span should be located on top.

8.4.15.3 *Cutoff amount*—Negative moment reinforcement at the locations indicated in 8.6.5 or 8.7.5 may be cut off, except cantilever negative moment reinforcement is not allowed to be cut off. Where adjacent spans are unequal, negative moment reinforcement cutoff points should be based on the longer span.

8.4.15.4 *Reinforcement splicing*—Negative moment reinforcement between cutoff point and the support should not be lap spliced.

8.4.15.5 *End anchorage*—Negative moment reinforcement at the end of a girder, beam, or joist should end in a standard hook at the far edge of the supporting girder, beam, column, or reinforced concrete wall, complying with anchorage distance described by 5.8.3. At the external edge of cantilevers, negative moment reinforcement should end in a standard hook.

8.4.15.6 *Stirrup support*—In areas where no negative reinforcement is needed, top bars should be provided for attachment and anchorage of stirrups. The diameter of these top bars should be equal to or greater than the stirrup bar diameter. Minimum lap length of these top bars should be 6 in. (150 mm).

8.5—Transverse reinforcement

8.5.1 *Description*—Transverse reinforcement for girders, beams, and joists should consist of stirrups that enclose the longitudinal reinforcement and are placed perpendicular to the longitudinal axis of the member at varying intervals.

The main functions for transverse reinforcement in girders, beams, and joists are:

(a) Contribute to member shear strength

(b) Provide lateral support for longitudinal reinforcement subjected to compression stresses

(c) Act as hanger reinforcement in girders, supporting beams and joists

(d) Contribute to member torsion strength

(e) Provide confinement to the concrete in seismic zones at selected locations within the member

8.5.2 *Stirrup shape*—A stirrup should consist of single or multiple vertical legs. Each vertical leg should engage





Fig. 8.5.2a—Typical stirrup shapes for girders and beams.



Fig. 8.5.2b—*Typical stirrup shape for joists, in addition to Fig.* 8.5.2a.

a longitudinal bar either by bending around it when the stirrup continues or by using a standard stirrup hook (5.6) to surround the longitudinal bar at the stirrup end (Fig. 8.5.2a and 8.5.2b).

8.5.2.1 *Permitted stirrup shape for girders and beams*—All stirrups in girders and beams should be closed stirrups with 135-degree hooks, as shown in Fig. 8.5.2a(a). The other stirrup shapes are in common use but are shown to clarify that they are not to be used with this guide. In seismic areas, the stirrup shape is further limited.

8.5.2.2 *Permitted stirrup shape for joists*—All stirrup types shown in Fig. 8.5.2a and 8.5.2b may be used in joists.

8.5.2.3 *Minimum clear spacing between stirrups legs*—In girders, beams, and joists, the minimum clear space between stirrups or parallel legs in a stirrup should be 1 in. (25 mm).

8.5.2.4 *Stirrup support*—Stirrups should be attached to longitudinal bars so that stirrups do not displace during concrete placement (8.4.15.6).

8.5.2.5 *Stirrup leg splicing*—Stirrup bars should not be lap spliced.

8.5.3 *Location of transverse reinforcement*—Stirrup spacing intervals *s* should comply with 8.5.4.5 (Fig. 8.5.3).

8.5.4 Contribution of transverse reinforcement to shear strength

8.5.4.1 *General*—Beam-action shear accompanies flexural moments and occurs in girders, beams, and joists along their length, and is of greater magnitude in the vicinity of supports and concentrated loads.

8.5.4.2 Design shear strength—Design shear strength ϕV_n of a girder, beam, or joist section should be computed following the procedure in 5.13.4 for beam-action shear as

$$\phi V_n = \phi (V_c + V_s) \tag{8.5.4.2}$$

where ϕV_c is the concrete contribution to the design shear strength; ϕV_s is the shear reinforcement contribution to the design shear strength; and $\phi = 0.75$.

8.5.4.3 Contribution of concrete to shear strength—At each location to be investigated (Fig. 8.5.4.3), the concrete contribution of the web of the girder, beam, or joist should

be taken into account (Fig. 8.5.3) and should be computed using Eq. (8.5.4.3), with $\phi = 0.75$.

$$\Phi V_c = \Phi 2 \sqrt{f_c' b_w d}$$

$$\left[\Phi V_c = \Phi 0.17 \sqrt{f_c' b_w d} \quad (SI) \right]$$
(8.5.4.3)

8.5.4.4 Contribution of transverse reinforcement to shear strength—For reinforcement perpendicular to the member axis, its contribution to design shear strength should be

$$\phi V_s = \phi \frac{A_v f_{yt} d}{s} \tag{8.5.4.4a}$$

where A_v is the area of shear reinforcement perpendicular to the member axis (the stirrup bar area A_b multiplied by the number of vertical stirrup legs) within a distance *s*; f_{yt} is the yield strength of the shear reinforcement steel; and $\phi = 0.75$.

