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Report on Torsion in Structural Concrete

Reported by Joint ACI-ASCE Committee 445

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A clear understanding of the effects of torsion on concrete members is essential to the safe, economical design of reinforced and prestressed concrete members. This report begins with a brief and systematic summary of the 180-year history of torsion of structural concrete members, new and updated theories and their applications, and a historical overview outlining the development of research on torsion of structural concrete members. Historical theories and truss models include classical theories of Navier, Saint-Venant, and Bredt; the three-dimensional (3-D) space truss of Rausch; the equilibrium (plasticity) truss model of Nielson as well as Lampert and Thürlimann; the compression field theory (CFT) by Collins and Mitchell; and the softened truss model (STM) by Hsu and Mo.

This report emphasizes that it is essential to the analysis of torsion in reinforced concrete that members should: 1) satisfy the equilibrium condition (Mohr's stress circle); 2) obey the compatibility condition (Mohr's strain circle); and 3) establish the constitutive relationships of materials such as the "softened" stress-strain relationship of concrete and "smeared" stress-strain relationship of steel bars.

The behavior of members subjected to torsion combined with bending moment, axial load, and shear is discussed. This report deals with design issues, including compatibility torsion, spandrel beams, torsional limit design, open sections, and size effects. The final two chapters are devoted to the detailing requirements of transverse and longitudinal reinforcement in torsional members with detailed, step-by-step design examples for two beams under torsion using ACI (ACI 318-11), European (EC2-04), and Canadian Standards Association (CSA-A23.3-04) standards. Two design examples are given to illustrate the steps involved in torsion design. Design Example 1 is a rectangular reinforced concrete beam under pure torsion, and Design Example 2 is a prestressed concrete girder under combined torsion, shear, and flexure.

Keywords: combined action (loading); compatibility torsion; compression field theory; equilibrium torsion; interaction diagrams; prestressed concrete; reinforced concrete; shear flow zone; skew bending; softened truss model; spandrel beams; struts; torsion detailing; torsion redistribution; warping.

CONTENTS

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CHAPTER 1—INTRODUCTION AND SCOPE

1.1—Introduction

Accounting for the effects of torsion is essential to the safe design of structural concrete members, requiring a full knowledge of the effects of torsion and a sound understanding of the analytical models that can easily be used for design. For over three decades, considerable research has been conducted on the behavior of reinforced concrete members under pure torsion and torsion combined with other loadings. Likewise, analytical models have been developed based on the truss model concept. Several of these models were developed to predict the full load history of a member, whereas others are simplified and used only to calculate torsional strength. Many models developed since the 1980s account for softening of diagonally cracked concrete.

This report reviews and summarizes the evolution of torsion design provisions in ACI 318, followed with a summary of the present state of knowledge on torsion for design and analysis of structural concrete beam-type members. Despite a vast amount of research in torsion, provisions of torsion design did not appear in ACI 318 until 1971 (ACI 318-71), although ACI 318-63 included a simple clause regarding detailing for torsion. Code provisions in 1971 were based on Portland Cement Association (PCA) tests (Hsu 1968b).

These provisions were applicable only to rectangular nonprestressed concrete members. In 1995, ACI 318-95 adopted an approach based on a thin-tube, space truss model previously used in the Canadian Standards Association (CSA-A23.3-77) code and the Comité Euro-International du Béton (CEB)-FIP code (1978). This model permitted treatment of sections with arbitrary shape and prestressed concrete (Ghoneim and MacGregor 1993; MacGregor and Ghoneim 1995). The ACI 318-02 code extended the application of the (ACI 318) 1995 torsion provisions to include prestressed hollow sections. ACI 318 allows the use of alternative design methods for torsional members with a cross section aspect ratio of 3 or greater, like the procedures of pre-1995 editions of ACI 318 or the Prestressed Concrete Institute (PCI) method (Zia and Hsu 1978).

This report reviews and summarizes the present state of knowledge on torsion and reviews their use as a framework for design and analysis of structural concrete beam-type members. Chapter 3 presents a historical background outlining the development of research on torsion of structural concrete members. The general behavior of reinforced and prestressed concrete members under pure torsion is discussed in Chapter 4. In Chapter 5, the compression field theory (CFT) and softened truss model (STM) are presented in detail. Chapter 5 also includes a description of two graphical methods (Rahal 2000a,b; Leu and Lee 2000). The behavior of members subjected to torsion combined with shear, flexure, and axial load is discussed in Chapter 6. Chapter 7 introduces additional design issues related to torsion, such as precast spandrel beams, torsion limit design, size effect, open sections, and torsional moment distribution. Detailing of torsional members is described in Chapter 8. Chapter 9 covers detailed design examples of several beams subjected to torsion using ACI 318, EC2-04, and CSA-A23.3-04 design equations, and additional graphical design methods reported by researchers.