Reinforcement contribution to the design shear strength should not be taken greater than

$$\Phi V_s \le \Phi 8 \sqrt{f_c'} b_w d = 4 \Phi V_c$$

$$\left[\Phi V_s \le \Phi 0.66 \sqrt{f_c'} b_w d = 4 \Phi V_c \quad (SI) \right] \qquad (8.5.4.4b)$$

8.5.4.5 *Design of shear reinforcement*—Shear reinforcement in girders, beams, and joists should be provided using stirrups perpendicular to the member axis with a maximum spacing *s*, measured along member axis:

(a) Where the factored shear V_u is less than $\phi V_c/2$, the use of shear reinforcement may be waived.

(b) Where V_u exceeds $\phi V_c/2$ and is less than ϕV_c , a minimum area of shear reinforcement should be provided as specified by Eq. (8.5.4.5). The spacing *s* along the member axis should not exceed the smaller of d/2 and 24 in. (600 mm) (Fig. 8.5.4.5).

$$A_{v} = 0.75 \sqrt{f_{c}'} \frac{b_{w}s}{f_{yt}}$$

$$\left[A_{v} = 0.062 \sqrt{f_{c}'} \frac{b_{w}s}{f_{yt}} \quad (SI)\right]$$
(8.5.4.5)

where A_v is A_b multiplied by the number of stirrup legs.

(c) Where V_u exceeds ϕV_c , the difference $(V_u - \phi V_c)$ should be provided for by shear reinforcement, using Eq. (8.5.4.3) and Eq. (8.5.4.4a), and the limitations (i) through (iv) should apply (Table 8.5.4.5):



Fig. 8.5.3—*Typical stirrup spacing along the girder, beam, or joist.*



Fig. 8.5.4.3—*Contribution of concrete to beam-action shear strength in girders, beams, and joists.*

i. Minimum shear reinforcement area should not be less than that determined using Eq. (8.5.4.5).

ii. Where the value of ϕV_s , calculated using Eq. (8.5.4.4a), is less than $2\phi V_c$, the spacing limits of (b) should be used.

iii. Where the value of ϕV_s , calculated using Eq. (8.5.4.4a), is greater than $2\phi V_c$, spacing limits should be half the values specified in (b).

iv. The value of ϕV_s , calculated using Eq. (8.5.4.4a), should not be taken greater than $4\phi V_c$.

8.5.4.6 Shear diagram—The value of V_u at the support face should be determined in conformance with 8.6 or 8.7. A diagram showing the shear variation within the span should be constructed, with the value of V_u at the left support face taken as positive. The shear from this point proceeding to the right should be decreased at a rate equal to

$$\frac{\left[(V_u)_{\text{left supp.}} + (V_u)_{\text{right supp.}} - \sum P_u\right]}{\ell_n} \qquad (8.5.4.6)$$

where $\sum P_u$ corresponds to the sum of all factored concentrated loads on the span. At the point where a concentrated load is applied, the value of P_u should be subtracted from the value of shear at the left of the load point. For beams with point loads, proceeding to the right, at the face of the right support, the negative value of V_u is reached (Fig. 8.5.4.6). At all sections within the span, the value of ϕV_n , as determined from Eq. (8.5.4.2), should be equal to or greater than the absolute value of $V_u(x)$ as shown in Fig. 8.5.4.6.

Limits for ϕV_n , as defined in Table 8.5.4.5, should be marked in the shear diagram, and stirrup spacing *s* should be defined for different regions within the shear diagram. The first stirrup should be placed not further than s/2 from

the face of the supporting member, with *s* being the stirrup spacing at the support. The minimum stirrup spacing should comply with 8.5.2.3. If the computed *s* is less than 2 in. (50 mm), using stirrups with more vertical legs or a larger bar diameter should be investigated.

8.5.5 *Hanger stirrups*—Where a beam is supported by a girder of similar depth, hanger reinforcement should be provided in the joint. The reaction from the supported beam tends to push down the bottom of the supporting girder. This reaction should be resisted by hanger reinforcement in the form of closed stirrups placed in both members. Hanger stirrups are in addition to stirrups needed for shear (Fig. 8.5.5) and should comply with 8.5.5.1 and 8.5.5.2.