1.2—Scope

Theories presented in this report were developed and verified for building members of typical size. For application to large-scale members, size effects should be considered. They could present a serious safety issue when using the shear strength equations provided in the design standard, which cannot take into account the shear strength reduction in large-scale members caused by loss of aggregate interlock behavior. Experimental information on large-scale torsional members is lacking.

CHAPTER 2—NOTATION AND DEFINITIONS

The material presented is a summary of research carried out worldwide and spanning more than four decades, making unification of the symbols and notations used by the various researchers and design codes a challenge. In some cases, mostly for graphs and figures, the notation is kept as originally published.

2.1—Notation

a = moment arm for bending, mm (in.)
a_c = geometric property index
a_o = depth of equivalent rectangular stress block in concrete strut of torsional member, mm (in.)
A = area of yield surface, mm^2 (in.^{2})
A_{cp} = area enclosed by outside perimeter of concrete cross section, mm^2 (in.^{2})
A_ℓ = total area of longitudinal reinforcement to resist torsion, mm^2 (in.^{2})
A_o = gross area enclosed by shear flow path, mm^2 (in.^{2}) (noted as A_{tb} in Eq. (7.2.6))
A_{oh} = area enclosed by centerline of outermost closed transverse torsional reinforcement, mm^2 (in.^{2})
A_{ps} = area of prestressing reinforcement in flexural tension zone, mm^2 (in.^{2})
A_s = area of nonprestressed longitudinal tension reinforcement, mm^2 (in.^{2})
A_s' = area of longitudinal compression reinforcement, mm^2 (in.^{2})
A_t = area of one leg of a closed stirrup resisting torsion within spacing s, mm^2 (in.^{2}) (noted as A_{tb} in Eq. (7.2.6))
b = width of compression face of member, mm (in.)
b_c = width of stirrups, mm (in.)
B = integral of T_w
C = cross-sectional constant to define torsional properties of a beam
d_v = distance between top and bottom longitudinal reinforcement, mm (in.)
D = cross-sectional depth used in fracture mechanics calculations, mm (in.)
D_0 = size effect constant for computing σ_N for plain concrete section
D_1 = normalized constant to represent characteristic structural dimensions used in fracture mechanics calculations
D_b = size effect constant for computing σ_N for reinforced concrete section
D_c = total energy dissipated on discontinuous concrete yield surface
D_s = total energy dissipated by reinforcement
e = moment arm for torsion, mm (in.)
E_c = modulus of elasticity of concrete, MPa (psi)
E_{ps} = modulus of elasticity of prestressed reinforcement in flexural tension zone, MPa (psi)
E_{ps}' = tangential modulus of Ramberg-Osgood curve at zero load MPa (psi)
E_s = modulus of elasticity of reinforcement and structural steel, MPa (psi)
EJ_w = rigidity of beam under warping torque, N·m^2 (lb-in.^{2})
f_c' = characteristic concrete cylinder compressive strength, MPa (psi)
f_c^* = concrete effective (plastic) compressive stress, MPa (psi)
f_{ck} = characteristic compressive strength of concrete, MPa (psi); $f_{ck} = f_{cm} - 8$ MPa ($f_{ck} = f_{cm} - 1200$ psi)
f_{cm} = mean compressive strength of concrete, MPa (psi)
f_d = diagonal concrete stress, MPa (psi)
f_{ds} = diagonal concrete stress corresponding to strain ε_{ds}, MPa (psi)
f_ℓ = reinforcement stress in ℓ direction, MPa (psi)
$f_{\ell p}$ = prestressing reinforcement stress in the ℓ direction, MPa (psi)
$f_{\ell y}$ = specified yield strength of longitudinal reinforcement, MPa (psi)
f_p = stress in prestressing reinforcement; f_p becomes $f_{\ell p}$ or f_{tp} when applied to longitudinal and transverse reinforcement, respectively, MPa (psi)
$f_{p0.1}$ = characteristic yield strength of prestressing reinforcing strands, MPa (psi); $f_{p0.1} = 0.9f_u$
f_{pc} = compressive stress in concrete due to prestress, MPa (psi)
f_{pk} = characteristic tensile strength of prestressing reinforcing strands, MPa (psi); $f_{pk} = f_{pu}$
f_{po} = effective prestress after losses in prestressing reinforcement, MPa (psi)
f_{pu} = specified tensile strength of prestressing reinforcement, MPa (psi)
$f_{p,ud}$ = design ultimate strength of prestressing reinforcing strands, MPa (psi); $f_{p,ud} = f_{pk}/\gamma_s$ ($\gamma_s = 1.15$)
f_r = modulus of rupture of concrete, MPa (psi)
f_t = reinforcement stress in t direction, MPa (psi)
f_t' = uniaxial tensile strength of concrete, MPa (psi)
f_t^* = concrete effective (plastic) tensile stress, MPa (psi)
f_{tp} = prestressing reinforcement stress in t direction, MPa (psi)