8.5.5.1 Hanger stirrup area

(a) Provide hanger stirrups where V_u from the supported beam at the interface is equal to or greater than $\phi 3\sqrt{f_c'}b_w d$ [$\phi 0.25\sqrt{f_c'}b_w d$ (SI)], where $\phi = 0.75$.

(b) Provide hanger stirrups where h_b is equal to or less than one-half the total depth of the supporting girder, where h_b is the vertical dimension from the bottom of the supporting girder to the bottom of the supported beam (Fig. 8.5.5).

(c) The area of hanger reinforcement, A_i , should be determined from Eq. (8.5.5.1).

$$A_{i} \ge \frac{\left[1 - (h_{b}/h_{g})\right]V_{u}}{\Phi f_{yt}}$$
(8.5.5.1)

where V_u is the beam factored shear at the support face; A_i is the total area of hanger stirrups; h_g is the girder height; f_{yt} is the stirrup specified yield strength; and $\phi = 0.75$.

8.5.5.2 Hanger stirrup placement—At least two-thirds of A_i should be evenly distributed within the supported beam width b_{w} , plus h_b at each side. The remaining area of hanger stirrups, not more than one-third of A_i , should be evenly distributed within d/4 from the supporting girder face, where d is the supported beam effective depth. Beam bottom longitudinal bars should be placed above the girder bottom longitudinal bars.

8.6—Joists and beams supported by girders

8.6.1 *General*—Section 8.6 applies to joists and beams, monolithic with and supported by girders. Two-way joist systems or waffle-on-beams systems, as described in 6.1.3.3, should also comply with 8.6. Waffle-slab systems without beams spanning between columns as described in 6.1.4.5 should be designed using Chapter 9 for slab-column systems.





Fig. 8.5.4.5—*Minimum shear reinforcement in girders, beams, and joists when* $(\phi V_0/2 \le V_u \le \phi V_0)$.

Table 8.5.4.5—Shear reinforcement in girders, beams, and joists, maximum spacing s

Value of factored required shear strength V_u	Limiting value of ϕV_s	Minimum area of shear reinforcement A _v within a distance s	Maximum spacing s
$\frac{\phi V_c}{2} > V_u$	_	Not needed	_
$\phi V_c > V_u \ge \frac{\phi V_c}{2}$	_	$A_{v} = 0.75 \sqrt{f_{c}'} \frac{b_{w}s}{f_{yt}}$ $\left[A_{v} = 0.062 \sqrt{f_{c}'} \frac{b_{w}s}{f_{yt}} (SI)\right]$	$s \le \min . \text{ of } \begin{cases} d/2\\ 24 \text{ in. (600 mm)} \end{cases}$
	$2\phi V_c > \phi V_s$	$A_{v} = \frac{(V_{u} - \phi V_{c})s}{\phi f_{yt}d}$	$s \le \min . \text{ of } \begin{cases} d/2 \\ 24 \text{ in. (600 mm)} \\ A_v f_{yt} / (50b_w) & [A_v f_{yt} / (0.35b_w) & (SI)] \end{cases}$
$V_u \ge \phi V_c$	$4\phi V_c > \phi V_s \ge 2\phi V_c$	$A_{v} = \frac{\left(V_{u} - \phi V_{c}\right)s}{\phi f_{yt}d}$	$\leq \min . \text{ of } \begin{cases} /4 \\ 12 \text{ in. (300 mm)} \\ A_v f_{yt} / (50b_w) \ [A_v f_{yt} / (0.35b_w) \ (\text{SI})] \end{cases}$
	$\phi V_s \ge 4\phi V_c$	Not permitted	_



Fig. 8.5.4.6—Calculation of the shear diagram of a girder, beam, or joist.

8.6.2 Dimensional limits

8.6.2.1 *Joists*—In addition to Chapter 8, joists should comply with the dimensional limits of 1.3 and the restrictions of 6.1.3.1. Ducts, shafts, and openings should comply with 6.8. Minimum depth should comply with 6.5.3 for one-way joists and 6.5.4 for two-way joists.

8.6.2. *Beams*—In addition to Chapter 8, beams supported by girders should comply with the dimensional limits of 1.3 and the restrictions of 6.1.2. Ducts, shafts, and openings should comply with 6.8. Minimum depth should comply with 6.5.3. Beam web width b_w should not be less than 8 in. (200 mm). Maximum spacing between lateral supports of

isolated beams should be 50 times the least width b of the compression flange.

8.6.2.3 *Cantilevers*—All cantilevers of joists or beams should be continuous with at least one interior span. A double cantilever without an interior span is not permitted.

8.6.3 Required moment strength

8.6.3.1 *Cantilevers*—The factored negative moment M_u (required moment strength) for beam and joist cantilevers supported by girders, beams, or reinforced concrete walls should be calculated using Eq. (8.6.3.1), assuming:

(a) One-half of the distributed factored load W_u acts as a concentrated load at the cantilever tip along with all concentrated loads that act on the cantilever span $\sum P_u$.

(b) One-half W_u acts as uniformly distributed load over the full span.

$$M_{u}^{-} = \frac{3W_{u}\ell_{n}^{2}}{4} + \ell_{n}\Sigma P_{u}$$
(8.6.3.1)

The cantilever-required negative moment strength at the support should equal or exceed the maximum negative factored moment at the first interior support and one-third the positive factored moment of the first interior span.

8.6.3.2 Single-span joists and beams supported by beams, girders, or reinforced concrete walls—Factored positive and negative moment M_u (required moment strength) for





(a) beams

Fig. 8.5.5—Hanger reinforcement.

Table 8.6.3.2—Factored moment for single-span beams and joists



Table 8.6.3.3—Factored moment for beams and one-way joists with two or more spans

Positive moment at end spans: $M_{u}^{+} = \frac{W_{u}\ell_{n}^{2}}{11} + \frac{\ell_{n}}{9}\Sigma P_{u} \qquad (8.6.3.3a)$ Interior spans: $M_{u}^{+} = \frac{W_{u}\ell_{n}^{2}}{16} + \frac{\ell_{n}}{5}\Sigma P_{u} \qquad (8.6.3.3b)$ Negative moment at interior face of external support: $M_{u}^{-} = \frac{W_{u}\ell_{n}^{2}}{24} + \frac{\ell_{n}}{16}\Sigma P_{u} \qquad (8.6.3.3c)$ Exterior face of first internal support, only two spans: $M_{u}^{-} = \frac{W_{u}\ell_{n}^{2}}{9} + \frac{\ell_{n}}{6}\Sigma P_{u} \qquad (8.6.3.3d)$ Faces of internal supports, more than two spans:

$$M_{u}^{-} = \frac{W_{u}\ell_{n}^{2}}{10} + \frac{\ell_{n}}{7}\Sigma P_{u}$$
(8.6.3.3e)

Faces of all supports for joists with spans not exceeding 10 ft (3 m):

$$M_{u}^{-} = \frac{W_{u}\ell_{n}^{2}}{12} + \frac{\ell_{n}}{8}\Sigma P_{u}$$
(8.6.3.3f)

single-span beams and single-span one-way joists should be calculated using Table 8.6.3.2, where $\sum P_u$ is the sum of all factored concentrated loads that act on the span.

8.6.3.3 Multi-span joists and beams supported by beams, girders, or walls—Factored positive and negative moment M_u (required moment strength) for beams and one-way joists, with two or more spans, supported by beams, girders, or reinforced concrete walls, should be calculated using



Table 8.6.4.3—Factored shear for beams and one-way joists with two or more spans

Exterior face of first interior support:	
$V_{u} = 1.15 \frac{W_{u}\ell_{n}}{2} + 0.8\Sigma P_{u}$	(8.6.4.3a)
Faces of all other supports:	
$V_u = \frac{W_u \ell_n}{2} + 0.75 \sum P_u$	(8.6.4.3b)

Table 8.6.3.3, where $\sum P_u$ is the sum of all factored concentrated loads that act on the span.

8.6.3.4 Use of frame analysis—Frame analysis, which meets 8.1.2, may be used to determine factored moments and shears as a substitute for values in 8.6.3.1 to 8.6.3.3.

8.6.3.5 *Two-way joists supported by beams, girders, or walls*—Required moment strength for two-way joists supported by beams, girders, or structural walls may be obtained using 7.9.1 and 7.9.2, ignoring the minimum depth of the supporting beams or girders as given by 7.9.1(c) and 6.1.3.3.

8.6.4 *Required shear strength*

8.6.4.1 Cantilevers of joists and beams supported by beams, girders, or walls—Factored shear V_u at the cantilever support should be calculated using Eq. (8.6.4.1).

$$V_u = W_u \ell_n + \sum P_u \tag{8.6.4.1}$$

where $\sum P_u$ is the sum of all factored concentrated loads that act on the span.