f_{ty} = specified yield strength of transverse reinforcement, MPa (psi)
f_y = specified yield strength of reinforcement, MPa (psi)
f_{yd} = design yield strength reinforcing steel, MPa (psi); $f_{yd} = f_y/\gamma_s$ ($\gamma_s = 1.15$)
$f_{y\ell}$ = yield strength of the torsional longitudinal reinforcement, MPa (psi)
f_{yv} = torsional hoop yield strength reinforcement, MPa (psi)
G = shear modulus, MPa (psi)
h = overall thickness or height of a member, mm (in.)
H_o = horizontal force in radial direction, N (lb) (Chapter 7)
I_p = polar moment of inertia, mm^4 (in.^{4})
k_1 = ratio of average stress to peak stress
K = value from Mohr-Coulomb yield criterion
K_f = flexural stiffness of floor beams, N·m^2 (lb-in.^{2})
K_{ts} = torsional stiffness of spandrel beam, N·m/rad (in.-lb/rad)
ℓ = span length of beam, mm (in.)
ℓ_f = length of flexural beam, mm (in.)
ℓ_q = width of shear flow q along top wall (Fig. 4.2(a) and (b)), mm (in.)
m = ratio of effective (plastic) compressive stress to effective (plastic) tensile stress of concrete
M = applied flexural moment at section, N·m (in.-lb)
M_o = pure flexural strength of section, N·m (in.-lb)
n = integer value
n_R = number of redundants
n_V = coefficient describing an under-reinforced, partially under-reinforced, or completely over-reinforced section
N = applied axial load at section, N (lb)
N_o = pure axial strength of section, N (lb)
p_h = perimeter of centerline of outermost closed transverse torsional reinforcement, mm (in.)
p_o = perimeter of outer concrete cross section, mm (in.) (sometimes noted as p_{cp})
P = applied concentrated load, N (lb)
q = shear flow, N/m (lb/in.)
r = ratio of top-to-bottom yield forces of the longitudinal reinforcement
r = size effect constant for computing σ_N
R = shape parameter used in Ramberg-Osgood
s = center-to-center spacing of longitudinal and transverse reinforcements, mm (in.)
s_1 = center-to-center spacing of longitudinal reinforcement, mm (in.)
s_t = center-to-center spacing of transverse reinforcement, mm (in.)
t = wall thickness of hollow section, mm (in.)
t_d = thickness of shear flow zone, mm (in.)
T = applied torsional moment at section, N·m (in.-lb)
T_c = nominal torsional strength provided by concrete, N·m (in.-lb)
T_{cr} = torsional cracking resistance of cross section, N·m (in.-lb)
T_f = applied torsional moment, N·m (in.-lb) (Chapter 9)
T_{max} = maximum torsional moment, N·m (in.-lb) (Chapter 7)
T_n = nominal torsional moment strength, N·m (in.-lb)
T_o = pure torsional strength of section, N·m (in.-lb)
T_s = nominal torsional strength provided by reinforcement, N·m (in.-lb)
T_u = factored torsional moment at section, N·m (in.-lb)
T_w = warping torsional moment, N·m (in.-lb)
T_{xu} = factored balanced torsional strength, N·m (in.-lb)
T_{xub} = balanced torsional strength, N·m (in.-lb)
$\overline{T}_{xub}$ = nondimensional balanced torsional strength, N·m (in.-lb)
v = shearing stress due to shear, MPa (psi)
v^* = plastic flow rate (Chapter 7)
v_u = ultimate shear stress, MPa (psi)
V = applied shear force at section, N (lb)
V_c = nominal shear strength provided by concrete, N (lb)
V_o = pure shear strength of section, N (lb)
V_u = factored shear force at section, N (lb)
w = ultimate distributed load on helical stair, N/m (lb/ft) (Chapter 7)
W = external work, N/m (lb/ft)
x = shorter overall dimension of rectangular part of cross section, mm (in.)
x_1 = distance section centroid and an infinitesimally small area of yield surface, mm (in.)
y = longer overall dimension of rectangular part of cross section, mm (in.)
z = distance along axis of beam, mm (in.)
α, β = Saint-Venant's coefficients for homogeneous torsional section
α^*, β^* = rotational angles in beam subjected to torsion (Chapter 7)
α_1 = stress block factor given as ratio of f_d to f_c' (Chapter 5)
β = factor relating effect of longitudinal strain on shear strength of concrete (American Association of State Highway and Transportation Officials (AASHTO) LRFD (general message)
β_1 = factor relating depth of equivalent rectangular compressive stress block to neutral axis depth; also, block factor given as ratio of a_o to t_d (Fig. 4.5)
γ_1 = angle along helical stair (in plan) at which maximum torsional moment is assumed to occur
γ_2 = angle along helical stair (in plan) at which vertical moment is assumed to be zero
$\gamma_{\ell t}$ = shear strain
ε_d = strain in d direction
ε_{dec} = strain in prestressing reinforcement at decompression of concrete
ε_{ds} = maximum strain at concrete strut surface (Fig. 4.3)
ε_h = strain in hoop direction ε_ℓ
$\varepsilon_{\ell y}$ = yield strain in ℓ direction
ε_o = strain at peak compressive stress f_c' in concrete
ε_p = peak strain in concrete
ε_r = strain in r direction
ε_s = strain in nonprestressed reinforcement; ε_s becomes ε_ℓ or ε_t when applied to longitudinal or transverse reinforcement, respectively
ε_t = strain in t direction
ε_{ty} = yield strain in t direction

ε_{uk} = characteristic total elongation of reinforcing steel at ultimate load
ε_x = longitudinal strain at midheight of concrete section
ζ = softening coefficient of concrete strut
η_ℓ = normalized reinforcement ratio of longitudinal reinforcement
η_{lb} = balanced normalized reinforcement ratio of longitudinal reinforcement
η_t = normalized reinforcement ratio of transverse steel reinforcement
η_{tb} = balanced normalized reinforcement ratio of transverse steel reinforcement
θ = angle between axis of strut, compression diagonal, or compression field and tension chord of the member; also, the angle between ℓ-t direction/axis and d-r direction/axis, radians
ξ = coefficient equal to 1 for rectangular sections and to $\pi/4$ for circular cross sections; ξ can be taken as unity for all shapes of cross sections with only negligible loss of accuracy for A_o and p_o
ρ_ℓ = reinforcement ratio in ℓ direction
$\rho_{\ell p}$ = prestressing reinforcement ratio in ℓ direction
ρ_t = reinforcement ratio in t direction
ρ_{tp} = prestressing reinforcement ratio in t direction
σ = compressive stress acting in combination with torsional moment, psi (MPa)
σ_0 = nominal torsional strength according to the current code specifications based on plastic limit analysis, MPa (psi)
σ_d = principal stress in d direction for concrete struts, MPa (psi)
σ_ℓ = normal stress in longitudinal direction for reinforced concrete, MPa (psi)
σ_{max} = maximum principal tensile stress, MPa (psi)
σ_N = nominal strength of structure, MPa (psi)
σ_r = principal stress in r direction for the concrete struts, MPa (psi)
σ_t = normal stress in the transverse direction for reinforced concrete, MPa (psi)
σ_∞ = strength of plain beams according to elastic analysis with maximum stress limited by material strength, MPa (psi)
τ = shearing stress due to torsion and shear, MPa (psi)
τ_{max} = maximum shear stress, MPa (psi)
$\tau_{\ell t}$ = applied shear stress in ℓ-t coordinate for reinforced concrete, MPa (psi)
ν = uniform plastic effectiveness factor (Chapter 7)
ν_c = plastic effectiveness factor for compression (Chapter 7)
ν_t = plastic effectiveness factor for tension (Chapter 7)
φ = friction angle
ϕ = strength reduction factor
ϕ_c = strength reduction factor for concrete (0.65 for cast-in-place, 0.70 for precast concrete)
ϕ_p = strength reduction factor for prestressing tendons (0.90)
ϕ_s = strength reduction factor for nonprestressed reinforcing bars (0.85)
Φ = angle of twist in torsional beam, radians/m (radians/in.)
Φ' = second derivative of rotation with respect to beam's axis z
Φ'' = third derivative of rotation with respect to beam's axis z
Ψ = bending curvature of concrete strut
ω_ℓ = reinforcement index in ℓ direction
ω_s = functional indicator of an index of reinforcement
$\omega_{s\ell}$ = reinforcement ratio index
ω_t = reinforcement index in t direction

2.2—Definitions

ACI provides a comprehensive list of definitions through an online resource, "ACI Concrete Terminology," http://terminology.concrete.org.