8.6.4.2 Single-span joists and beams supported by beams, girders, or walls—Factored shear V_u for single-span beams and single-span one-way joists should be calculated using Eq. (8.6.4.2).

$$V_u = \frac{W_u \ell_n}{2} + 0.8 \sum P_u$$
 (8.6.4.2)

where $\sum P_u$ is the sum of all factored concentrated loads that act on the span.

8.6.4.3 Joists and beams supported by beams, girders, or walls, with two or more spans—Factored shear V_u for beams and one-way joists with two or more spans supported by beams, girders, or structural walls should be calculated using equations given in Table 8.6.4.3, where $\sum P_u$ corresponds to the sum of all factored concentrated loads on the span.





Fig. 8.6.5.1—*Reinforcement for beams and joists supported by beams or girders.*

8.6.4.4 Use of frame analysis—Frame analysis, meeting 8.1.2, may be used to determine factored shear as a substitute for the values determined from 8.6.4.1 to 8.6.4.3.

8.6.4.5 *Two-way joists supported by beams, girders, or walls*—Factored shear for two-way joists supported by beams, girders, or structural walls may be determined using 7.9.1 and 7.9.4, ignoring the minimum depth of the supporting beams or girders indicated in 7.9.1(c) and 6.1.3.3.

8.6.5 Reinforcement

8.6.5.1 Positive moment reinforcement—The positive moment reinforcement area should be determined for the calculated value of M_u^+ . When a slab is present in the upper part of the section or when the beam or joist is T-shaped, the T-beam effect may be used. Positive moment reinforcement should comply with 8.4.14. At internal supports, at a distance equal to $\ell_n/8$ measured from the face of the supports into the span, up to one-half the positive moment reinforcement may be cut off if there are no concentrated loads within that distance (Fig. 8.6.5.1). For single-span beams and joists, positive moment reinforcement should not be cut off.

8.6.5.2 Negative moment reinforcement—The negative moment reinforcement area should be determined for the larger value of M_u^- calculated for both sides of the support. This reinforcement should comply with 8.4.15. When a slab is present in the upper part of the section or when the beam or joist is T-shaped, negative moment reinforcement should comply with 8.4.11.1. At a distance equal to $\ell_n/4$ for external supports and $\ell_n/3$ for internal supports, measured from the internal face of the support into the span, all negative moment reinforcement may be cut off (Fig. 8.6.5.1). Negative moment reinforcement should not be cut off in cantilevers.

8.6.5.3 *Transverse reinforcement*—Values of V_u at the right and left support faces should be determined by the appropriate equation from 8.6.4. The transverse reinforcement should comply with 8.5.

8.6.6 Reactions on beams and girders

8.6.6.1 One-way joists—Factored reaction on the joist system support may be considered as uniformly distributed. Factored reaction on the supports, r_u , per unit length, should be the value determined from Eq. (8.6.6.1) plus the



Fig. 8.7.2.2—Limits on girder depth and width.

uniformly distributed reaction from any cantilever spanning from that support.

$$r_u = \frac{V_u \ell_s}{s_j \ell_n} \tag{8.6.6.1}$$

where V_u is the factored shear from 8.6.4; ℓ_s is the center-tocenter span of the joist; ℓ_n is the clear span of the joist; and s_j is the center-to-center spacing between parallel joists.

8.6.6.2 *Two-way joists supported by beams, girders, or walls*—Factored reactions for two-way joists supported by beams, girders, or structural walls may be calculated using 7.9.1 and 7.9.5, ignoring minimum depth of the supporting beams or girders given in 7.9.1(c) and 6.1.3.3.

8.6.6.3 *Beams*—Factored reactions on the supports, R_u , should be the values determined from Eq. (8.6.6.3) plus the factored reaction from any cantilever spanning from that support.

$$R_u = \frac{V_u \ell_s}{\ell_n} \tag{8.6.6.3}$$

aci

where V_u is the factored shear from 8.6.4; ℓ_s is the center-tocenter span; and ℓ_n is the clear span of the beam.



Table 8.7.3.1—Factored moment for girders of frames

Positive moment at end spans:				
$M_u^+ = \frac{W_u \ell_n^2}{14} + \frac{\ell_n}{6} \sum P_u$	(8.7.3.2a)			
Interior spans:				
$M_u^+ = \frac{W_u \ell_n^2}{16} + \frac{\ell_n}{7} \sum P_u$	(8.7.3.2b)			
Negative moment at supports at				
Interior face of external column or perpendicular structural wall:				
$M_u^- = \frac{W_u \ell_n^2}{16} + \frac{\ell_n}{10} \sum P_u$	(8.7.3.2c)			
Exterior face of first internal column or perpendicular structural wall, only two spans:				
$M_u^- = \frac{W_u \ell_n^2}{9} + \frac{\ell_n}{6} \sum P_u$	(8.7.3.2d)			

Faces of internal columns or perpendicular structural walls, more than two spans:

$$M_{u}^{-} = \frac{W_{u}\ell_{n}^{2}}{10} + \frac{\ell_{n}}{6.5}\Sigma P_{u}$$
(8.7.3.2e)

Faces of structural walls parallel to the plane of the frame:

$$M_{u}^{-} = \frac{W_{u}\ell_{n}^{2}}{12} + \frac{\ell_{n}}{7}\Sigma P_{u}$$
(8.7.3.2f)

Support of girder cantilevers:

$$M_{u}^{-} = \frac{3W_{u}\ell_{n}^{2}}{4} + \ell_{n}\Sigma P_{u}$$
 (8.7.3.2g)

8.7—Girders that are part of a frame

8.7.1 *General*—Section 8.7 applies to girders that are part of a moment-resisting frame where the girders are monolithic and are directly supported by columns or reinforced concrete walls.

8.7.2 Dimensional limits

8.7.2.1 *General*—In addition to Chapter 8, girders that are part of a frame should comply with the dimensional limits set forth in 1.3. Embedded conduits and pipes should comply with 6.8.

8.7.2.2 *Girder depth and width*—The girder should be prismatic without haunches, brackets, or corbels. The height *h* should comply with the minimum depth of 6.5.3. Clear span of the member should not be less than four times its height *h*. The width-to-height ratio $b_w h$ should not be less than 0.3. The width b_w should not be less than 8 in. (200 mm), nor exceed the corresponding width of the supporting column plus 3/4h on each side of the supporting column (Fig. 8.7.2.2).

8.7.2.3 *Girders supported by reinforced concrete walls*— Girders, supported by a reinforced concrete wall located in the plane of the frame, should continue along the full horizontal wall length. Girder width should not be less than wall thickness. Where girders are supported by walls perpendicular to the longitudinal axis of the girder a beam should

Table 8.7.4.1—Factored shear for girders of frames

Exterior face of first interior column:

$$V_u = 1.15 \frac{W_u \ell_n}{2} + 0.8 \sum P_u$$
 (8.7.4.1a)

Faces of all other columns:

$$=\frac{W_u \ell_n}{2} + 0.75 \Sigma P_u \tag{8.7.4.1b}$$

Supports of girder cantilevers:

 V_{u}

 $V_{\mu} = W_{\mu}\ell_{\mu} + \Sigma P_{\mu} \tag{8.7.4.1c}$

run along the full horizontal wall length at the same level and have the same height as the girder. Beam width should not be less than wall thickness or 8 in. (200 mm). Vertical reinforcement of the wall should pass through the girder or beam, as indicated in Chapter 12.

8.7.2.4 *Lateral support*—For girders not continuously laterally supported by the floor slab or secondary beams, the clear distance between lateral supports should not exceed 50 times the least width b of compression flange or face.

8.7.2.5 *Restrictions*—The following restrictions should be in effect for girders of frames designed under 8.7:

(a) There are two or more spans

(b) Spans are approximately equal, with the shorter of two adjacent spans greater than or equal to 80 percent of the larger span (1.3)

(c) Loads are uniformly distributed and adjustments for concentrated loads are performed

(d) Unfactored unit live load w_{ℓ} does not exceed three times unfactored unit dead load w_d

(e) Girders should not have a slope exceeding 15 degrees **8.7.3** *Required moment strength*

8.7.3.1 Factored positive and negative moment—Factored positive and negative moment M_u (required moment strength) for girders and beams that are part of a frame whose vertical members are columns and concrete structural walls should be calculated using equations in Table 8.7.3.1, where $\sum P_u$ is the sum of all factored concentrated loads that act on the span.

8.7.3.2 *Girders parallel to one-way joist systems*—For girders parallel to joists, the assumed tributary width for the calculation of girder factored loads should be twice the joist spacing plus the girder width.