CHAPTER 3—HISTORICAL OVERVIEW OF TORSION THEORIES AND THEORETICAL MODELS

3.1—Navier's theory

A theory for torsion of elastic homogeneous members was first developed by C. L. Navier (1826) for circular cross sections. His theory, which was based on equilibrium conditions, compatibility conditions, and a linear stress-strain relationship like Hooke's Law, has guided the development of various theories about the behavior of reinforced concrete members subjected to torsion after cracking.

3.2—Thin-tube theory

Navier's torsion theory for members of circular sections was followed by Saint-Venant's (1856) solution for rectangular sections. Saint-Venant's torsional constants considered warping of rectangular cross sections. According to Saint-Venant's circulatory shear flow theory, the most efficient cross section to resist torsion is a thin tube. Bredt (1896) was able to derive simple equations for thin tubes. His thin-tube theory states that the shear stress multiplied by wall thickness has a constant value around the perimeter and that this shear flow is found by dividing the torsion by twice the area enclosed by the shear flow path. Bredt's theory has served as the basis for modern theories of cracked reinforced concrete members subjected to Saint-Venant torsion.

3.2.1 *Two- and three-dimensional plane truss models*—The first theoretical models for shear in cracked reinforced concrete members date back to the turn of the century when Ritter (1899) and Mörsch (1902) formulated the two-dimensional (2-D) plane truss model concept, where reinforced concrete members were modeled as an assembly of two types of linear elements—concrete struts and reinforcement ties. The axis of concrete struts in the model was assumed to be inclined at 45 degrees to longitudinal members, and shear strength was assumed to be controlled by the yielding of transverse reinforcement ties. By extending the 2-D plane truss model, Rausch (1929) developed a three-dimensional (3-D) space truss model for torsion that consisted of longitudinal and hoop reinforcement-resisting tension and concrete struts-resisting compression. He also assumed that the shear flow path would follow the centerline of the hoop reinforcement.

3.2.2 *Skew-bending and space truss theories*—In later years, research on torsion followed two theoretical tracks—skew-bending and space truss. Lessig (1959) first proposed a skew-bending theory for reinforced concrete members with two modes of failure, Mode 1 and Mode 2, as explained in 3.3.5. The skew-bending theory used only equilibrium equations and assumed that all reinforcement yielded before failure. Lessig's research was followed by the skew-bending theory of Walsh et al. (1966) and Collins et al. (1968a,b), who proposed a third failure mode, Mode 3, and used all three modes to derive nondimensional torsion-bending moment interaction equations (Walsh et al. 1967), as described in 3.3.5. Based on three modes of failure, a nondimensional interaction surface of torsion, shear, and flexure was derived by Elfgren (1972a,b) and Elfgren et al. (1974a,b). Rausch's space truss theory for torsion was generalized by Lampert and Thürlimann (1969, 1971), who showed how the angle of inclination of the compression diagonals at failure could be determined from equilibrium if both the hoops and longitudinal reinforcement were assumed to yield. Lampert and Collins (1972) showed that predictions of skew-bending and space truss theories were in close agreement.

3.2.3 *Compression field theory (CFT)*—The truss model with linear elements developed by Rausch was replaced in the 1960s by a new type of truss model with membrane elements that were subjected to in-plane normal and shearing stresses. In determining the torsional strength of members where some reinforcement does not yield, consider compatibility conditions. Such conditions were introduced by Baumann (1972) for shear and by Collins (1973) for torsion. Mitchell and Collins (1974) incorporated compatibility conditions in their CFT, which also relied on equilibrium equations and nonlinear material models for concrete and reinforcement. Unlike previous models, the CFT calculates cracked member torsional behavior up to the peak torque. A compatibility condition derived by minimizing the strain energy in the system is used to calculate the angle of inclination in the truss model struts.

3.2.4 *Softened truss model (STM)*—In 1985, Hsu and Mo (1985a,b,c) proposed the STM by softening the concrete stress-strain curve. All of the aforementioned models satisfy Navier's theory. Earlier models overestimated test strengths (Hsu 1968c), whereas the CFT, which uses spalling of concrete cover, and STM, which uses softening of concrete, have been shown to predict test results accurately (McMullen and El-Degwy 1985).

3.3—Historical development of theories for reinforced concrete members subjected to torsion

3.3.1 *General*—Section 3.1 summarizes the historical models developed to describe reinforced concrete members subjected to torsion, covering almost two centuries of research from 1826 to the early twenty-first century (2007). Classical theories include Navier (1826), Saint-Venant (1856), Bredt (1896), and Bach (1911). This review shows that the three principles of mechanics of materials (equilibrium, compatibility conditions, and materials stress-strain relationships) have been the basis for research in torsion of reinforced concrete members. (Equation notation in this section are provided in Chapter 2.)

Fig. 3.3.2—ACI Committee 438—Torsion, Mexico City, October 1976: Tom Hsu, Lennart Elfgren, Phil Ferguson, Art McMullen, Emory Kemp, Gordon Fisher, Paul Zia, and Michael Collins.

3.3.2 *Twentieth century*—In the first 60 years of the twentieth century, progress in reinforced concrete theories was made primarily on flexure. Early flexural theories for reinforced concrete assumed plane sections remained plane and stress-strain relationships of concrete and reinforcement were linear. Equilibrium conditions for longitudinal stresses were used to determine the location of the neutral axis and stresses in concrete and reinforcement caused by the moment. The contribution of concrete tensile stresses was disregarded if concrete cracking was expected. Later flexural theories accounted for the nonlinear stress-strain response of the concrete and steel reinforcement so that the complete moment-curvature relationship for a section could be predicted. In terms of shear and torsion research, a significant achievement was made with the development of truss models (Ritter 1899; Mörsch 1902; Rausch 1929). Research in torsion of reinforced concrete underwent significant advances during the last 40 years of the twentieth century. Two theories were developed—skew-bending and truss models with membrane elements. Skew-bending includes the theories of Lessig (1959), Yudin (1962), Collins et al. (1968a), Hsu (1968a), and Elfgren (1972a,b). Truss models include the theories of Nielsen (1967), Lampert and Thürlimann (1968, 1969), Collins (1973), Mitchell and Collins (1974), Elfgren et al. (1974a,b), Collins and Mitchell (1980), and Hsu and Mo (1985a,b,c). Several researchers involved in the development of theories for torsion were members of ACI Committee 438 for Torsion, which is now the Joint ACI-ASCE Committee 445 for Shear and Torsion (Fig. 3.3.2).

Development of these modern truss models was based on the same three principles of mechanics, which, in terms of torsion and shear, include the softened stress-strain relationship of concrete.

3.3.3 *Classical torsion theory for homogeneous members*—Navier (1826) derived a theory for torsion of homogeneous elastic members with circular cross sections. His theory is based on the three principles of mechanics of materials:

equilibrium, compatibility conditions, and Hooke's Law. Navier's work also includes the linear theory for flexure. His book is recognized as the first on the mechanics of materials The three principles of the mechanics of materials have become well known as Navier's theory. His theory defines the torsional rigidity for circular sections as GI_p. By extending the formulas for the polar moment of inertia of circular sections to square sections, Navier found that the calculated strength for specimens tested by Duleau (1820) overestimated measured values by approximately 20 percent. He acknowledged that "the formulae for square members do not depict as accurately the behavior as those for circular members." This inconsistency was explained three decades later by Saint-Venant (1856), who recognized that Navier's polar moment of inertia could not reflect the warping deformation of rectangular cross sections. To obtain the correct solution, Saint-Venant developed the semi-inverse method to solve all 15 differential and algebraic equations in the theory of elasticity developed by Cauchy (1828). By satisfying equilibrium, compatibility, and Hooke's Law at each differential element of a member, Saint-Venant developed a solution that considered the warping displacements of rectangular cross sections. In Saint-Venant's rigorous derivation, torsional rigidity is defined as GC. The torsional constant C is taken as $\beta x^3 y$, where the coefficient β is a function of the ratio y/x and varies between 0.141 (y/x = 1) and 0.333 ($y/x = \infty$). The maximum shear stress τ_{max} occurs on the outside face of the rectangular section at midpoint of each long side and is equal to $T/\alpha x^2 y$, where α is a coefficient that varies between 0.208 (y/x = 1) and 0.333 ($y/x = \infty$). The relationship between α and β is $\alpha = \beta/k$, where

$$k = 1 - \frac{8}{\pi^2} \sum_{n=1,3,5,...}^{\infty} \frac{1}{n^2 \cosh \frac{n\pi y}{2x}}$$

According to Saint-Venant's circulatory shear flow pattern, the maximum shear stresses occur at the outer periphery of a cross section, and the most efficient cross section to resist torsion is a thin tube, as shown in Fig. 3.3.3.