8.7.3.3 Use of frame analysis—Frame analysis, meeting 8.1.2, may be used to determine factored moments and shears as a substitute for values computed from 8.7.3.1 and 8.7.4.1.

8.7.4 Required shear strength

8.7.4.1 *Factored shear*—Girder V_u should be calculated at support faces using equations of Table 8.7.4.1, where $\sum P_u$ is the sum of all factored concentrated loads that act on the span. Refer to 8.7.3.2 for assumed tributary width.

8.7.4.2 *Use of frame analysis*—Frame analysis, meeting 8.1.2, may be used to determine factored shear as a substitute for values computed from 8.7.4.1.

8.7.5 Reinforcement

8.7.5.1 *Positive moment reinforcement*—The positive moment reinforcement area should be determined using the





Fig. 8.7.5.1*a*—*Reinforcement in girders that are part of a moment-resisting frame supported by columns or reinforced concrete walls.*



Fig. 8.7.5.1b—Reinforcement in girders that are part of the perimeter frame.

calculated value of M_u^+ . If a slab exists in the upper part of the girder, the T-beam effect may be taken into account. Positive moment reinforcement should comply with 8.4.14. At internal supports, at a distance equal to $\ell_n/8$ measured from the support face into the span, up to one-half of the positive moment reinforcement may be cut off if there are no concentrated loads within that distance (Fig. 8.7.5.1a and 8.7.5.1b).

8.7.5.2 Negative moment reinforcement—The negative moment reinforcement area should be determined using the larger value of M_u^- computed at both sides of the support. This reinforcement should comply with 8.4.15. When a slab is present in the upper part of the section or when the girder is T-shaped, negative moment reinforcement should comply with 8.4.11.1. For perimeter beams, as indicated in 6.3.2, one-fourth of the negative moment reinforcement should either be continuous or spliced at midspan (Fig. 8.7.5.1b). Negative moment reinforcement in cantilevers should not be cut off. For other beams, all negative moment reinforcement reinforcement may be cut off at a distance equal to $\ell_n/4$ for external supports, measured from the support face into the span (Fig. 8.7.5.1a).

8.7.5.3 Transverse reinforcement—Values of V_u at the right and left faces of the supports should be determined

using the appropriate equation from 8.7.4. Transverse reinforcement should meet 8.5.

8.7.5.4 *Hanger stirrups*—Where a beam is supported by a girder of similar depth, the use of hanger stirrups as indicated by 8.5.5 should be investigated.

8.7.6 Reactions on columns and reinforced concrete walls

8.7.6.1 Vertical reactions at columns and walls—Factored reactions at the supports, R_u , should be the values determined from Eq. (8.7.6.1) plus the factored reaction from any cantilever spanning from that support.

$$R_u = \frac{V_u \ell_s}{\ell_n} \tag{8.7.6.1}$$

(aci)

where V_u is the factored shear from 8.7.4; ℓ_s is the center-tocenter span; and ℓ_n is the clear span of the girder.

8.7.6.2 Unbalanced moment from vertical loading— Moment reaction on columns should be evaluated using the unbalanced factored moment ΔM_u , caused by the factored vertical loads on girders that span from the column in the plane of the frame. The unbalanced moment should be distributed to the column above and below the girder in proportion to their relative stiffness. To calculate the unbal-





Fig. 8.7.6.2—Girder unbalanced moment to be transferred to columns.



Fig. 8.7.6.3—Joint types for determination of column moments.

anced moment, (a), (b), and (c) should apply, or a frame analysis complying with 8.1.2 should be performed:

(a) The unbalanced moment ΔM_u is the largest difference in girder factored negative moments at the column when load cases in (b) and (c) are evaluated.

(b) In the first case (Case A of Fig. 8.7.6.2), the entire girder should be loaded with the factored dead load and alternate odd spans should be loaded with the factored live load.

(c) In the second case (Case B of Fig. 8.7.6.2), the entire girder should be loaded with the factored dead load, and the alternate even spans should be loaded with the factored live load.

8.7.6.3 Unbalanced moment distribution—To distribute the unbalanced moment to columns or walls above and below the girder, (a) applies to roof joints, and (b) through (d) apply to joints with columns top and bottom, or a frame analysis complying with 8.1.2 should be performed:

(a) For roof joints (Types D, E, and F of Fig. 8.7.6.3), the column factored-moment should correspond to ΔM_u .