Bredt (1896) was able to derive a simple equilibrium equation for thin tubes by assuming the entire tube to be uniformly and fully stressed

$$q = \frac{T}{2A_o} \tag{3.3.3}$$

The area A_o is formed by sweeping the lever arm (symbol a in Fig. 3.3.3) around the axis of twist, a term later called the "lever arm area" (Hsu 1988). The torsional constant C for thin tubes with constant thickness was also simplified to $C = 4A_o^2 t/p_o$. Bach's formula (1911) is a simplification of Saint-Venant's theory for thin-walled open sections, such as T, L, and I sections. Because the coefficient β for each rectangular component of such sections can be approximated as 1/3, the torsional constant for the entire section can be taken as the sum (Σ) of the components ($C = 1/3\Sigma x^3 y$). The theories of

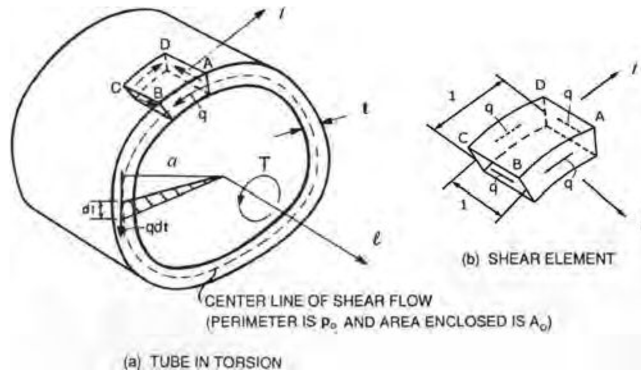

Fig. 3.3.3—Torsion of thin tube and lever arm area A_o *(Hsu 1993 after Bredt 1896).*

Navier, Saint-Venant, Bredt, and Bach are applicable to reinforced concrete beams before cracking. They also laid the foundation for developing theories to calculate the behavior of cracked reinforced concrete members subjected to torsion.

3.3.4 *Theories for reinforced concrete under flexure, shear, and torsion*—Reinforced concrete was first developed in 1867 when Joseph Monier obtained a patent for reinforcing his concrete flowerpots with wrought iron wires. The concept of using steel reinforcement to overcome the weakness of concrete in tension was quickly adapted to buildings and bridges, making the use of reinforced concrete for construction a widely accepted application in the last quarter of the nineteenth century. Such growth in applications gave rise to the demand for theories to analyze and design reinforced concrete structures.

3.3.4.1 *Flexure theory*—As reported by Delhumeau (1999), the first flexure theory to emerge was the linear flexure theory developed by Hennebique's firm near the end of the nineteenth century. This theory served as the basis for the allowable stress design method addressed in the first ACI code [National Association of Cement Users (NACU) 1910]. Theories for nonlinear flexure occupied the attention of researchers until 1963 when strength design was incorporated in ACI 318 (ACI 318-63). Both linear and nonlinear theories for flexure satisfy the three principles of the mechanics of materials: equilibrium of parallel coplanar forces, Bernoulli's linear strain compatibility, and the constitutive laws of materials. A linear stress-strain curve of reinforcement and concrete is used for the linear flexure theory and a nonlinear stress-strain curve of concrete is used for the nonlinear flexure theory.

3.3.4.2 *Torsion and shear theory*—Following the adoption of flexural strength design in ACI 318-63, the attention of researchers turned to more complex problems of torsion and shear in beams. Computer analysis of structures was becoming available, making it feasible for design engineers to compute the magnitude of torsions in their buildings. In addition, concrete box-girder bridges, which were becoming a competitive bridge type, needed to be designed for torsion. Shear is essentially a two-dimensional problem, requiring an understanding of the interaction of two principal stresses and strains in a membrane element. Torsion is complicated because it is a three-dimensional problem involving