(b) For joints with columns top and bottom (Types A, B, and C of Fig. 8.7.6.3), the unbalanced moment should be distributed to the column, or wall, above using Eq. (8.7.6.3a).

$$(M_u)_{up} = \Delta M_u \frac{(I_c/h_{pi})_{up}}{(I_c/h_{pi})_{up} + (I_c/h_{pi})_{down}}$$
(8.7.6.3a)

(c) For joints with columns top and bottom (Types A, B, and C of Fig. 8.7.6.3), the unbalanced moment should be distributed to the column, or wall, below using Eq. (8.7.6.3b).

$$(M_u)_{down} = \Delta M_u \frac{(I_c/h_{pi})_{down}}{(I_c/h_{pi})_{up} + (I_c/h_{pi})_{down}}$$
(8.7.6.3b)

(d) In Eq. (8.7.6.3a) and Eq. (8.7.6.3b), I_c should be evaluated using Eq. (8.7.6.3c) (Fig. 8.7.6.3).

$$I_c = \frac{b_c h_c^3}{12}$$
 (8.7.6.3c)

where b_c is the column dimension, or wall section in the direction perpendicular to the girder span; h_c is the column dimension, or wall section in the direction parallel to the girder span; and h_{pi} is the column or wall story height.

CHAPTER 9—SLAB-COLUMN SYSTEMS

9.1—General

Slabs in slab-column systems as described in 6.1.4 should be designed using Chapter 9. The design of waffle slabs as described in 6.1.4.5 is included.

9.2-Loads

9.2.1 Loads to be included—Service loads for slabs that are part of slab-column systems should be established from Chapter





Fig. 9.3.2—Column and middle strips.



Fig. 9.3.9.3—Effect of openings in the slab.

(a) Dead loads: Slab self-weight, flat nonstructural elements, standing nonstructural elements, and fixed equipment loads, if any

(b) Live loads

(c) Where the slab is part of the roof system, appropriate values of roof live load, rain load, and snow load should be used

9.2.2 Dead load and live load—Values of q_d for dead load should include slab self-weight and the weight of the horizontal and vertical nonstructural elements as defined in 4.5.3. The value q_ℓ for live load should be determined by 4.6. Where the slab is part of the roof system, roof live load given in 4.7, rain load in 4.8, and snow loads in 4.9 should be included, as appropriate.

9.2.3 *Factored loads*—Factored load q_u should be the largest value determined using load factors and load combinations in 4.2.

9.3—Dimensional limits

9.3.1 *General*—In addition to Chapter 9, the slab in slabcolumn systems should comply with the dimensional limits in 1.3, and the restrictions of 6.1.4 for slab-column systems.

9.3.2 Column strip—A column strip is a design strip with a width on each side of a column centerline equal to $\ell_2/4$ or $\ell_1/4$, whichever is less (Fig. 9.3.2). Where the strip is adjacent and parallel to an edge, the column-strip width on the external side should be the distance from the column centerline to the edge, without exceeding the width of the internal side. For waffle slabs, the column strip comprises all joists that engage the column capital. Refer to 6.1.4.5 for the minimum number of joists that should engage the capital.





9.3.3 *Middle strip*—A middle strip is a design strip bounded by two column strips (Fig. 9.3.2).

9.3.4 *Definition of a panel*—A panel is bounded by column or wall centerlines on all sides (Fig. 9.3.2).

9.3.5 *Minimum slab thickness*—Minimum slab thickness to meet the serviceability limit state of slab-column systems should conform to 6.5.5.

9.3.6 *Restriction on column dimensions*—In slab-column systems, the ratio of long to short cross-sectional column dimensions should not exceed 2.

9.3.7 Support dimensions and clear span—For a slab system supported by columns or walls, the clear span ℓ_n should extend from face-to-face of columns, capitals, brackets, or walls.

9.3.8 *Other restrictions*—Slab-column systems should comply with (a) through (f):

(a) There should be a minimum of three continuous spans in each direction.

(b) Panels should be rectangular, with a ratio of longer to shorter span center-to-center of supports within a panel not greater than 2.

(c) Successive span lengths measured center-to-center of supports in each direction should not be smaller than 80 percent of the adjacent larger span, except in elevator and stair cores (1.3.6).

(d) Columns may be offset by a maximum of 10 percent of the span (in direction of offset) from either axis between centerlines of successive columns.

(e) All loads should be due to gravity only and be uniformly distributed over an entire panel.

(f) Unfactored live load q_{ℓ} should not exceed twice the unfactored dead load q_{d} .

aci