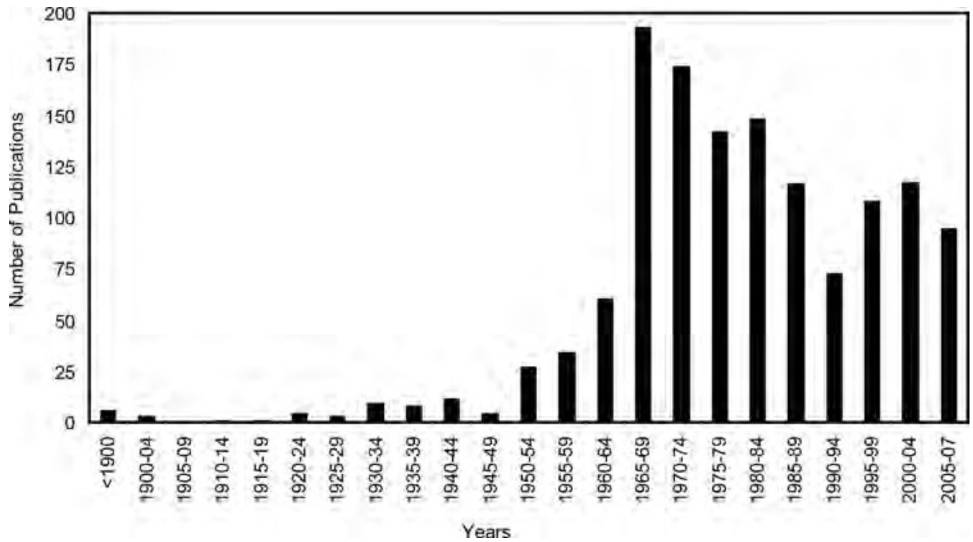

Fig. 3.3.4.2—Number of published papers on torsion from 1900 to 2007 (plot prepared by A. Belarbi).

the shear problem of membrane elements in a tube and the warping of tube walls that cause flexure in concrete struts. Figure 3.3.4.2 shows that the number of globally published papers on torsion began to surge around 1960 and peaked around 1970. The principles of equilibrium, compatibility conditions, and materials stress-strain relationships that were needed to solve torsion problems in reinforced concrete members were primarily developed between 1960 and 1985 (Lampert and Thürlimann 1971; Lampert and Collins 1972; Elfgren et al. 1974a,b; Collins and Mitchell 1980; Hsu and Mo 1983). Theories and tests produced before 1980 are summarized in detail by Hsu (1984).

By 1985, researchers solved the basic problems of reinforced concrete design by applying Navier's theory. Further research was necessary to refine the constitutive laws of materials for torsion and shear. The experimental work needed to generate new advancements is tedious and requires highly sophisticated testing equipment. Only two universities in North America—the University of Toronto and University of Houston—are capable of studying the behavior of reinforced concrete shell and panel elements subjected to in-plane shear and normal stresses. The study of softened concrete in shear elements, which has been the subject of extensive research worldwide in the last three decades, continues to be a major research topic. The need for larger and more complex specimens has increased ongoing work at both universities. The high cost of experimental research needed for new developments in torsion imposes a limiting constraint on new research. For example, studying the behavior of full-scale girders with open sections that involve Saint-Venant and warping torsion is expensive. The future of torsion research is largely tied to available equipment or to the combined efforts of many institutions, or both.

3.3.5 *Space truss model using struts and ties*—The first theory for shear design of reinforced concrete was developed at the turn of the twentieth century when Ritter (1899) and Mörsch (1902) formulated the concept of plane trusses with struts and ties. They modeled a reinforced concrete member as a truss with two types of linear elements: struts made out of concrete and ties made out of steel reinforcement. The Ritter and Mörsch model represents the struts and ties as lines without cross-sectional dimensions, where forces satisfy equilibrium at points of intersection—a model with the advantage of conceptual clarity. Extending the 2-D plane truss model to a 3-D space truss model, Rausch (1929) developed a theory for torsion of reinforced concrete. Rausch's space truss model, as shown in Fig. 3.3.5a, is made up of 45-degree diagonal concrete struts, longitudinal reinforcing bars, and hoop reinforcing bars connected at the joints by hinges. Torsional moment is carried by the concrete struts in axial compression (dotted lines), and by the straight reinforcing bars in axial tension (solid lines) in the longitudinal (horizontal) and lateral (hoop) directions. Equilibrium of the joints in the longitudinal, lateral, and radial directions requires that the forces in the longitudinal bars (X), in the hoop bars (Q), and in the inclined struts (D) should be evenly distributed among all cells and joints. To satisfy equilibrium, the relationship between these forces should be $X = Q = D/\sqrt{2}$. As shown in Fig. 3.3.5a, the series of hoop forces Q at the joints constitute a shear flow $q = Q/s$. Using Bredt's lever arm area concept, T can be related to q (or Q/s) by $2A_o$, as expressed by Bredt's equation (Eq. (3.3.3)). The term A_o refers to the area enclosed by a series of straight lines connecting joints of the cross section.

Assuming that ultimate torque is reached when the forces in the transverse reinforcement reach the yield stress, then $q = Q/s = A_t f_{ty}/s_t$ and Eq. (3.3.3) becomes

$$T_n = 2A_{oh}\frac{A_t f_{ty}}{s_t} \qquad (3.3.5)$$

Although the space truss model has the advantage of conceptual clarity in terms of simple assemblage of compression struts and tension ties, Rausch's equation (Eq.